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A NEW THEORY OF JUPITER AND SATURN.

BY

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ASSISTANT AMERICAN EPHEMERIS.

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UNDER THE DIRECTION OF

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PROFESSOR UNITED STATES NAVY

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PREFATORY NOTE.

The objects of the series of papers of which the publication is commenced in the present volume, are, a systematic determination of the constants of astronomy from the best existing data, a re-investigation of the theories of the celestial motions, and the preparation of tables, formulæ, and precepts for the construction of ephemerides, and for other applications of the results. The adopted policy, which is more fully set forth in the Introduction, contemplates the subdivision of the work and the publication of each part as soon as completed, in such a way as to render easy the subsequent combination of the whole.

It is not intended to include any papers in the series but such as conduce to the objects in view.

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NOTE.

Part IV of this series of papers is identical with the catalogue of 1098, Standard Stars, of which the copies were issued in a bound form. This fact is mentioned to correct misapprehension among recipients of the publications respecting the completeness of their sets.

Volume I is completed with Part VI.

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INTRODUCTION.

It is well known to all astronomers who have given attention to the subject that meridian observations of the moon and planets are not completely represented by any of the existing tables, and that the deviation of prediction from observation is constantly increasing. It is true that, so far as the current requirements of astronomy are concerned, the state of the case may be considered as not unsatisfactory. Not only may the planets be found and eclipses predicted for many years to come by the present tables, but, with the exception of the moon, there is every reason to suppose that the tabular positions will serve the purposes for which they are immediately required in navigation and practical astronomy. But when we take a wider view and consider the general wants of science both now and in the future, we find that in the increasing discordance between theory and observation there is a field which greatly needs to be investigated.

If mutual gravitation according to the law of NEWTON is the only cause which changes the motions of the planets, then it is mathematically possible to construct tables which shall represent observations with the last degree of precision and through any period of time. It is quite possible that the discordances alluded to proceed solely from the imperfections in the mathematical theory, and do not indicate any unknown cause affecting the celestial motions. But when we investigate more closely, and seek to ascertain the cause of such discordances, we find a state of things which renders it impossible to draw any definite conclusions respecting the ultimate possibility of representing observations by existing physical and mathematical theories. This state of things has its origin in the comparative brevity of the period during which accurate observations have been made, and in the difficulty of conducting, on a systematic plan, mathematical investigations having in view the perfection of astronomy.

One point in which the requirements of astronomy differ from those of physics is that the element of time enters into the former much more than into the latter. The experimental investigation of forces which act on the surface of the earth requires only the time necessary to make and perfect the experiments. There is no one research of which we can say that it will necessarily require a definite number of years or centuries for its completion. But since astronomical generalizations rest, not upon experiments, but upon observations, it is always necessary to wait for the recurrence of the phenomena on which the conclusions are to depend. The main object of investigation being the forces which change the motions of the planets we must observe these motions during a sufficient period to make evident the action of the forces. The longer the time which elapses the more material we have for reaching conclusive results. It is generally considered that accurate observations commenced with BRADLEY in the

middle of the last century. The period during which they have continued is therefore about a century and a third. But there are many exceptions in the case of different classes of observations. The places of the moon have been traced backward with a nearly modern precision through the century preceding BRADLEY's observations, while the observations of the Babylonians and the Arabs are still of the greatest value in the lunar theory. On the other hand none of BRADLEY's instruments fulfill the requirements of the present time, and his observations were in many cases extremely defective as compared with our own. If, therefore, we attempt to learn what conclusions can be reached in the present state of astronomy we must consider each object of observation separately with reference to its general place in a comprehensive scheme.

But time is not the only element which comes in. If we are to determine what unknown causes affect the motions of the planets the first step is to prove that there is really a discordance between the results of observations and the results of the theory of gravitation. The first step towards establishing such a discordance is the construction of tables and formulæ of which we can say that they are beyond reasonable doubt the results and the only results of the gravitation of the known bodies of the solar system. The necessary conditions which such tables and formulæ must satisfy are that they shall be founded upon uniform elements and data, and that the results of employing the adopted elements shall be carried out with all necessary precision. Now, not only has this requirement never been fulfilled, but the effect of recent advances in exact astronomy has rather been to carry us away from its fulfillment.

It is scarcely possible for a year to pass without some new investigation or series of observations which shall materially add to the precision with which we can determine some astronomical constant. Each astronomer who finds material to be used in this way is naturally desirous of utilizing it to its fullest extent, and is therefore under a temptation to introduce each new improvement into his investigations without respect to their consistency with the investigations of others which have been made with the older data. Sometimes, too, the object of constructing an astronomical formula is to correct it from time to time, and the very object of the constructor may tend to destroy its consistency. A brief glance at some features of the existing planetary tables will illustrate the point in question.

LAPLACE, in the third volume of his *Mécanique Céleste*, constructs, by the most rigorous and complete methods then known to science, a complete theory of the planetary perturbations, founded on elements and masses which are quoted in Chapter VI of his work. From his results tables were constructed by LINDENAU and BOUVARD during the early years of the present century.

In order to give the tables the required precision it was necessary to correct the elements by a comparison with observation. Thus, the new tables no longer corresponded to the original formulæ of LAPLACE. Moreover, the theory was in many respects so imperfect that no certain conclusion could be drawn from a comparison with observation. This was notably the case with the perturbations of the second order. It was therefore necessary to make a complete reconstruction of the theory. Nevertheless, such was the labor and difficulty of constructing new tables that those of

Lindenau and Bouvard remained the standards for use in the preparation of ephemerides during nearly half a century.

The next complete reconstruction of the theories and tables of the planetary motions was that of Leverrier. His work on this subject forms the most important part of the fourteen volumes which he published under the title *Annales de l'Observatoire de Paris.* The first of these volumes appeared in 1855, the last in 1877.

Some consideration of the circumstances under which this great work was carried out and of the objects at which it aimed may not be out of place as showing how it happens that more remains to be done in the same direction. When Leverrier commenced his work, the most striking feature which presented itself was the imperfections of the tables of Lindenau and Bouvard. The formulæ on which they were constructed, though fully up to the science of the time in which they were formed, was far behind modern requirements in generality and rigor. Better tables and formulæ constituted one of the most pressing wants of exact astronomy. Both his position and his previous works marked Leverrier as the one to undertake the work of constructing such tables and formulæ. Naturally desirous of beginning to reap the results of his labor as soon as possible, he investigated the elements of the planets and published the corresponding tables one or two at a time. This course did not detract from from his main object, that of constructing improved planetary tables. But there was another object, the desirableness of which was not immediately felt, but which must be more and more felt in the not distant future, namely, the attainment of uniformity in adopted astronomical data. So far was Leverrier from aiming at this object, in its entirety, that his tables do not, in all cases, embody his final results. The consequence is, that notwithstanding that his work makes a greater epoch in astronomy than any of his immediate successors can hope to make, it does not wholly supply the wants of science in the immediate future. In many of his tables large and increasing deviations from observation already exhibit themselves. This is most notably the case with the planet Saturn, the theory of which he did not succeed in bringing to a satisfactory conclusion. The geocentric places of Mars and Venus are also largely in error at the time of nearest approach to the earth. The earlier tables, those of the Sun and Mercury, are the only ones which can be regarded as entirely satisfactory in their agreement with observations, with the possible exception of Uranus and Neptune.

What has been said of Leverrier's tables applies with yet greater force to the tables of Uranus and Neptune by the present writer. Their main object was to supply an immediate astronomical want. The data on which they were found could not be regarded in any respect as definitive, nor were the adopted masses absolutely uniform. The formulæ of perturbations on which they depend are also such that we cannot say with certainty whether the deviations from observations which they exhibit arise from any other cause than the imperfections of the theories on which they are founded.

Now, the material available for the accurate determinations of the fundamental elements of astronomy has increased many fold since the conclusion of Leverrier's work on the four inner planets. The recurrence of transits of Venus and Mercury, the perfection of astronomical instruments, the employment of improved places of the fixed stars, the introduction of more systematic methods of research, and the rein-

vestigation of older observations have all combined to bring precise astronomy to a higher plane than it ever before occupied. Supposing that their mutual gravitation is really the only cause which disturbs the elliptic motion of the planets around the sun, it is now theoretically possible to construct tables of all the large planets, except Neptune, from exact data, which shall represent observations within their probable errors until the middle of the next century. The desirableness of having such tables founded on one consistent and fully elaborated theory, hardly needs to be insisted on. Only in this way can it be decided whether deviations from theory arise from its imperfections, or from the action of unknown and, perhaps, unsuspected causes.

A more detailed survey of the field will bring to light other reasons for placing the results of past observations and researches in such a form that they may be utilized in the future.

We first remark that the existing data in the form of observations lie in great part unused, and are in danger of never being used, unless discussed and condensed in such a way as to render them manageable. Long series of observations made during the present century by eminent astronomers, and with the best appliances, lie idle in the volumes which embody them, never having appeared in any of the existing tables. In order to be utilized to the best extent they need to be rediscussed by modern methods and with modern places of the fixed stars. The labor of doing this is such that we only find it performed in sporadic cases by individual astronomers. One of two courses must now be adopted. We must either suffer this great mass of material, collected in many cases by the life labors of eminent observers, and published at great expense, to go to utter waste, or we must speedily put it in a shape to be utilized for present and future purposes. It is true that if nothing were to be added to the mass we might safely leave it in confidence that future astronomers would give it more attention than we have. But so rapidly does it increase that it is even now entirely beyond the power of individual management, and the longer it is left the less hope there is that it ever will be managed. The required work must be that of an organization rather than that of an individual. All that the head of an organization can do is to plan the work, investigate the formulæ and data by which it is to be done, devise the checks which are to guard against error, discuss the results, arrange them for the press, and see that every operation is conducted on correct principles and by the best methods.

Not only should the work be founded on all the observations which it is practicable to employ as its basis, but a necessary feature is a utilization, so far as possible, of all discussions by other astronomers. Although the work may become less individual in character, it has greater claims to consideration on the score of embodying the labors of the leading astronomers of the time.

On assuming the superintendency of the American Ephemeris in 1877, the writer determined to employ the resources at his disposal to carry out, or at least to enter upon, a long cherished plan of executing the work in question. No published announcement of his programme was, however, made, owing to the ease of making such a programme alongside the difficulty of executing it. There are, however, two reasons for no longer maintaining this reserve. One is that although what has been done is

only a commencement, the prospects of being able to carry it through are fairly good. Both Congress and the Navy Department have supplied all the assistance which has been asked for, and a force of from eight to twelve computers, some of the highest order of mathematical ability, has been actually employed during the past year, and may, if necessary, be increased in the future. Another and more cogent reason for announcing the programme is that much duplication of work may thus be avoided. Astronomers in other parts of the world are from time to time undertaking investigations already in hand and sometimes announce their intention in private correspondence where nothing has appeared in print.

This remark is not made to discourage such attempts, because, owing to the magnitude of the work, it is desirable to utilize all investigations, wherever made, which will in any way contribute to its completion. It is, however, essential that such investigations should be made in such a way as to adapt themselves to the general plan, and that they should be completed so far as practicable. With a view of enabling those interested to form the best judgment of the situation a statement of the unpublished work now in hand, with a general programme for its continuance, is here presented.

The theories of the four inner planets naturally claim the first attention as embodying most of the fundamental elements of astronomy. This branch of the work includes not only the masses of the planets and the elements of the respective orbits, but the constants connected with the rotation of the earth on its axis, namely, the annual precession, the obliquity of the ecliptic and its secular variation, the position of the equinox among the stars, and, indirectly, the positions of the fundamental stars. To these may be added the solar parallax and the mass of the moon, as well as a number of quantities connected with those already mentioned. In the determination of these constants the plan, as already mentioned, contemplates the utilization and combination of all valuable data.

Besides what is found on the general subject in the present volume the following works are finished or in progress:

LEVERRIER'S tables of the Sun, Mercury, Venus, and Mars have been partially reconstructed with a view of making them more convenient in use. His theory, however, remains unaltered in the manuscript tables. A comparison of the Greenwich, Paris, and Washington meridian observations of Mercury with those tables has been commenced and is approaching completion. Similar comparisons for the Sun, Venus, and Mars have not been seriously commenced, but it is expected to commence them in the course of the year 1883.

A discussion of the corrections required by the older Greenwich observations up to 1830, as published by Professor AIRY, in order to reduce the results to a uniform system, is nearly completed, and is expected to appear as Volume II, Part I, of these Papers.

General tables and formulæ for forming the differential coefficients for correcting the elements of the inner planets have been prepared, and it is intended to publish them in the next volume.

Although the final completion of the theories of the other planets must follow the work on the interior planets, it is advisable to begin it without delay, owing to the great labor which it involves. The general perturbations of Jupiter and Saturn were

therefore taken up by Mr. GEORGE W. HILL in 1877, but they are still unfinished. It is now expected that Mr. HILL's work will be completed about the end of 1883. The computation has been made principally by the methods of HANSEN.

Much attention has also been paid to the subject of the moon's motion. The first object has been the continuance of the discussion of eclipses and occultations previous to 1750 up to the present time. The reason for laying so much stress upon occultations is that notwithstanding the irregularity with which they are observed, their considerable accidental errors, and the labor of reducing them, they constitute the only observations of the moon which are free from systematic error, and which can therefore be used with safety to compare the mean longitudes of the moon at wide intervals of time.

Tabular positions of the moon, as well as those of the fixed stars, are now complete for all the more important occultations since 1750, and the reductions for parallax are in progress. Should the work not be intentionally delayed in order to bring it up to the date at which new tables of the moon shall be actually constructed, it may be expected that this particular discussion will be terminated by the end of 1884.

Although the theory of Jupiter's satellites does not form an essential part of the proposed investigations, the motions of the first satellite are intimately connected with the general subject, owing to the light which they may throw upon the question of the uniformity of the earth's rotation. All the observed and recorded eclipses of the satellites have, therefore, been computed from DAMOISEAU's tables up to the early part of the present century. The work is discontinued for the present, owing to the difficulty of introducing and discussing the various corrections which will be required to the observations on account of different apertures of telescopes employed, the different distances of the planet from Jupiter, etc. It is a matter of regret to me, as it must be to all astronomers interested in this matter, that Mr. GLASENAPP has not continued the very thorough discussion of observations of these satellites which he published some six years ago.

An essential and very laborious and difficult part of the work is that of preparing formulæ and tables for computing the general perturbations of all the planets. A problem which has taxed the powers of the greatest mathematicians of modern times, and the solution of which is still, after all their work, in an unsatisfactory state, is one which the writer feels most hesitation in approaching. He has, however, devised a method which he hopes may prove convenient in practice for the general development of the disturbing function and its derivatives. Whether any improvements can be devised in the method of integrating must be left to the future.

In the future work it is intended to combine the data in a way different from that generally adopted. When all four of the inner planets are considered together it is possible greatly to strengthen the results on special points. An example of this is afforded by the relation of observations on Mercury and Venus to the obliquity of the ecliptic and the position of the equinox. Hitherto these quantities have been made to depend solely upon observations of the sun. Were the sun a point of light which could be observed in the same way as a fixed star, the results from this method would be so far beyond doubt that we should have no occasion to look further. But there are

several causes which diminish the value of solar observations. In the first place, the sun being a round body and not a point of light, it is well known that large personal differences exist in the observations of its position by different observers. Again, it has always to be observed at mid-day, when the atmosphere is most disturbed by its rays, and when the roof of the observing-room is heated from the same cause. Moreover, there is always more or less danger of systematic deformation of the instrument produced by the concentration of the solar rays in the focus. It is therefore impossible to view observations of the sun without a strong suspicion of systematic errors existing among them.

Now, geometrically considered, observations of Mercury may be utilized for determining the position of the earth's orbit relative to the equator almost as well as observations of the sun itself. If we supposed the elements of the orbit of Mercury perfectly known it would be easy to reduce each observation of Mercury to the center of motion. But since the elements of the planet are to be considered unknown, the question arises whether these elements and those of the earth's motion can be independently determined. That they can, to a certain extent, will be evident by the following considerations.

Let us imagine the observer to be in any fixed position on the orbit of the earth, and to observe Mercury from time to time through several revolutions around the sun. It is evident that from these observations the orbit of the planet, and the position of the observer relative to it, could both be determined. By supposing him to move around the earth's orbit to different positions, and to repeat the determinations, we see that any number of separate determinations of the elements of the planet could be made. The several determinations would then be combined and reconciled by attributing suitable elements to his own motion around the earth's orbit.

This is substantially the actual case except that the observations from any one point of the earth's orbit do not embrace the whole orbit of Mercury, but only those portions of it not very near the points of conjunction with the sun. Although this circumstance detracts from the completeness of the determinations it does not detract from the accuracy with which the main problem, that of the obliquity of the ecliptic and position of the equinox, can be solved. It is therefore possible, from meridian observations of Mercury alone, to obtain the principal elements of the earth's orbit around the sun, including the absolute longitude of the sun itself, and hence a separate determination for the position of the equinox. It is true that some of the elements, especially the eccentricity and longitude of the perihelion, may prove to have small weight, but this is because what is most accurately given by the observations will be a linear function of the corrections to these elements. But even such a result will furnish valuable data for the final values of the necessary quantities.

Nearly the same remarks apply to the meridian observations of Venus, and, to a limited extent, to those of Mars. Indeed it is evident that what is given by planetary observations generally is not the absolute position of the planet but the direction of the line joining the earth and planet, which direction is equally available for the determination of the elements of either of the two bodies. Whether it is advisable to employ it in determining both sets of elements must depend upon circumstances. If it

were possible to determine the solar elements by observations of the sun with an accuracy far exceeding the joint determination by observations of the planet, the latter might be entirely omitted. But, for reasons already pointed out, this is far from being the case. It seems better, therefore, under the circumstances, to ascertain what functions of the corrections to the two sets of elements can best be determined from observations of the planets, and to supply whatever is weak in the combination by observations of the sun itself.

In theory, observations of the moon might also be utilized for an absolutely independent determination of the equinox and of the obliquity of the ecliptic. In fact the mean orbit of the moon during a period of one revolution of the node is the ecliptic itself, and therefore exact observations of its position through one period will give the position of the ecliptic. But the rapid motion of the moon in declination when near either equinox introduces a large probable systematic error into the measures made upon it at any definite moment. No weight can therefore properly be assigned to a position of the equinox by meridian observations of the moon. The obliquity derived from such observations may, however, be worthy of more consideration.

The position of the sun among the stars may, however, be determined through the aid of the moon with a considerable approach to precision. The direct comparison of the sun and stars through the sidereal clock is uncertain, from the causes already pointed out, namely, the effect of the sun's rays in disturbing the air and instruments, and personality in observing a limb. Now, by observations of eclipses, especially at the beginning and ending of totality, the exact moment when the sun and moon are in conjunction is determined with great precision. By observations of occultations the mean position of the moon among the stars is determined with yet greater precision. Hence, by a combination of the two we have a result for the position of the sun among the stars which may possibly be entitled to considerable weight. It is, however, a drawback to the method that few observations of eclipses having any claim to precision were made between 1720 and 1800, while those made before 1720 are of course subject to more or less suspicion of systematic error.

It is worthy of note that this method of determining the position of the sun among the stars is, in principle, that adopted by HIPPARCHUS and PTOLEMY.

The above are the leading features in which the plan of the proposed work differs from that hitherto followed. The objects are also somewhat different, in that they include a basis for future conclusions as well as the determination of astronomical constants and the construction of new tables. It is hoped, should the work be completed on the proposed plan, that for a miscellaneous and frequently inconsistent combination of astronomical constants there will be substituted a consistent set, and that the result of this substitution will be to make it easy to determine, from any future deviation between theory and observation which may show itself, in what direction we are to look for the cause.

SIMON NEWCOMB.

WASHINGTON, 1882, *September* 16.

ON THE

RECURRENCE OF SOLAR ECLIPSES

WITH

TABLES OF ECLIPSES

FROM B.C. 700 TO A.D. 2300

BY

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WASHINGTON
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PREFACE.

The following paper presents a new theory of the recurrence of solar eclipses, founded on some hitherto unnoticed properties of the 18-year eclipse cycle. This theory has been utilized in the formation of tables whereby the solar eclipses of any class which have occurred during the past twenty-five centuries, or are to occur during the next five centuries, may be determined and approximately computed with great rapidity. The tables are founded on the mean motions and other elements of the sun and moon given in Hansen's Tables, the mean motion of the moon and of its nodes being corrected to accord with the results deduced in the author's *Researches on the Motion of the Moon.*

In the concluding section, the eclipses most remarkable for the duration of total phase are pointed out, and the conditions for their occurrence briefly discussed.

A considerable part of the work of constructing the tables has been performed by Mr. John Meier, assistant in this office.

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THE RECURRENCE OF SOLAR ECLIPSES.

§ 1.

GENERAL THEORY.

It has been known from ancient times that eclipses both of the sun and moon generally repeat themselves in a cycle of 18 years and 11 or 12 days, known as the Saros. This cycle is due to the circumstance that 242 revolutions of the moon relatively to either of its nodes require nearly the same period with 19 revolutions of the sun relatively to the same node. The time required for either of these returns is $6585\frac{1}{3}$ days. Hence, if we note the relative positions of the sun and moon at any moment, and then count forward through this period, we shall, at the end of it, find them in nearly the same position, both relative to each other and relative to the node. If we start from the centre of an eclipse, when the two bodies are nearly in the same straight line, we shall, at the end of the period, find another eclipse very similar in its character. This relation affords a very simple and easily applied method of finding the series of eclipses which occur during any period of 18 years, from those which occurred during the cycle previous.

There are, however, two remarkable chance relations connected with the Saros, which, so far as I know, have never been remarked, and without which the period would not have served the purpose of foreseeing eclipses so well as it actually does. The cycle takes account only of the mean motions of the sun and moon. But in consequence of the eccentricity of the orbits, the sun may be 2 degrees on either side of its mean place and the moon 5 degrees. The relative position of the two bodies may therefore vary 7 degrees from their mean position at any time; this extreme variation would change the time of an eclipse by half a day and the distance from the node at which it occurred about 2 degrees. If the corresponding eclipses in two successive cycles were subject to these independent variations, their circumstances might differ so widely that the recurring eclipse would differ considerably from its predecessor, and might be nearly a day later or earlier than the mean length of the cycle in its recurrence. A partial eclipse might fail entirely to recur, and a total one might become partial at the first recurrence and then total again at the second one. But, as a matter of fact, the irregularities of this class are reduced almost to nothing by two other remarkable relations. At the end of a Saros, not only are the sun, the moon, and the node found nearly in their original relation, but the mean anomaly of the moon has also the same value to less than 3 degrees, and the mean anomaly of the sun to some 12 degrees. There is no *a priori* reason that this should be the case: it arises only from

the fact that 18 years is a close multiple, not only of the times of revolution of the sun and moon, but also of the times of revolution of the moon's node and perigee. The following is a more exact statement of the changes at the end of the Saros. Taking as a period the time required for 223 lunations, the changes in the elements at the end of the period will be as follows:—

In the argument of latitude, - - - - - - - - - -	$-28'.6$
In the moon's mean anomaly, - - - - - - - - - -	$-2^\circ.831$
In the sun's mean anomaly, - - - - - - - - - -	$+10^\circ.494$
In the distance of the lunar perigee from the node, -	$+2^\circ.353$
In the distance of the solar perigee from the node, -	$-10^\circ.971$

In consequence of the minuteness of these changes, not only the mean place of the moon, but all its larger inequalities, will return nearly to their original values at the end of the period. This will hold true, not only with respect to the time of the eclipse, but also with respect to its character, since the parallax and semi diameter of the moon must also return nearly to their original values. If the eclipse is of a remarkable character with respect to duration, the corresponding ones of succeeding cycles will be of the same character.

An interesting illustration of this fact is found in a series of total eclipses now in progress, namely, those of 1850, 1868, 1886, etc., in which the duration of totality is greater than in any others which have occurred for several centuries. This series will be investigated in the course of the present paper.

Owing to the mean retrocession of 28′ from the node in each cycle, the corresponding eclipses in successive cycles are subject to a progressive change. A series of such eclipses commences with a very small eclipse near one pole of the earth. Gradually increasing for about eleven recurrences, it will become central near the same pole. Forty or more central eclipses will then recur, the central line moving slowly toward the other pole. The series will then become partial, and finally cease entirely. The entire duration of the series will be more than a thousand years. A new series commences, on the average, at intervals of thirty years.

It follows from this that all eclipses may be divided into sets, the separate eclipses of each set being separated by intervals of one 18-year cycle, and extending through sixty or seventy cycles. Moreover, from the elements of the central eclipse of each set, those of any other of the same set may be readily found by applying the changes corresponding to the number of intervals which separate it from the central one. It is now proposed to utilize this circumstance by the formation of a series of tables, by which the approximate elements of any solar eclipse between the years B. C. 700 and A. D. 2300 may be found with a few minutes' calculation, and by which any such eclipse occurring during this period may be promptly identified. The principles on which the most important of these tables are constructed may be readily comprehended by a conception of movable conjunction points reached in the following manner.

Let us suppose the mean motions, n and n', of two bodies, planets for instance, revolving round a common centre, to be so related that

$$i'n - in' = 0,$$

i and i' being integers. Then, i' revolutions of the first will require the same period as i revolutions of the second, so that at the end of this period, which we may call P, they will have returned to their original positions. During the period P they will have been in conjunction $i - i'$ times at the same number of equidistant points of either orbit. Every subsequent mean conjunction will occur at these same points. We shall call them *conjunction points*, and shall represent their number, $i - i'$, by ν.

If we suppose these points to be numbered, in the order of longitude, 0, 1, 2 .... $\nu - 1$, and suppose the two bodies to start out from the point 0, the number of revolutions which each body must severally make to reach the point p will be found by solving the indeterminate equation

$$i' x - i y = \pm p.$$

x will then be the entire number of revolutions of the one planet and y that of the other before the required conjunction will occur; that is, the one planet will then have passed over $\nu x + p$ intervals between the conjunction points, and the other over $\nu y + p$. The condition that these two quantities shall be in the ratio $i : i'$ gives the above indeterminate equation. In order to avoid the ambiguous sign, we may suppose $n > n'$, which will make $i > i'$. This will make the equation

$$i' x - i y = p.$$

In what precedes, we have supposed the mean motions of the two bodies to be exactly in the ratio of the entire numbers i and i'. This is never the case in nature, if we reckon the mean longitudes from a fixed point of departure; but we may always assign such a uniform progressive motion to this point that the condition shall be fulfilled. Let us put k for the progressive motion required. The mean motions relative to the moving departure point will then be $n - k$ and $n' - k$ respectively. The condition that these shall be in the ratio $i : i'$, or

$$\frac{n - k}{n' - k} = \frac{i}{i'},$$

gives

$$k = \frac{i n' - i' n}{i - i'} = \frac{i n' - i' n}{\nu}.$$

The conjunction points, being fixed relatively to the departure point, will have this same mean motion k; that is:—

By assigning to the ν conjunction points the uniform mean motion k, the conjunctions of the two bodies will always take place at these points.

This conception of movable conjunction points is of great assistance in representing and investigating the relations of the two bodies through many revolutions. For instance, in the case of Jupiter and Saturn, taking $i = 5$ and $i' = 2$, there will be three conjunction points having a direct mean motion of 489″ per annum relative to a fixed equinox. Their successive passages through a fixed point occur at intervals of

883 years, and we may consider the great inequality between the two planets as depending on the position of the conjunction points relative to their perihelia.

Theoretically, the values of i and i' may be regarded as entirely arbitrary. But to obtain the advantage of the conception, we take them as nearly as practicable in the ratio of the mean motions. Even with this limitation we have a choice of systems, an increase in the assumed values of i and i' having the disadvantage of increasing the number of points to be considered, and the advantage of diminishing their mean motion. The most advantageous systems will of course be found by developing the ratio of the mean motions as a continued fraction, and taking the successive converging fractions which approach to the ratio. Between two such successive systems the following relation subsists:—

The interval between the successive transits of the conjunction points of one system over any one of the next higher, and therefore more slowly moving system, is equal to the time required for the conjunctions to occur at all the points of this latter system.

Commencing with the higher system, and supposing the mean motions n and n' to be counted from a point of this system, and to be in the ratio $j : j'$, we shall have

$$j' n - j n' = 0.$$

The mean motion of the points of the next lower system relatively to the higher one will then be,

$$k = \frac{i n' - i' n}{i - i'};$$

the time required for a complete revolution of the lower system will be,

$$\frac{2\pi}{k} = \frac{360^\circ (i - i')}{i n' - i' n};$$

and the intervals between successive passages of its $i - i'$ points over a fixed point of the other system will be,

$$\frac{2\pi}{\nu k} = \frac{360^\circ}{i n' - i' n}.$$

Since n and n' are in the ratio $j : j'$, we may put

$$n = \alpha j,$$
$$n' = \alpha j',$$

which will make

$$i n' - i' n = (i j' - i' j) \alpha.$$

But, by the properties of continued fractions, the value of the coefficient of α in this expression is ± 1. Hence, the sign being indifferent, as expressing only the direction of the motion, the interval between successive passages of the conjunction points becomes

$$\frac{360^\circ}{\alpha}.$$

In order that the conjunctions may occur at all points of the higher system, it is necessary that the one planet should make j and the other j' revolutions. The time required for this will be,

$$\frac{j}{n}\,360^\circ = \frac{j'}{n'}\,360^\circ = \frac{360^\circ}{\alpha},$$

the same as the interval just found.

Let us now apply these methods to the problem now under consideration, that of the recurrence of solar eclipses. Let us put

g, the mean anomaly of the moon;
g', that of the sun;
ω, the distance of the lunar perigee from the node;
ω', that of the solar perigee from the moon's node;
T, the number of Julian centuries after 1800.

Applying to the elements given by Hansen (*Tables de la Lune*, p. 15) the corrections to the mean longitude and the longitude of the node given in my *Researches on the Motion of the Moon*, p. 268 and p. 274, the numerical expressions for g, ω, g', and ω' will become:—

$$g = 110^\circ\ 19'\ 32''.50 + (1325^r + 715807''.98)\,T + 45''.58\,T^2 + 0''.050\,T^3.$$
$$\omega = 192^\circ\ \ 7'\ 21''.91 + (\ \ 16^r + 875512''.07)\,T - 44''.32\,T^2 - 0''.044\,T^3$$
$$g' = \ \ 0^\circ\ 24'\ 28''.22 + (\ 100^r - \ \ 3392''.18)\,T - \ 0''.56\,T^2$$
$$\omega' = 246^\circ\ 13'\ 50''.28 + (\ \ \ 5^r + 489088''.09)\,T - \ 6''.52\,T^2 - 0''.007\,T^3.$$

Epoch, + 1800.0, Jan. 0, Greenwich mean noon.

In dealing with a subject of this kind, the entire revolution is a more convenient unit than the angular denominations usually adopted. We therefore transform these angles into revolutions and fractions, with the following results:—

$$\begin{aligned} g = {}^r.30646026 &+ 1325^r.55232097\ T \\ &+ \ \ 0^r.00003517\ T^2 \\ &+ \ \ 0^r.000000039\ T^3 \\[6pt] \omega = {}^r.53367431 &+ \ \ 16^r.67554944\ T \\ &- \ \ 0^r.00003420\ T^2 \\ &- \ \ 0^r.000000034\ T^3 \\[6pt] g' = {}^r.00113289 &+ \ \ 99^r.99738258\ T \\ &- \ \ 0^r.00000043\ T^2 \\[6pt] \omega' = {}^r.68397398 &+ \ \ 5^r.37738278\ T \\ &- \ \ 0^r.00000503\ T^2 \\ &- \ \ 0^r.000000005\ T^3. \end{aligned}$$

In the construction of the present tables we shall use the Julian calendar, it being more convenient to change the dates from this calendar to the Gregorian than to take account of the complexities of the latter. We shall therefore take, as our fundamental epoch,

1800, Jan. 1, Greenwich mean noon of the Julian calendar,
= 1800, Jan. 12, Greenwich mean noon of the Gregorian calendar.

Transferring to this epoch, the constants of the four principal elements will become,

$$g_0 = 0^r.74196000,$$
$$\omega_0 = 0^r.53915294,$$
$$g'_0 = 0^r.03398624,$$
$$\omega'_0 = 0^r.68574068,$$

while the coefficients of the powers of T will remain unaltered.

We shall count the time from this epoch in Julian centuries or in equal Julian years of 365.25 days each. This reckoning of time will hereafter be called a fictitious one to distinguish it from the civil reckoning. The expression for the mean distance of the two bodies from the ascending node of the moon's orbit, which we shall represent by u and u', putting

$$u = g + \omega,$$
$$u' = g' + \omega',$$

will now be

$$u = 0^r.281112{,}94 + 1342^r.227870{,}41\,T + 0{,}96\,T^2 + 0{,}005\,T^3,$$
$$u' = 0^r.719726{,}92 + \ \ 105^r.374765{,}36\,T - 5{,}46\,T^2 - 0{,}005\,T^3.$$

The comma in these expressions is used to cut off six places of decimals.

If we differentiate these expressions with respect to T, and then put T = 0 and T = − 25, we have the following expressions for the mean motions from the node at the epochs − 700.0 and + 1800.0:—

	Epoch, − 700.0,	+ 1800.0,
Mean motion of u, $= \mu$,	$1342^r.227832$	$1342^r.227870{,}41$
Mean motion of u', $= \mu'$,	$105^r.375028$	$105^r.374765{,}36$.

Developing the ratios of these two quantities into a continued fraction, we have,

For − 700.0,

$$\frac{\mu}{\mu'} = 12 + \frac{1}{1+}\,\frac{1}{2+}\,\frac{1}{1+}\,\frac{1}{4+}\,\frac{1}{3+}\,\frac{1}{3+}\,\frac{1}{7}.$$

For + 1800.0,

$$\frac{\mu}{\mu'} = 12 + \frac{1}{1+}\,\frac{1}{2+}\,\frac{1}{1+}\,\frac{1}{4+}\,\frac{1}{3+}\,\frac{1}{5+}\,\frac{1}{1+}\,\frac{1}{1+}\,\frac{1}{2}.$$

The several converging fractions, so far as it is worth while to carry them, are:—

For $-$ 700.0: $\frac{12}{1}$, $\frac{13}{1}$, $\frac{38}{3}$, $\frac{51}{4}$, $\frac{242}{19}$, $\frac{777}{61}$, $\frac{2573}{202}$, etc.

For $+$ 1800.0: $\frac{12}{1}$, $\frac{13}{1}$, $\frac{38}{3}$, $\frac{51}{4}$, $\frac{242}{19}$, $\frac{777}{61}$, $\frac{4127}{324}$, $\frac{4904}{385}$, etc.

Of these systems the one which offers the greatest advantages is $\frac{242}{19}$, which will give us 223 conjunction points, each having (relative to the node) a retrograde motion such that it would, if constant, make a revolution in about 14,000 years. This time, however, varies with the mean motion of the moon and its node. From the formulæ for k, already given, we find,

$$\text{Epoch, } -\ 700.0\text{: } k = -.0007050,$$
$$\text{Epoch, } +\ 1800.0\text{: } k = -.0007338.$$

The distance apart of two consecutive conjunction points is,

$$K = \frac{1^r}{223} = 0^r.0044843o4 = 1^\circ.614350;$$

and they pass the node at the following intervals:—

At the epoch $-$ 700.0, interval $= 63^y.607 = 785$ lunations.
At the epoch $+$ 1800.0, interval $= 61^y.111 = 756$ lunations.

Between these two fundamental epochs there will be 40 passages of conjunction points through the node.

We next investigate the positions of the conjunction points at the first of these epochs. We note that a conjunction (new moon) occurred $7^d.01670$ before the first epoch, when

$$\begin{aligned} u = u' &= 0^r.327024 \\ &= 73\ K - 0^r.000330 \\ &= \left(73 - \frac{1}{14}\right) K. \end{aligned}$$

We conclude that the node is very near the 73d conjunction point back from that at which the new moon just found occurred, and that this point passed the node about $\frac{1}{14}$th of an interval, or $4\frac{1}{2}$ years before the epoch. We shall take this as the zero conjunction point, and count the others in the order of longitude. Their successive passages across the ascending node will then occur at the times shown in the left-hand half of the following table. The intervals between consecutive passages, as just shown, will diminish from $63^y.607$ at $-$ 700.0 to $61^y.111$ at $+$ 1800.0.

Passages of Conjunction Points through Nodes.

Conj. Point.	Ascend. Node.	Conj. Point.	Ascend. Node.	Conj. Point.	Descend. Node.	Conj. Point.	Descend. Node.
	y.		y.		y.		y.
0	− 704.82	25	865.93	112	− 673.03	137	896.94
1	− 641.25	26	927.96	113	− 609.50	138	958.94
2	− 577.74	27	989.92	114	− 546.02	139	1020.87
3	− 514.29	28	1051.82	115	− 482.60	140	1082.74
4	− 450.90	29	1113.66	116	− 419.24	141	1144.55
5	− 387.58	30	1175.44	117	− 355.95	142	1206.30
6	− 324.32	31	1237.15	118	− 292.72	143	1267.97
7	− 261.12	32	1298.80	119	− 229.55	144	1329.59
8	− 197.98	33	1360.39	120	− 166.44	145	1391.15
9	− 134.91	34	1421.92	121	− 103.40	146	1452.66
10	− 71.90	35	1483.39	122	− 40.42	147	1514.09
11	− 8.95	36	1544.79	123	22.50	148	1575.46
12	53.94	37	1606.13	124	85.36	149	1636.77
13	116.77	38	1667.41	125	148.16	150	1698.02
14	179.54	39	1728.63	126	210.90	151	1759.20
15	242.25	40	1789.78	127	273.58	152	1820.32
16	304.90	41	1850.87	128	336.20	153	1881.38
17	367.49	42	1911.90	129	398.75	154	1942.39
18	430.01	43	1972.87	130	461.24	155	2003.32
19	492.47	44	2033.77	131	523.67	156	2064.19
20	554.87	45	2094.61	132	586.04	157	2125.00
21	617.21	46	2155.39	133	648.34	158	2185.75
22	679.48	47	2216.11	134	710.59	159	2246.42
23	741.69	48	2276.77	135	772.76	160	2307.03
24	803.84	49	2337.37	136	834.88	161	2367.58

At the first of the above epochs the descending node will fall between the 111th and the 112th conjunction point, and the passages will occur midway between those of the ascending node. These times are shown in the right-hand portion of the table.

A new moon occurs at each conjunction point at equal intervals of 223 lunations; and, according to the system adopted, eclipses are classified according to the conjunction point at which they occur, those of each series being separated by intervals of 223 lunations. The middle eclipse of each series will be that which occurs nearest the time when the conjunction passes the node; and we now wish to find when these successive middle eclipses occur. We have just seen that the sun and moon were together at the 73d conjunction point on the 7th day before − 700.0. We wish to find when they were together at the zero point, which is 150 points farther advanced. Each new moon occurs at an interval of 19 conjunction points past the preceding one; therefore, if i be the number of lunations required, we must have

$$19\,i \equiv 150 \pmod{223}.$$

This gives:—

$$i = 137, \text{ or } i = -86.$$

The required conjunctions at the zero point are therefore the 137th following and the 86th preceding that of $-700^y - 7^d$, from which we started. The latter, of course, is nearest the node.

The number of lunations between a conjunction at any point and the first following conjunction at the next point in order is given by the congruence,

$$19\,i \equiv 1 \pmod{232},$$

the solution of which is,

$$i = 47.$$

We shall therefore have a conjunction at point $n + 1$ at an interval of 47 lunations after any conjunction at point n, whatever be n. The intervals between consecutive middle eclipses must therefore be of the form,

$$223\,x + 47,$$

x being an integer. The mean interval must be the same as that between two passages of the node over a conjunction point; that is, 785 lunations about the epoch -700.0 and 756 lunations about the epoch $+1800.0$. The actual intervals are there fore found by putting $x = 3$ and $x = 4$, so that they must be either

716 or 939 lunations.

§ 2.

DATA FOR TABLES OF ECLIPSES.

When the possible solar eclipses which may have occurred during any period are to be investigated, it is convenient to have tables by which we can at once find the limits of time within which their occurrence is possible. A central eclipse can occur only within eleven or twelve days of the time when the sun passes the moon's node, and therefore only at the new moon nearest such passage. A partial eclipse may occur at any time within eighteen days of such passage: there may, therefore, be two partial eclipses; one at the new moon preceding, and the other at the new moon following, the passage of the sun through the node. Our first problem is, therefore, to find the dates of passage of the sun through the nodes of the moon's orbit, which gives us at once the middle of what we may call an eclipse season This is effected by two tables, of which the first gives the dates at which the ascending node has the same longitude that the sun has at the beginning of the fictitious Julian year, and the second the changes in the times of passage for the 19 years following these dates.

The data for the construction of the first table are as follows. Hansen's longitude of the node, corrected, is,

$$\theta = 33^\circ\ 16'\ 31''.15 - 6962929''.61\ \mathrm{T} + 8''.19\ \mathrm{T}^2 + 0''.007\ \mathrm{T}^3,$$

Epoch, $+1800.0$, Gregorian calendar.

Reduced to the Julian epoch, 12 days later, it becomes approximately,

$$\theta = 32° \ 38'.47 - 116048'.827 \, T + 0'.136 \, T^2.$$

The sun's mean longitude at the beginning of the fictitious Julian year is,

$$291° \ 44' + 46'.13 \, T + 0'.021 \, T^2.$$

The distance of the node from the chosen departure point is,

$$100° \ 54' - 116094'.96 \, T + 0'.116 \, T^2.$$

The annual motion and period are,

Epoch, $-$ 700.0,	$m = -116100'.76$;	Period $= 18^y.60453$:
Epoch, $+$ 1800.0,	$m = -116094'.96$;	Period $= 18^y.60546$.

The first passage through the departure point after $+$ 1800.0 is at the epoch

1805.2147.

The 135th passage preceding is at the epoch

$-$ 706.460.

The times of the intermediate passages are then interpolated from the known periods.

Table II gives the days of the fictitious year at which conjunctions of the mean sun with either node occur. The argument is the interval which must elapse after the beginning of the year under examination before the next following conjunction in Table I. The units of the argument are on the left hand, and the tenths on the top of the table. Eclipses can occur only near one of the two or three epochs found in this table, unless a conjunction has occurred near the end of the year preceding or shortly after the beginning of the year following.

Table III, on the same page, gives the reduction from the time of mean to that of true conjunction of the sun with the node, which reduction arises from the eccentricity of the earth's orbit. This table is used only to make more definite the eclipse limits by enabling us to decide whether an eclipse could or could not occur at a given conjunction in cases where the mean values of the argument might leave the question doubtful.

Table IV enables us to find the moon's mean age at any fictitious Julian date. To the fictitious day of the year we add the value of D corresponding to the century, and that corresponding to the year, and subtract the greatest multiple of Period. We may also subtract the next greatest multiple, and thus obtain a negative value of D, counted backward from the next following conjunction.

By taking, for the required date, that of conjunction of the mean or true sun with the node, we are enabled to judge whether an eclipse of given character could or could not have occurred at the preceding or following new moon.

Tables V and VI give the approximate arguments for the central eclipse of each series from — 700.0 to + 2300.0, a period of thirty centuries. To understand its construction we call to mind that, on the system adopted, the moon's orbit is conceived as divided into 223 equal parts by that number of conjunction points; that this whole system of points has a very slow retrograde motion relative to the moon's nodes, such that 61 years elapse between the passages of two consecutive points; that all mean new moons occur at some one of these 223 points; that those at any one point are separated by intervals of 223 lunations, or one Saros or cycle; that if we isolate every 47th lunation, we shall find these isolated lunations to occur at consecutive conjunction points in the order of longitude.

When a conjunction point, by the slow motion already described, approaches within about 18° of the node, there will be an eclipse of the sun at every new moon which occurs at that conjunction point. The series of eclipses will become central within 10° or 12° of the node, and will continue unbroken until the conjunction point has got 18° beyond the node. We shall thus have a series of central eclipses, generally between 45 and 50 in number, with about 15 partial eclipses on each side of it. The total number will generally range between 75 and 80. Since the conjunction point moves about 0°.48 between the consecutive eclipses of each series, some one eclipse must occur within 0°.24 of the node. This nearest eclipse we have sought to take as the central eclipse of the series; but, in some cases, that chosen is not absolutely the nearest. The numbers in Tables V and VI correspond to the eclipse of each series chosen as the central one.

The intervals between the passages of consecutive conjunction points through the node are about 61 years at the present time, and were 63.6 years 25 centuries ago. This must be the mean interval between consecutive central eclipses. But it has been shown that this interval, expressed in lunations, is necessarily of the form $223\,x + 47$, x being an integer, and must be either 716 or 939 lunations, the former being the more frequent value.

From the mean motions already given we derive the following numbers and periods for the two fundamental epochs, — 700.0 and + 1800.0, which have served as the basis of Tables V and VI.

	Epoch, — 700.0;	+ 1800.0.
One mean lunation, in days, - - - - -	29.53059562	29.53058844
Length of Saros, in Julian years, - - - -	18.02963127	18.02962689
Length of Saros, in days, - - - - - - -	6585.322823	6585.321222
Annual motion of mean anomaly, in rev., -	13.25550638	13.25552321
Motion of mean anom. in one Saros, in rev., -	238.9918923	238.9921377
Change of the same, in degrees, - - - -	— 2.9188	— 2.8304
Centennial motion of conj. points, in rev., -	— 0.007050	— 0.007338
Centennial motion of conj. points, in degrees,	— 2.5380	— 2.6417
Motion of conj. points in one Saros, in degrees,	— 0.45758	— 0.47628
Motion of ☉'s mean anomaly in one Saros, in degrees, - - - - - - - - - - - -	+ 10.4980	+ 10.4947
Motion of ☉'s mean long. in one Saros, in deg.,	+ 10.8025	+ 10.8037

The necessary explanation of the principal columns in Tables V and VI will next be given. The conjunction point at which the new moon of — 689, January 20th, occurred, is arbitrarily taken as the zero one. The others are counted from it in the order of longitude. The slow retrograde motion of the whole system relatively to the node causes them to cross the node in the same order.

In column T is found the fictitious Julian date of the central eclipse of each series, already described. Any one of these dates being found, the next following is derived by adding to it the time either of 716 or 939 lunations; such intervals being chosen as would keep the dates near the times of passages of the corresponding conjunction point through the node. A table of these passages has already been given. Near the beginning and end of the table, the regular order has been deviated from, for the reason that it was supposed that there would be no occasion to use the tables for epochs outside the limiting dates, — 700.0 and + 2300.0, while it was desirable to be able to compute all the partial or total eclipses within these dates. Many of these eclipses would, however, take place at conjunction points the central eclipse of which might take place several centuries without the limits. Instead of choosing the central eclipse of the series, one occurring near the limiting epoch was chosen in each case.

It may also be noted that the years before Christ are reckoned in the usual astronomical way; the year immediately preceding the first of the Christian era being considered as zero, the next preceding being — 1, etc The days are, however, considered as positive; so that if we express any one of these dates in years and fractions, the integer number of years would be one less than in the table.

The reckoning of fictitious time throughout the table is that already explained, namely, taking Greenwich mean noon of 1800, January 12th, as the epoch, we call this epoch 1800.0, and count backward and forward by years of $365\frac{1}{4}$ days each. The days are, therefore, not always reckoned from noon, but from noon, 6 hours, 12 hours, or 18 hours, according to the number of the year. A correction is therefore required to reduce to the time of noon, and, since 1582, a still further correction to reduce from the Julian to the Gregorian calendar. These corrections are shown in Table XIII *b*.

The times of mean conjunction correspond to Hansen's mean motion and secular variation, with the corrections given in my *Researches on the Motion of the Moon*, page 268,* the periodic terms being omitted.

The times of mean conjunction, as given, are generally accurate to one or two units in the last place of decimals, or to 8″ or 10″ of arc in the relative positions of the sun and moon. Their errors, therefore, fall far within the necessary uncertainty of the lunar theory in past and future centuries.

The moon's mean anomaly, g, has been divided from — 180° to + 180°, for greater convenience in the selection of total eclipses. It is derived from Hansen's tables, applying the same correction as to the mean longitudes.

The sun's mean anomaly, g', and the mean longitude, L, do not seem to require any special explanation.

The moon's mean argument of the latitude, u_0, has been derived from the difference between the date of each central eclipse and the passage of the conjunction point

* Washington Observations for 1875, Appendix II.

through the node, and is equal to the motion of the conjunction points during this interval.

Table VI, which gives the mean elements for eclipses at the descending node, is constructed on the same principles. Here the argument of latitude, $u_0 - 180°$, is of course counted from the descending node.

Table VII gives the reduction of the arguments in Tables V and VI for other eclipses of the same series. In the use of the tables for calculating a particular eclipse, it is necessary to find the date of the central eclipse of the series to which the one under consideration belongs, as given in Tables V and VI. This is readily done by the precepts given in the tables. Having found the central eclipse, the elements for the required eclipse are deduced by adding the corrections for the number of periods elapsed, as given in Table VII. Owing to the secular changes in the motion of the arguments, these motions are given for three epochs, namely, the year 0, the year 1000, and the year 2000 of our era. For greater facility in the use of the tables, the change in the last place of decimals for intervening centuries is added wherever it is necessary. In using these tables, the number must be taken out for an epoch midway between that of the central eclipse and that of the eclipse to be computed.

Having found the arguments for the moment of mean conjunction, the next step is to deduce the elements for the moment of true conjunction. The theory of this process has been fully developed by Hansen in his *Analyse der ecliptischen Tafeln.** The same author has given tables for the approximate computation of eclipse elements, which are of direct application to the problem as here presented. These tables are, however, rather meagre, and can only be used in connection with the author's tables of the moon. On the other hand, the formulæ in the later paper are developed with such fullness that it is not necessary to go over them. I shall, therefore, accept Hansen's results, with such modifications as are necessary to make them applicable to the form of tables now proposed. The following are the modified expressions.

A general remark, applicable to the tables, is, that the quantities required are given for the moment of true conjunction in ecliptic longitude, but are expressed in terms of the values of g, g', etc., at the moment of mean conjunction.

(1) *Reduction from time of mean conjunction to that of true ecliptic conjunction.*

$$\begin{aligned}\delta T = &- 0^d.4089 \sin g\\ &+ 0^d.0161 \sin 2g\\ &- 0^d.0004 \sin 3g\\ &+ 0^d.1743 \sin g'\\ &+ 0^d.0021 \sin 2g'\\ &- 0^d.0051 \sin (g + g')\\ &+ 0^d.0075 \sin (g - g')\\ &+ 0^d.0104 \sin 2u.\end{aligned}$$

(2) *True argument of latitude, reduced to the ecliptic, for the moment of true conjunction.*

In the special form of tables adopted, it is necessary to reduce the mean longitudes of the two bodies at the moment of mean conjunction to their true longitudes at the

* Berichte über die Verhandlungen der Königlich-Sächsischen Gesellschaft der Wissenschaften, Bd. XV, Leipzig, 1863, and Bd. IX, 1857.

moment of true conjunction. If the expression for the elapsed time between mean and true conjunction is correct, this reduction ought to be the same for both bodies. Hansen gives its expression for the moon; the corresponding correction for the sun is,

Mean motion during interval + Equation of centre for true conjunction.

Putting δT for the elapsed interval, and g'_1 for the mean anomaly at the moment of true conjunction, the required reduction will be,

$$n'\delta T + 1^\circ.922 \sin g'_1 + 0^\circ.020 \sin 2g'_1,$$

where we must put

$$g'_1 = g' + n'\delta T.$$

Substituting this value, and developing, the expression will be,

$$n'\delta T\,(1 + 0.0335 \cos g') + 1^\circ.922 \sin g' + 0^\circ.020 \sin 2g'.$$

Substituting the value of $n\delta T$ just given, we find the following expression for the true ecliptic distance of both bodies, counted from the node, at the moment of true conjunction, u being the mean distance at the moment of mean conjunction:—

$$\begin{aligned} u_1 = u &- 0^\circ.403 \sin g & &= u - .00703 \sin g \\ &+ 0^\circ.016 \sin 2g & &+ .00028 \sin 2g \\ &+ 2^\circ.094 \sin g' & &+ .03655 \sin g' \\ &+ 0^\circ.027 \sin 2g' & &+ .00047 \sin 2g' \\ &- 0^\circ.012 \sin (g+g') & &- .00021 \sin (g+g') \\ &+ 0^\circ.010 \sin 2u & &+ .00017 \sin 2u. \end{aligned}$$

(3) *Vertical distance of the axis of the moon's shadow from the centre of the earth at the moment of ecliptic conjunction.*

The expression for this element is,

$$y_2 = \frac{\sin \text{☽'s latitude}}{\sin(\pi - \pi')}.$$

Hansen puts it into the form,

$$B = P \cos u + Q \sin u = y_2.$$

Its numerical expression will, however, be a little more simple by substituting u_1, the true argument of latitude, for u, the mean argument. Hansen's expressions for P and Q are nearly as follows, some very small terms being omitted:—

$$\begin{aligned} P = &- .0392 \sin g & Q = &+ 5.2207 \\ &+ .0116 \sin 2g & &- 0.3299 \cos g \\ &+ .2080 \sin g' & &- 0.0048 \cos g' \\ &+ .0024 \sin 2g' & &+ 0.0020 \cos 2g' \\ &- .0073 \sin (g+g') & &- 0.0060 \cos (g+g') \\ &+ .0067 \sin (g-g') & &+ 0.0041 \cos (g-g'). \\ &+ .0118 \sin 2u. & & \end{aligned}$$

If we suppose

$$y_2 = P_1 \cos u_1 + Q_1 \sin u_1,$$

we shall have,

$$P_1 = P \cos (u_1 - u) - Q \sin (u_1 - u),$$
$$Q_1 = Q \cos (u_1 - u) + P \sin (u_1 - u).$$

From the preceding expression for u_1 we find,

$$\begin{aligned} \cos (u_1 - u) = 1 &- .00036 \\ &- .00034 \cos 2g' \\ &+ .00014 \cos (g + g') \\ &- .00014 \cos (g - g'), \end{aligned}$$

while we may suppose

$$\sin (u_1 - u) = u_1 - u.$$

We then find,

$$\begin{aligned} Q \sin (u_1 - u) = &- 0386 \sin g \\ &+ .0025 \sin 2g \\ &+ .1917 \sin g' \\ &+ .0022 \sin 2g' \\ &- .0070 \sin (g + g') \\ &+ .0060 \sin (g - g'). \end{aligned}$$

$$\begin{aligned} Q \cos (u_1 - u) = Q &- .0019 \\ &- .0018 \cos 2g' \\ &+ .0007 \cos (g + g') \\ &- .0007 \cos (g - g'). \end{aligned}$$

$$\begin{aligned} P \sin (u_1 - u) = &+ .0039 \\ &- .0002 \cos 2g \\ &- .0038 \cos 2g' \\ &+ .0014 \cos (g + g') \\ &- .0014 \cos (g - g'). \end{aligned}$$

$$P \cos (u_1 - u) = P.$$

$$\begin{aligned} P_1 = &- .0006 \sin g \\ &+ .0091 \sin 2g \\ &+ .0163 \sin g' \\ &+ .0002 \sin 2g' \\ &- .0003 \sin (g + g') \\ &+ .0007 \sin (g - g') \\ &+ .0118 \sin 2u. \end{aligned}$$

$$\begin{aligned} Q_1 = &+ 5.2227 \\ &- 0.3299 \cos g \\ &- 0.0048 \cos g' \\ &- 0.0036 \cos 2g' \\ &- 0.0039 \cos (g + g') \\ &+ 0.0020 \cos (g - g'). \end{aligned}$$

The value of P_1 is so small that we may suppose $\cos u_1 = \pm 1$ in multiplying it by this quantity. In a total eclipse, the value of u_1 can differ from 0° or 180° by only about 11°, and that of u by only about 16°. We shall, therefore, obtain a result nearly accurate to the third place of decimals by replacing $0.0118 \sin 2u$ by $0.022 \sin u_1$. A sufficiently accurate expression for y_2 will therefore be,

$$y_2 = P_1 + Q_1 \sin u_1;$$

or, omitting terms which will not change y_2 by .001,.

$$y_2 = \pm(-.0006 \sin g + .0091 \sin 2g + .0163 \sin g') + (5.245 - 0.330 \cos g) \sin u_1,$$

the upper sign being used at the ascending and the lower at the descending node. An error of a unit in the third place of decimals corresponds to one of about 5′ in the position of the shadow-path on the earth's surface; the probable error of the shadow-path, on account of the quantities neglected in y_2, will therefore not exceed 10 or 15 miles.

(4) *For the hourly motion of the axis of the shadow along the fundamental plane,** Hansen's expression is equivalent to

$$x'_2 = \frac{dx_2}{dt} = +0.5410 + 0.0397 \cos g - 0.0010 \cos g' + 0.0006 \cos(g+g') - 0.0004 \cos(g-g').$$

$$y'_2 = \frac{dy_2}{dt} = (.0540 + .0034 \cos g) \cos u_1.$$

We may, without an error exceeding 0.001, regard $\frac{dy_2}{dt}$ as equal to $\frac{1}{10} \cdot \frac{dx_2}{dt}$.

(5) *The radius of the shadow on the fundamental plane and the angle of the shadow-cone* are given by the formulæ,

$$\rho = +0.0059 - 0.0182 \cos g + 0.0004 \cos 2g + 0.0046 \cos g' - 0.0005 \cos(g+g').$$

$$\sin f = +0.004653 + 0.000078 \cos g'.$$

When ρ is positive, the eclipse will be annular; when negative, total.

The value of ρ for external contact may be found by increasing the above by 0.5460, which will make the constant term 0.5519. The same value of $\sin f$ may be used in the two cases.

Circumstances of an Eclipse on the Earth's Surface—Our next step is to find the relation of any point on the earth's surface to the shadow. Several systems of co-ordinates may be adopted for this purpose, which vary with the adopted direction of the axis of X on the fundamental plane. We have the choice of three systems, depending on the following three positions of this axis of X in the fundamental plane:—

(1) The intersection of the earth's equator with the fundamental plane;

(2) The intersection of the ecliptic with the same plane;

(3) A line in the same plane parallel to the path of the axis of the shadow along it.

The first system is that of Bessel, while the second and third have been used by Hansen.

* It will be remembered that the fundamental plane in the theory of eclipses passes through the centre of the earth perpendicular to the axis of the shadow.

Let us put

$\odot$, the sun's true longitude, or, more exactly, the longitude of the sun as seen from the moon;

ε, the obliquity of the ecliptic;

a, d, the right ascension and declination of the sun as seen from the moon, for which, in the present case, we may take the geocentric direction of the sun;

α, the angle of the shadow-path along the fundamental plane with the intersection of the ecliptic with the same plane;

$\left.\begin{array}{l} h = \rho \cos \varphi' \\ k = \rho \sin \varphi' \end{array}\right\}$ ρ being here the earth's radius and φ' the geocentric latitude of any point on its surface.

For the present we shall represent the co-ordinates corresponding to these various systems by subscript numbers. It will be remarked that in all the systems the axis of Z passes through the centre of the earth parallel to the line joining the centres of the moon and sun.

The value of the equation of the centre by which $\odot$ is found may be obtained from Table XXVI. The expression is, $\odot = \mathrm{L} +$ Equation of centre.

For the relations between systems (1) and (2), we determine the angle p from any or all of the equations,

$$\begin{aligned} \cos d \sin p &= \sin \varepsilon \cos \odot, \\ \cos d \cos p &= \cos \varepsilon, \\ \sin d &= \sin \varepsilon \sin \odot. \end{aligned}$$

The required relations will then be,

$$\begin{aligned} x_2 &= \quad x_1 \cos p + y_1 \sin p, \\ y_2 &= -x_1 \sin p + y_1 \cos p, \\ z_2 &= \quad z_1; \end{aligned}$$

or, reversing them,

$$\begin{aligned} x_1 &= x_2 \cos p - y_2 \sin p, \\ y_1 &= x_2 \sin p + y_2 \cos p. \end{aligned}$$

For the use of the third system, it will be sufficiently accurate to suppose that the axis of x_3 makes an angle of $5^\circ\ 30'$ with that of x_2. With a little greater probable accuracy we may determine the angle α by the condition, $\alpha = \pm\ 5^\circ.7 \cos u_1$, this angle being positive at the ascending and negative at the descending node. Then, putting

$$p' = p + \alpha,$$

we shall have,

$$\begin{aligned} x_3 &= \quad x_1 \cos p' + y_1 \sin p' = \quad x_2 \cos \alpha + y_2 \sin \alpha, \\ y_3 &= -x_1 \sin p' + y_1 \cos p' = -x_2 \sin \alpha + y_2 \cos \alpha. \end{aligned}$$

Tables XVIII to XX give the value of y_2 for the shadow-axis at the moment of conjunction in longitude, when $x_2 = 0$; and Tables XXI and XXII give the hourly

variation of x_2, from which that of y_2 may be obtained by multiplying by tan α or by cos u_1 tan $(5^\circ\ 42')$. The expressions for x_2 and y_2 will therefore be,

$$x_2 = x'_2\, t,$$
$$y_2 = y_2^\circ + x'_2\, t \tan \alpha = y_2^\circ + y'_2\, t.$$

Here t is the time after true conjunction, T, expressed in hours as the unit.

To refer the shadow-axis to either of the other systems, we shall then have,

$$\begin{aligned}
x_3 &= y_2^\circ \sin \alpha + x'_2\, t \sec \alpha, \\
y_3 &= y_2^\circ \cos \alpha, \\
x_1 &= - y_2^\circ \sin p + x'_2\, t\, (\cos p - \tan \alpha \sin p), \\
&= - y_2^\circ \sin p + x'_2\, t \sec \alpha \cos\, (p + \alpha), \\
y_1 &= y_2^\circ \cos p + x'_2\, t\, (\sin p + \tan \alpha \cos p), \\
&= y_2^\circ \cos p + x'_2\, t \sec \alpha \sin\, (p + \alpha).
\end{aligned}$$

The values of the coefficients for x_1 and y_1 may be taken from Table XXVIII, where we have put

$$\begin{aligned}
a &= - \sin p, \\
a' &= \cos p, \\
b &= \sec \alpha \cos\, (p \pm \alpha), \\
b' &= \sec \alpha \sin\, (p \pm \alpha);
\end{aligned}$$

so that the expressions for x_1 and y_1 are,

$$x_1 = a\ y_2^\circ + b\ x'_2\, t,$$
$$y_1 = a'\ y_2^\circ + b'\, x'_2\, t.$$

We now require the corresponding co-ordinates for the point on the earth's surface, expressed by the quantities h and k. If we represent these by ξ, η, and ζ, Bessel's eclipse formulæ give,

$$\xi_1 = h \sin \mathrm{H},$$
$$\eta_1 = k \cos d - h \sin d \cos \mathrm{H},$$

H being the hour-angle of the sun, or, to speak more exactly, the hour-angle of that point of the sphere representing the direction of the sun as seen from the moon. The other co-ordinates of the place will then be,

$$\begin{aligned}
\xi_2 &= k \cos d \sin p\ \ + h\, (\cos p\ \sin \mathrm{H} - \sin d \sin p\ \cos \mathrm{H}), \\
\eta_2 &= k \cos d \cos p\ \ - h\, (\sin p\ \sin \mathrm{H} + \sin d \cos p\ \cos \mathrm{H}), \\
\xi_3 &= k \cos d \sin p' + h\, (\cos p' \sin \mathrm{H} - \sin d \sin p' \cos \mathrm{H}), \\
\eta_3 &= k \cos d \cos p' - h\, (\sin p' \sin \mathrm{H} + \sin d \cos p' \cos \mathrm{H}).
\end{aligned}$$

The angle H is expressed in terms of t, as follows:—From the conjunction tables V–VII we have, by the corrections from Tables VIII–XII and by correcting for the fictitious date, the fraction of a day of Greenwich mean time at the moment of true

ecliptic conjunction. Multiplying this fraction by 360° (Table XIV), we have the hour-angle of the mean sun for the meridian of Greenwich at the moment of conjunction.

Let us put

H_0, this hour-angle;

λ, the longitude of the place west from Greenwich;

E, the equation of time, to be *added* to apparent time in order to obtain mean time, expressed in arc.

Then, at conjunction, the hour-angle of the mean sun will be $H_0 - \lambda$, and that of the apparent sun will be $H_0 - \lambda - E$. The expression for H, as a function of t, will then be

$$H = H_0 - \lambda - E + 15^\circ \times t.$$

We have now all the data for proceeding with the computation of the eclipse in any of the usual ways.

The data of the present tables are of such accuracy that we may generally expect to predict the phases of an eclipse, by means of them, within one or two minutes of time, and to determine the shadow-path in total or annular eclipses to coarse fractions of a degree. In fact, supposing the tables perfect to the last place of decimals, the probable error of this path should not exceed two or three tenths of a degree, unless near the north or south pole of the earth; but small errors of theory are possible, leading to larger errors in the shadow-path.

The following are the formulæ for the computation of the path of a central eclipse. They are applied by computing the longitude and latitude of the point in which the axis of the shadow intersects the earth's surface at any assumed moment of Greenwich mean time.

Compute the values of x_1 and y_1 for the assumed moment. Table XXVIII is designed to facilitate this computation.

If it is desired to take into account the ellipticity of the earth, the neglect of which will introduce a probable error of perhaps 10′ in the point required (which amount, however, is hardly greater than the necessary uncertainty of the results from the preceding tables), we compute ρ_1 and d_1 from the formulæ,

$$\rho_1 \sin d_1 = \sin d,$$
$$\rho_1 \cos d_1 = \sqrt{1 - e^2} \cos d = [9.99855] \cos d,$$
$$y'_1 = \frac{y_1}{\rho_1}.$$

The values of ρ_1 and d_1 may be taken at once from Table XXIX with the argument $\odot = L +$ Equation of centre. If, however, we neglect the ellipticity, we put d for d_1, y_1 for y'_1, and φ for φ_1 in the following formulæ, which are a continuation of the preceding ones.

From

$$c \sin C = y'_1,$$
$$c \cos C = \sqrt{1 - x_1^2 - y'^2_1},$$

find c and C. The last quantity may be computed by the auxiliary angle β, thus:—

$$\sin\beta\sin\gamma = x_{\prime\prime}$$
$$\sin\beta\cos\gamma = y'_{\prime\prime}$$
$$c\cos C = \cos\beta.$$

Then, from the equations,

$$\cos\varphi_1\sin H = x_{\prime\prime}$$
$$\cos\varphi_1\cos H = c\cos(C + d_1),$$
$$\sin\varphi_1 = c\sin(C + d_1),$$
$$\tan\varphi = [0.00145]\tan\varphi_{\prime\prime}$$

find φ and H. The former will be the latitude of the point required, and the latter the local hour-angle of the shadow-axis. The Greenwich hour-angle is found by the formula,

$$H_1 = H_0 - E + 15^\circ t;$$

H_0 being the Greenwich mean time of true conjunction in longitude, expressed in arc; E, the equation of time (Tables XXVI and XXVII); t, the interval of the assumed time after that of true conjunction, expressed in hours The west longitude of the point sought will then be:—

$$\lambda = H_1 - H.$$

§ 3.

RECURRENCE OF REMARKABLE ECLIPSES.

The occurrence of eclipses approaching the maximum length of totality is a subject of astronomical interest. We have already shown that the successive eclipses at the same conjunction point, occurring at intervals of 18 years, are nearly of the same character. Consequently, if we have at any time an eclipse in which the duration of totality approaches the maximum, we shall have a similar one after a lapse of one period, and the duration will vary but slowly from period to period. We shall therefore search in our tables, not for single eclipses with long duration of totality, but for series of such eclipses, the distinctive mark of each series being the conjunction point at which it occurs.

The conditions necessary to the greatest duration of totality, considered individually, are the following:—

I. The moon must be near its perigee at the time of conjunction. In other words, its mean anomaly, positive or negative, must be small in absolute value.

II. The sun must be near its apogee in order that its semi-diameter may be small; hence its mean anomaly must differ little from 180°. It is, however, to be remarked that a deviation of the sun's mean anomaly from 180° will produce only about one fourth the effect of an equal deviation of the moon's anomaly from 0°.

III. The preceding two conditions give the maximum breadth of shadow on the fundamental plane, passing through the centre of the earth at right angles to the axis of the shadow. But, the shadow being conical, its diameter increases as we approach the moon. The observer should therefore be as near the moon as possible. In other words, at the moment of central eclipse, the sun and moon should be near his zenith. The diameter of the shadow at his station will then be nearly one third greater than on the fundamental plane. In order that these conditions may be fulfilled, it is necessary that the observer should be within the tropics and that the conjunction should take place near the node. For, the two bodies being in the zenith, the effect of parallax is zero, and the eclipse must be central at the centre of the earth, which can only occur when the conjunction coincides with the node. These conditions will be most nearly fulfilled by the central eclipses, the dates of which are given in Tables V and VI.

IV. The diurnal motion of the observer must be as great as possible, because by this motion he is carried along in the same direction with the axis of the shadow, and thus the time which he remains within it is increased. This condition is best fulfilled when he is upon the equator.

V. The direction in which the observer is carried by the diurnal motion must be parallel to the direction of the shadow. This condition demands that the direction of the axis of the shadow shall be near the great circle joining the pole of the earth and the pole of the moon's orbit. This condition can be fulfilled only when the sun's longitude is near 90° or 270°; in other words, near the times of the two solstices.

It is impossible that all the preceding conditions can be simultaneously fulfilled, owing to the obliquity of the ecliptic. The 4th condition can be fulfilled only at the equinoxes, and the 5th only at the solstices. Also, since the sun's apogee has, during each century, a nearly definite longitude (at present about 90°), it is only near 90° of the sun's longitude that the 2d condition can at present be fulfilled. In former ages, the case was somewhat different. But the distance of the epoch from these solstices was only about 20° at the beginning of the Christian era. We may see, therefore, that during historic times, and for several centuries to come, the solar eclipses of greatest duration can occur only near the summer solstice. If the eclipse at this time occurs also exactly at the node it will be central in the zenith of the Tropic of Cancer Conditions 3d and 5th will therefore be fulfilled, but condition 4th will not. If the latitude of the moon be north at the moment of conjunction, conditions 3d and 4th will both be less favorable. If the latitude be south, condition 4th may be more favorable, because the shadow will then be thrown further towards the equator, but condition 3d will be less favorable. Condition 4th will be best fulfilled by south latitude of about 24′, or by an argument of latitude of $-4°$ or $-5°$ near the ascending node and $+4°$ or $+5°$ near the descending node. Beyond these points of latitude, both conditions are unfavorable. We conclude, therefore, that when the two first conditions are properly fulfilled, the most favorable eclipses will be the ten or twelve which follow the central one of each series at the ascending node and the ten or twelve which precede it at the descending node. The most favorable will generally fall between the fourth and the seventh from the central eclipse; and the first two conditions require

that we should then have $g = 0$ and $g' = 0$. In order that these conditions should be fulfilled at the sixth eclipse, we should have, near the time of central eclipse, the following system of values of g, g', and L :—

Ascending Node.	Descending Node.
$g = 17^\circ$	$g = -17^\circ$
$g' = 120^\circ$	$g' = 240^\circ$
$L = 25^\circ$	$L = 155^\circ$

An examination of Table V will now enable us to select series of remarkable total eclipses almost by inspection. We see that the moon's mean anomaly is repeated within 12° or 15° at every third conjunction point. Considering, first, eclipses at the ascending node, we perceive that the moon's mean anomaly is small at the 10th, 13th, 15th, etc., conjunction points, and that the condition, $g = 17^\circ$, is most nearly fulfilled at the 16th and 19th points. The conditions, $g' = 120^\circ$ and $L = 25^\circ$, though not fulfilled at either point, are so near fulfillment that there were then two series of total eclipses nearer the maximum duration than any which occurred for several subsequent centuries. The last of the most favorable series were in the years 663, 681, 699, etc.

To find other series approaching the maximum of totality, we have to pass over more than a thousand years, until the 42d and 45th conjunction points approach the node. To the 42d conjunction point belong the series of great eclipses of 1832, 1850, 1868, 1886, etc., of which the maximum was that of 1832 or 1850. The position of the solar perigee is unfavorable at this point; otherwise the duration would have gone on increasing through the next century. But at the 45th conjunction point, the conditions are more nearly fulfilled than they have been for at least twenty centuries, and we shall therefore have a series of eclipses approaching within a very few seconds of the maximum duration of totality. These will occur in the years 2150, 2168, etc.

Passing now to the descending node, we see that in a general way the series occur in the same order. The favorable conjunction points near the present epoch seem to be the 149th, 152d, 155th, etc. In the first two of these, the sun's mean anomaly is not favorable except when the moon's is unfavorable. The conditions are better fulfilled at the 155th conjuction point, the central eclipse of which takes place in the year 2009. The eclipses of maximum duration will occur two or three periods before the central eclipse, namely, in the years 1955 and 1973. To this series belong the total eclipses of 1865, 1883, etc. The successive eclipses of this series will therefore increase in duration for five or six periods to come, when the duration will probably be greater than that of any that have preceded them during the past thousand years.

TABLES OF SOLAR ECLIPSES.

B. C. 700 TO A. D. 2300.

TABLE I.—*Dates at which the Moon's Ascending Node has the same Longitude that the Sun has at the Beginning of the Fictitious Year.*

Year.	Year.	Year.	Year.	Year.
−780.878	−148.321	+484.244	+1116.818	+1749.398
−762.273	−129.716	502.849	1135.423	1768.004
−743.6[illegible]9	−111.112	521.454	1154.028	1786.609
−725.064	− 92.507	540.059	1172.633	1805.215
−706.460	− 73.902	558.664	1191.239	1823.820
−687.855	− 55.297	577.269	1209.844	1842.426
−669.251	− 36.693	595.874	1228.450	1861.031
−650.646	− 18.088	614.479	1247.055	1879.637
−632.042	+ 0.517	633.084	1265.660	1898.242
−613.437	19.122	651.689	1284.266	1916.848
−594.832	37.727	670.294	1302.871	1935.453
−576.228	56.332	688.899	1321.476	1954.059
−557.623	74.936	707.504	1340.080	1972.664
−539.019	93.541	726.109	1358.686	1991.270
−520.414	112.146	744.714	1377.291	2009.875
−501.809	130.751	763.319	1395.896	2028.481
−483.205	149.356	781.924	1414.502	2047.086
−464.600	167.961	800.529	1433.107	2065.692
−445.996	186.566	819.135	1451.712	2084.298
−427.391	205.171	837.740	14[illegible]0.318	2102.903
−408.786	223.775	856.345	1488.923	2121.509
−390.182	242.380	874.950	1507.528	2140.114
−371.577	260.985	893.555	1526.134	2158.720
−352.972	279.590	912.160	1544.739	2177.326
−334.368	298.195	930.765	1563.344	2195.931
−315.763	316.800	949.370	1581.950	2214.537
−297.158	335.404	967.976	1600.555	2233.142
−278.554	354.009	986.581	1619.160	2251.748
−259.949	372.614	1005.186	1637.766	2270.354
−241.344	391.219	1023.792	1656.371	2288.959
−222.740	409.824	1042.397	1674.977	2307.565
−204.135	428.429	1061.002	1693.582	2326.170
−185.530	447.034	1079.607	1712.187	2344.776
−166.926	+465.639	+1098.212	+1730.793	+2363.381

TABLE I.

The use of this and the next table is as follows:—Subtract the number of the year, neglecting fractions, from the next larger number (algebraically) in the table. With the excess enter Table II, the entire number being on the side and the tenths at the top. The corresponding number will be the day and tenths of a day of the fictitious Julian year at which the mean sun was in conjunction with the node indicated in the second column. Should the number be negative it will indicate days before the beginning of the year, and should it exceed 365.25 it will indicate a conjunction after the end of the year.

In general, a central eclipse of the sun can only occur within ten or twelve days of the times thus found, and a partial eclipse within eighteen or twenty days. It is to be remarked, however, that an eclipse may occur when the corresponding conjunction takes place near the end of the year preceding, or during the first seventeen days of the year following.

Table II.—*Days of the Fictitious Year when the Mean Sun is in Conjunction with either Node.*

Year.	Node.	.0	.1	.2	.3	.4	.5	.6	.7	.8	.9
		d.	d.	d.	d.	d.	d.	d.	d.	d.	d.
0	Asc.	0.0	1.9	3.7	5.6	7.4	9.3	11.2	13.0	14.9	16.8
	Desc.	173.3	175.2	177.0	178.9	180.7	182.6	184.5	186.3	188.2	190.1
	Asc.	346.6	348.5	350.3	352.2	354.0	355.9	357.8	359.6	361.5	363.4
1	Asc.	18.6	20.5	22.3	24.2	26.0	27.9	29.7	31.6	33.5	35.4
	Desc.	191.9	193.8	195.6	197.5	199.4	201.3	203.1	205.0	206.9	208.7
2	Asc.	37.3	39.2	41.0	42.9	44.7	46.6	48.4	50.3	52.2	54.0
	Desc.	210.6	212.5	214.3	216.2	218.0	219.9	221.7	223.6	225.5	227.3
3	Asc.	55.9	57.8	59.6	61.5	63.3	65.2	67.0	68.9	70.8	72.7
	Desc.	229.2	231.1	232.9	234.8	236.6	238.5	240.3	242.2	244.1	246.0
4	Asc.	74.5	76.4	78.2	80.1	82.0	83.9	85.7	87.6	89.5	91.3
	Desc.	247.8	249.7	251.5	253.4	255.3	257.2	259.0	260.9	262.8	264.6
5	Asc.	93.2	95.1	96.9	98.8	100.6	102.5	104.3	106.2	108.1	109.9
	Desc.	266.5	268.4	270.2	272.1	273.9	275.8	277.6	279.5	281.4	283.2
6	Asc.	111.8	113.7	115.5	117.4	119.2	121.1	122.9	124.8	126.7	128.5
	Desc.	285.1	287.0	288.8	290.7	292.5	294.4	296.2	298.1	300.0	301.9
7	Asc.	130.4	132.3	134.1	136.0	137.9	139.8	141.6	143.5	145.4	147.2
	Desc.	303.7	305.6	307.4	309.3	311.1	313.0	314.9	316.8	318.7	320.5
8	Asc.	149.0	150.9	152.8	154.7	156.5	158.4	160.2	162.1	164.0	165.8
	Desc.	322.3	324.2	326.1	328.0	329.8	331.7	333.5	335.4	337.3	339.1
9	Desc.	−5.6	−3.7	−1.9	0.0	1.8	3.7	5.5	7.4	9.3	11.1
	Asc.	167.7	169.6	171.4	173.3	175.1	177.0	178.8	180.7	182.6	184.4
	Desc.	341.0	342.9	344.7	346.6	348.4	350.3	352.1	354.0	355.9	357.7
10	Desc.	13.0	14.9	16.7	18.6	20.4	22.3	24.1	26.0	27.9	29.8
	Asc.	186.3	188.2	190.0	191.9	193.7	195.6	197.4	199.3	201.2	203.1
	Desc.	359.6	361.5	363.3	365.2	367.0	368.9	370.7	372.6	374.5	376.4
11	Desc.	31.6	33.5	35.3	37.2	39.1	41.0	42.8	44.7	46.6	48.4
	Asc.	204.9	206.7	208.6	210.5	212.4	214.3	216.1	218.0	219.9	221.7
12	Desc.	50.2	52.1	54.0	55.9	57.7	59.6	61.4	63.3	65.2	67.0
	Asc.	223.6	225.5	227.3	229.2	231.0	232.9	234.7	236.6	238.5	240.3
13	Desc.	68.8	70.7	72.6	74.5	76.3	78.2	80.0	81.9	83.8	85.6
	Asc.	242.2	244.1	245.9	247.8	249.7	251.6	253.4	255.3	257.2	259.0
14	Desc.	87.5	89.4	91.2	93.1	94.9	96.8	98.6	100.5	102.4	104.3
	Asc.	260.8	262.7	264.5	266.4	268.3	270.2	272.0	273.9	275.8	277.6
15	Desc.	106.1	108.0	109.9	111.8	113.6	115.5	117.3	119.2	121.1	122.9
	Asc.	279.4	281.3	283.2	285.1	286.9	288.8	290.6	292.5	294.4	296.2
16	Desc.	124.7	126.6	128.4	130.3	132.1	134.0	135.8	137.7	139.6	141.5
	Asc.	298.1	300.0	301.8	303.7	305.5	307.4	309.2	311.1	313.0	314.8
17	Desc.	143.4	145.3	147.1	149.0	150.8	152.7	154.5	156.4	158.3	160.2
	Asc.	316.7	318.6	320.4	322.3	324.1	326.0	327.8	329.7	331.6	333.5
18	Desc.	162.0	163.8	165.7	167.5	169.4	171.2	173.1	175.0	176.9	178.8
	Asc.	335.3	337.2	339.1	341.0	342.8	344.7	346.5	348.4	350.3	352.1

For hundredths of a year.	
y.	d.
0.01	0.2
0.02	0.4
0.03	0.6
0.04	0.7
0.05	0.9
0.06	1.1
0.07	1.3
0.08	1.5
0.09	1.7
0.10	1.9

Table III.—*Reduction to Time of True Conjunction of Sun with Node.*

Days.	d.	Days.	d.	Days.	d.	Days.	d.	Days.	d.	Days.	d.	Days.	d.	Days.	d.
0	−0.4	50	−1.6	100	−1.8	150	−0.7	200	+ 1.0	250	+ 1.9	300	+ 1.5	350	+0.1
10	0.7	60	1.8	110	1.7	160	−0.3	210	1.2	260	1.9	310	1.2	360	−0.3
20	1.0	70	1.9	120	1.5	170	0.0	220	1.5	270	1.9	320	1.0	370	−0.5
30	1.2	80	1.9	130	1.2	180	+0.3	230	1.7	280	1.8	330	0.7		
40	−1.5	90	−1.9	140	−1.0	190	+0.6	240	+ 1.8	290	+ 1.7	340	+ 0.4		

Table IV.—*To find the Age of the Moon at any Fictitious Julian Date.*

Century.	D.	Year.	D.	Year.	D.	Year.	D.	Year.	D.	Multiples of Period.	
	d.										
− 800	6.8	0	4.6	25	10.9	50	17.2	75	23.5	1	29.5
− 700	2.5	1	15.5	26	21.8	51	28.1	76	4.8	2	59.1
− 600	27.6	2	26.3	27	3.1	52	9.4	77	15.7	3	88.6
− 500	23.3	3	7.7	28	14.0	53	20.3	78	26.6	4	118.1
− 400	18.9	4	18.6	29	24.9	54	1.6	79	7.9	5	147.7
− 300	14.6										
− 200	10.3										
− 100	5.9	5	29.4	30	6.2	55	12.5	80	18.8	6	177.2
0	1.6	6	10.8	31	17.1	56	23.4	81	0.2	7	206.7
+ 100	26.7	7	21.7	32	28.0	57	4.8	82	11.1	8	236.2
200	22.4	8	3.0	33	9.3	58	15.6	83	21.9	9	265.8
300	18.0	9	13.9	34	20.2	59	26.5	84	3.3	10	295.3
400	13.7										
500	9.4	10	24.8	35	1.6	60	7.9	85	14.2	11	324.8
600	5.0	11	6.2	36	12.5	61	18.8	86	25.1	12	354.4
700	0.7	12	17.0	37	23.3	62	0.1	87	6.4	13	383.9
800	25.9	13	27.9	38	4.7	63	11.0	88	17.3	14	413.4
900	21.5	14	9.3	39	15.6	64	21.9	89	28.2	15	443.0
1000	17.2										
1100	12.8										
1200	8.5	15	20.2	40	26.5	65	3.2	90	9.5		
1300	4.2	16	1.5	41	7.8	66	14.1	91	20.4		
1400	29.4	17	12.4	42	18.7	67	25.0	92	1.8		
1500	25.0	18	23.3	43	0.1	68	6.4	93	12.6		
1600	20.7	19	4.6	44	10.9	69	17.2	94	23.5		
1700	16.3										
1800	12.0										
1900	7.7	20	15.5	45	21.8	70	28.1	95	4.9		
2000	3.3	21	26.4	46	3.2	71	9.5	96	15.8		
2100	28.5	22	7.8	47	14.1	72	20.5	97	26.7		
2200	24.2	23	18.6	48	24.9	73	1.7	98	8.0		
2300	19.9	24	0.0	49	6.3	74	12.6	99	18.9		

The age (D) of the mean moon at any time is found by taking the sum of the values of D corresponding to the century and to the year of the century, adding the fictitious Julian date and subtracting the greatest multiple of the period. Generally it will be better to subtract the multiple next smaller and next greater than the sum. The first remainder will then indicate the days which have elapsed since the preceding mean new moon, and the second those before the next following. The limits of D for a central eclipse will then be,

D between $\pm$ 14^d.2 : an eclipse certain. D between $\pm$ 8^d.0 : a central eclipse certain.
D between $\pm$ 20^d.8 : an eclipse possible. D between $\pm$ 14^d.3 : a central eclipse possible.

If the occurrence of the eclipse is still doubtful, the limits may be narrowed by applying to D the further correction taken from Table III, of which the argument is the day of the fictitious year, already found from Table II. This table only holds good within two or three centuries of the present time; but it may be used for other centuries by simply increasing the argument by one day for each century before the nineteenth. Using the corrected values of D, the limits will be,

$\pm$ 16^d.1 : an eclipse certain. $\pm$ 9^d.9 : a central eclipse certain.
$\pm$ 18^d.9 : an eclipse possible. $\pm$ 12^d 4 : a central eclipse possible.

To find the central eclipse of the series to which the required one belongs, take one of the values of P corresponding most nearly to D in the following table:—

D	0.5	1.0	1.4	1.9	2.4	2.9	3.4	3.8	4.3	4.8	5.3	5.8	6.2	6.7	7.2	7.7	8.2	8.6
P	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
T	18	36	54	72	90	108	126	144	162	180	198	216	234	252	270	288	307	325
D	9.1	9.6	10.1	10.6	11.1	11.5	12.0	12.5	13.0	13.5	13.9	14.4	14.9	15.4	15.8	16.3	16.8	17.3
P	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36
T	343	361	379	397	415	433	451	469	487	505	523	541	539	577	596	614	632	650

If D is positive, subtract one of the nearest values of T from the number of the year; if negative, add it, and we shall hit very nearly upon the date of central eclipse found in Table V or VI, according to the node, or upon a date differing by a multiple of 18 years. The corresponding value of P will be the number of periods of 18^y 11^d between the date in Table V or VI and the date of the eclipse sought.

It is to be noted that if the value of T thus found carries the central date without the limits − 770 and + 2360, a value of P and T small enough to bring the date within these limits will be necessary.

TABLE V.—*Mean Elements of Eclipses at Ascending Node.*

Conj. Point.	T Fictitious Date of Central Mean Conjunction.		g Moon's Mean Anomaly.	g' Sun's Mean Anomaly.	L Sun's Mean Longitude.	u_0 Moon's Mean Arg Lat.	Conj. Point.	T Fictitious Date of Central Mean Conjunction		g Moon's Mean Anomaly.	g' Sun's Mean Anomaly.	L Sun's Mean Longitude.	u_0 Moon's Mean Arg. Lat.
	y.	d.	°	°	°	°		y.	d.	°	°	°	°
214	− 724	313.2334	+ 68.73	344.67	221.25	−14.052	25	866	166.7316	+ 49.43	185.37	88.92	− 0.013
215	− 720	240.1713	− 157.00	272.63	149.27	−12.536	26	924	126.1349	+ 174.22	144.78	49.33	+ 0.094
216	− 716	167.1093	− 24.52	200.60	77.30	−11.021	27	1000	96.3598	− 63.83	114.75	20.57	− 0.269
217	− 712	94.0473	+ 108.85	128.54	5.30	− 9.505	28	1058	55.7627	+ 60.98	74.18	341.00	− 0.165
218	− 708	20.9853	− 117.77	56.48	293.33	− 7.988	29	1116	15.1656	− 174.20	33.63	301.43	− 0.062
219	− 705	313.1733	+ 15.61	344.43	221.35	− 6.472	30	1173	339.8183	− 49.37	353.08	261.87	+ 0.040
220	− 701	240.1112	+ 148.98	272.38	149.37	− 4.955	31	1249	310.0425	+ 72.61	323.02	233.10	− 0.333
221	− 697	167.0492	− 77.64	200.33	77.37	− 3.439	32	1307	269.4450	− 162.55	282.45	193.53	− 0.234
222	− 693	93.9872	+ 55.73	128.28	5.38	− 1.921	33	1365	228.8473	− 37.70	241.87	153.95	− 0.137
0	− 689	20.9252	− 170.89	56.25	293.40	− 0.403	34	1423	188.2496	+ 87.16	201.28	114.37	− 0.042
1	− 632	345.5815	− 46.27	15.67	253.82	− 0.259	35	1481	147.6517	− 147.98	160.75	74.80	+ 0.052
2	− 574	304.9079	+ 78.36	335.13	214.28	− 0.116	36	1539	107.0536	− 23.11	120.20	35.23	+ 0.145
3	− 516	264.3940	− 157.00	294.57	174.68	+ 0.025	37	1615	77.2768	+ 98.93	90.17	6.48	− 0.239
4	− 440	234.6227	− 35.27	264.52	145.93	− 0.294	38	1673	36.6783	− 136.19	49.58	326.90	− 0.150
5	− 382	194.0285	+ 89.38	223.97	106.35	− 0.156	39	1730	361.3299	− 11.29	9.02	287.33	− 0.062
6	− 324	153.4343	− 145.96	183.43	66.78	− 0.019	40	1788	320.7313	+ 113.60	328.47	247.77	+ 0.024
7	− 248	123.6626	− 24.20	153.35	38.00	− 0.345	41	1846	280.1325	− 121.49	287.88	208.19	+ 0.108
8	− 190	83.0682	+ 100.47	112.82	358.43	− 0.210	42	1904	239.5337	+ 3.42	247.32	168.62	+ 0.192
9	− 132	42.4736	− 131.85	72.25	318.85	− 0.078	43	1980	209.7557	+ 125.51	217.27	139.87	− 0.204
10	− 74	1.8789	− 10.17	31.73	279.30	+ 0.054	44	2038	169.1565	− 109.56	176.68	100.28	− 0.124
11	+ 1	337.3564	+ 111.63	1.67	250.52	− 0.279	45	2096	128.5573	+ 15.37	136.13	60.72	− 0.046
12	59	296.7615	− 123.67	321.08	210.93	− 0.151	46	2154	87.9579	+ 140.32	95.60	21.15	+ 0.031
13	117	256.1664	+ 1.04	280.53	171.35	− 0.024	47	2212	47.3584	− 94.73	55.03	341.58	+ 0.106
14	175	215.5712	+ 125.75	239.97	131.78	+ 0.102	48	2270	6.7587	+ 30.22	14.43	302.00	+ 0.180
15	251	185.7981	− 112.41	209.93	103.03	− 0.239	49	2327	331.4091	+ 155.19	333.87	262.43	+ 0.252
16	309	145.2026	+ 12.31	169.40	63.47	− 0.116	50	2331	258.3466	− 71.42	261.83	190.45	+ 1.763
17	367	104.6069	+ 137.04	128.85	23.90	+ 0.005	51	2335	185.2842	+ 61.98	189.79	118.47	+ 3.275
18	425	64.0112	− 98.21	88.28	344.30	+ 0.125	52	2339	112.2218	− 164.62	117.75	46.50	+ 4.787
19	501	34.2375	+ 23.66	58.23	315.55	− 0.223	53	2343	39.1593	− 31.22	45.70	334.52	+ 6.298
20	558	358.8915	+ 148.41	17.65	275.97	− 0.106	54	2346	331.3468	+ 102.18	333.65	262.53	+ 7.809
21	616	318.2953	− 86.83	337.10	236.40	+ 0.008	55	2350	258.2844	− 124.42	261.60	190.55	+ 9.321
22	674	277.6990	+ 37.94	296.53	196.82	+ 0.122	56	2354	185.2220	+ 8.98	189.55	118.57	+10.832
23	750	247.9246	+ 159.85	266.47	168.05	− 0.233	57	2358	112.1596	+ 142.38	117.50	46.58	+12.344
24	808	207.3282	− 75.37	225.93	128.50	− 0.123	58	2362	39.0972	− 84.22	45.45	334.59	+13.856

The second and third columns of Tables V and VI give, with some exceptions noted on page 18, the dates of those mean new moons which occur nearest to the nodes. The four columns following give the values of the four principal arguments for these dates.

The number of the conjunction point at which an eclipse occurs may be found from the results of Tables I, II, and IV, as follows:—Put t for the fraction of a century which has elapsed since the last preceding passage of a conjunction point through the corresponding node, as found on p. 14, and n_0 for the number of that point. Then, the quantity

$$n_0 + 1\tfrac{1}{3}\,t - 0.64\ \mathrm{D}$$

be nearly an integer, which integer will be the number of the point sought, and the argument for Table V or VI.

Table VI.—*Mean Elements of Eclipses at Descending Node*

Conj. Point.	T Fictitious Julian Date of Central Mean Conjunction.		g Moon's Mean Anomaly	g' Sun's Mean Anomaly.	L Sun's Mean Longitude.	$u_0 - 180°$ Moon's Mean Arg. Lat.	Conj. Point.	T Fictitious Julian Date of Central Mean Conjunction.		g Moon's Mean Anomaly	g' Sun's Mean Anomaly.	L Sun's Mean Longitude.	$u_0 - 180°$ Moon's Mean Arg. Lat.
	y.	d.	°	°	°	°		y.	d.	°	°	°	°
102	− 771	+322.7073	− 170.66	354.43	230.23	−13.665	136	837	+187.0300	+ 167.03	205.65	108.72	− 0.068
103	− 767	249.6452	− 37.28	282.40	158.26	−12.150	137	895	146.4333	− 68.17	165.07	69.12	+ 0.040
104	− 763	176.5832	+ 96.09	210.36	86.28	−10.635	138	953	105.8365	+ 56.63	124.53	29.57	+ 0.147
105	− 759	103.5212	− 130.53	138.30	14.30	− 9.119	139	1011	65.2396	− 178.56	84.00	350.00	+ 0.252
106	− 755	30.4592	+ 2.84	66.25	302.31	− 7.602	140	1069	24.6425	− 53.75	43.43	310.43	+ 0.357
107	− 752	322.6471	+ 136.22	354.21	230.33	− 6.086	141	1144	360.1170	+ 68.22	13.37	281.65	− 0.011
108	− 748	249.5851	− 90.41	282.17	158.35	− 4.569	142	1202	349.5197	− 166.95	332.82	242.10	+ 0.090
109	− 744	176.5231	+ 42.97	210.12	86.36	− 3.052	143	1260	278.9222	− 42.12	292.27	202.55	+ 0.189
110	− 740	103.4611	+ 176.34	138.08	14.38	− 1.535	144	1336	219.1462	+ 79.88	262.17	173.75	− 0.186
111	− 736	30.3990	− 50.28	66.03	302.40	− 0.018	145	1394	208.5485	− 155.27	221.58	134.17	− 0.090
112	− 679	355.0555	+ 74.33	25.45	262.82	+ 0.127	146	1452	167.9506	− 30.41	181.03	94.58	+ 0.005
113	− 603	325.2849	− 163.95	355.37	234.02	− 0.188	147	1510	127.3527	+ 94.46	140.48	55.02	+ 0.098
114	− 545	284.6908	− 39.32	314.82	194.43	− 0.046	148	1568	86.7546	− 140.67	99.90	15.43	+ 0.190
115	− 487	244.0970	+ 85.32	274.30	154.90	+ 0.095	149	1626	46.1564	− 15.79	59.38	335.90	+ 0.280
116	− 411	214.3254	− 152.94	244.25	126.13	− 0.225	150	1702	16.3792	+ 106.26	29.32	307.13	− 0.106
117	− 353	173.7314	− 28.29	203.70	86.57	− 0.087	151	1759	341.0306	− 128.85	348.73	267.55	− 0.019
118	− 295	133.1372	+ 96.37	163.15	47.00	+ 0.049	152	1817	300.4319	− 3.95	308.17	227.97	+ 0.066
119	− 237	92.5428	− 138.96	122.58	7.42	+ 0.184	153	1875	259.8331	+ 120.96	267.60	188.40	+ 0.150
120	− 161	62.7708	− 17.19	92.53	338.65	− 0.144	154	1933	219.2342	− 114.12	227.03	148.83	+ 0.233
121	− 103	22.1762	+ 107.49	51.98	299.07	− 0.012	155	2009	189.4562	+ 7.98	196.98	120.08	− 0.164
122	− 46	346.8314	− 127.82	11.45	259.50	+ 0.119	156	2067	148.8570	+ 132.91	156.42	80.50	− 0.085
123	+ 30	317.0589	− 6.02	341.37	230.72	− 0.215	157	2125	108.2576	− 102.15	115.87	40.93	− 0.008
124	88	276.4639	+ 118.68	300.83	191.17	− 0.087	158	2183	67.6582	+ 22.79	75.32	1.37	+ 0.068
125	146	235.8687	− 116.61	260.27	151.58	+ 0.039	159	2241	27.0586	+ 147.74	34.73	321.78	+ 0.142
126	204	195.2736	+ 8.11	219.70	112.02	+ 0.164	160	2298	351.7090	− 87.30	354.18	282.26	+ 0.214
127	280	165.5003	+ 129.95	189.67	83.25	− 0.177	161	2302	278.6465	+ 46.10	282.12	210.25	+ 1.727
128	338	124.9047	− 105.32	149.10	43.67	− 0.055	162	2306	205.5841	+ 179.50	210.07	138.27	+ 3.239
129	396	84.3091	+ 19.42	108.55	4.10	+ 0.065	163	2310	132.5217	− 47.10	138.02	66.28	+ 4.750
130	454	43.7133	+ 144.16	68.00	324.52	+ 0.184	164	2314	59.4592	+ 86.29	65.97	354.28	+ 6.263
131	530	13.9394	− 93.97	37.93	295.75	− 0.165	165	2317	351.6468	− 140.31	353.92	282.30	+ 7.773
132	587	338.5934	+ 30.79	357.38	256.18	− 0.049	166	2321	278.5844	− 6.91	281.85	210.32	+ 9.287
133	645	297.9972	+ 155.56	316.83	216.62	+ 0.066	167	2325	205.5219	+ 126.49	209.80	138.34	+10.800
134	703	257.4015	− 79.66	276.25	177.03	+ 0.179	168	2329	132.4595	− 100.11	137.74	66.35	+12.312
135	761	216.8046	+ 45.11	235.68	137.47	+ 0.290	169	2333	59.3971	+ 33.28	65.69	354.37	+13.823

In Table VI the values of u_0 are given in the last column as if counted from the descending node, but, to make the notation uniform, these are considered as values of $u_0 - 180°$, u_0 being always counted from the ascending node.

TABLE VII.—*Reduction of Quantities in the Preceding Tables to Corresponding Conjunctions in Other Cycles.*

Cycles.	Reduction of T. Year 0.		1000.	2000.	Change for— 100 y.	200 y.	300 y.	400 y.	500 y.	Reduction of g. Year 0.	1000.	2000.	Change for— 100 y.	200 y.	300 y.	400 y.	500 y.
	y.	d.	d.	d.						°	°	°					
1	18	10.8224	10.8217	10.8211	1	1	2	2	3	− 2.89	− 2.86	− 2.82	0	1	1	2	2
2	36	21.6447	21.6435	21.6422	1	3	4	5	6	5.78	5.72	5.64	1	1	2	3	4
3	54	32.4671	32.4652	32.4633	2	4	6	8	9	8.68	8.58	8.47	1	2	3	4	6
4	72	43.2895	43.2869	43.2844	3	5	8	10	13	11.58	11.43	11.29	1	3	4	6	7
5	90	54.1119	54.1086	54.1055	3	6	10	13	16	14.47	14.29	14.12	2	4	5	7	9
6	108	64.9342	64.9304	64.9265	4	8	11	15	19	− 17.36	− 17.15	− 16.94	2	4	6	9	11
7	126	75.7566	75.7521	75.7476	4	9	13	18	22	20.26	20.01	19.76	3	5	8	10	12
8	144	86.5790	86.5738	86.5687	5	10	15	20	26	23.15	22.87	22.58	3	6	8	11	14
9	162	97.4013	97.3956	97.3898	6	12	17	23	29	26.05	25.73	25.41	3	6	9	13	16
10	180	108.2237	108.2173	108.2109	6	13	19	26	32	28.94	28.59	28.23	4	7	11	14	18
11	198	119.0461	119.0390	119.0320	7	14	21	28	35	− 31.83	− 31.44	− 31.05	4	8	11	16	19
12	216	129.8684	129.8608	129.8531	8	15	23	31	38	34.73	34.30	33.88	4	8	13	17	21
13	234	140.6908	140.6825	140.6742	8	17	25	33	42	37.62	37.16	36.70	5	9	14	19	23
14	252	151.5132	151.5042	151.4953	9	18	27	36	45	40.52	40.02	39.52	5	10	15	20	24
15	270	162.3356	162.3259	162.3164	10	19	29	38	48	43.41	42.88	42.35	5	11	16	22	26
16	288	173.1579	173.1477	173.1374	10	21	31	41	51	− 46.30	− 45.74	− 45.17	6	11	17	23	28
17	306	183.9803	183.9694	183.9585	11	22	33	44	55	49.20	48.60	47.99	6	12	18	24	30
18	324	194.8027	194.7911	194.7796	12	23	35	46	58	52.09	51.45	50.81	6	13	19	26	31
19	342	205.6250	205.6129	205.6007	12	24	37	49	61	54.99	54.31	53.64	7	13	20	27	33
20	360	216.4474	216.4346	216.4218	13	26	38	51	64	57.88	57.17	56.46	7	14	21	29	35
21	378	227.2697	227.2563	227.2429	14	27	40	54	67	− 60.77	− 60.03	− 59.28	7	15	22	30	37
22	396	238.0921	238.0781	238.0640	14	28	42	56	71	63.67	62.89	62.11	8	15	23	32	39
23	414	248.9145	248.8998	248.8851	15	29	44	59	74	66.56	65.75	64.93	8	16	24	33	41
24	432	259.7369	259.7215	259.7062	15	31	46	62	77	69.45	68.61	67.75	8	17	25	34	42
25	450	270.5593	270.5432	270.5273	16	32	48	64	80	72.35	71.46	70.58	9	17	26	36	44
26	468	281.3817	281.3650	281.3483	17	33	50	66	83	− 75.24	− 74.32	− 73.40	9	18	27	37	45
27	486	292.2041	292.1867	292.1694	17	35	52	69	86	78.13	77.18	76.22	10	19	28	38	47
28	504	303.0264	303.0084	302.9905	18	36	54	72	90	81.03	80.04	79.05	10	20	29	39	49
29	522	313.8488	313.8302	313.8116	18	37	56	74	93	83.92	82.90	81.87	10	20	31	41	51
30	540	324.6712	324.6519	324.6327	19	38	58	77	96	86.82	85.76	84.69	11	21	32	42	53
31	558	335.4935	335.4736	335.4538	20	40	59	79	99	− 89.71	− 88.62	− 87.52	11	22	33	44	55
32	576	346.3159	346.2954	346.2749	20	41	61	82	102	92.61	91.47	90.34	11	23	34	45	56
33	594	357.1383	357.1171	357.0960	21	42	63	84	106	95.50	94.33	93.16	12	23	35	47	58
34	613	2.7107	2.6888	2.6671	22	44	65	87	109	98.39	97.19	95.98	12	24	36	48	60
35	631	13.5330	13.5105	13.4882	22	45	67	90	112	101.29	100.05	98.81	12	25	37	50	62
36	649	24.3554	24.3322	24.3092	23	46	69	92	115	−104.18	−102.91	−101.63	13	25	38	51	63
37	667	35.1778	35.1539	35.1303	24	47	71	95	119	107.07	105.77	104.45	13	26	39	52	65

*Having identified the central eclipse of the series to which the required one belongs, the times and arguments from Table V or VI are to educed to the required date, for the number of periods elapsed, by means of Table VII. Here the time is to correspond to the middle of the osed interval. If the eclipse examined precedes the central one in time, the signs of all the quantities are to be changed.

Table VII.—*Reduction of Quantities in the Preceding Tables, etc.*—Continued.

Cycles.	Reduction of g'. Year 0.	Reduction of g'. 2000.	Reduction of L. Year 0.	Reduction of L. 2000.	Reduction of u. Year 0.	Reduction of u. 1000.	Reduction of u. 2000.	Change for — 100 y.	Change for — 200 y.	Change for — 300 y.	Change for — 400 y.	Change for — 500 y.
	°	°	°	°	°	°	°					
1	+ 10.50	+ 10.49	+ 10.80	+ 10.80	− 0.463	− 0.470	− 0.478	1	2	2	3	4
2	20.99	20.99	21.61	21.61	0.926	0.940	0.956	1	3	4	6	7
3	31.49	31.48	32.41	32.41	1.389	1.411	1.433	2	4	7	9	11
4	41.99	41.98	43.21	43.22	1.851	1.881	1.911	3	6	9	12	15
5	52.48	52.47	54.01	54.02	2.314	2.351	2.389	4	7	11	15	19
6	+ 62.98	+ 62.97	+ 64.82	+ 64.82	− 2.777	− 2.822	− 2.867	4	9	13	18	22
7	73.48	73.46	75.62	75.63	3.240	3.292	3.345	5	10	16	21	26
8	83.98	83.96	86.42	86.43	3.703	3.762	3.822	6	12	18	24	30
9	94.48	94.45	97.23	97.24	4.166	4.233	4.300	7	13	20	27	34
10	104.97	104.94	108.03	108.04	4.628	4.703	4.778	7	15	22	30	37
11	+115.47	+115.44	+118.83	+118.84	− 5.091	− 5.173	− 5.256	8	16	25	33	41
12	125.96	125.93	129.64	129.65	5.554	5.644	5.734	9	18	27	36	45
13	136.46	136.43	140.44	140.45	6.017	6.114	6.211	10	19	29	39	48
14	146.96	146.92	151.24	151.25	6.480	6.584	6.689	11	21	31	42	52
15	157.45	157.42	162.04	162.06	6.943	7.054	7.167	11	22	34	45	56
16	+167.95	+167.91	+172.85	+172.86	− 7.406	− 7.525	− 7.645	12	24	36	48	60
17	178.45	178.41	183.65	183.67	7.868	7.995	8.123	13	25	38	51	64
18	188.95	188.90	194.45	194.47	8.331	8.465	8.600	13	27	40	54	68
19	199.44	199.39	205.26	205.27	8.794	8.936	9.078	14	28	43	57	71
20	209.94	209.89	216.06	216.08	9.257	9.406	9.556	15	30	45	60	74
21	+220.44	+220.38	+226.86	+226.88	− 9.720	− 9.876	−10.034	16	31	47	63	78
22	230.94	230.88	237.67	237.68	10.183	10.347	10.512	16	33	49	66	82
23	241.43	241.37	248.47	248.49	10.646	10.817	10.989	17	34	52	69	86
24	251.93	251.87	259.27	259.29	11.108	11.287	11.467	18	36	54	72	90
25	262.43	262.36	270.07	270.10	11.571	11.758	11.945	19	37	56	75	93
26	+272.92	+272.86	+280.88	+280.90	−12.034	−12.228	−12.423	19	39	58	78	97
27	283.42	283.35	291.68	291.70	12.497	12.698	12.901	20	40	61	81	101
28	293.92	293.85	302.48	302.51	12.960	13.168	13.378	21	42	63	84	104
29	304.41	304.34	313.29	313.31	13.422	13.639	13.856	22	43	65	87	108
30	314.91	314.83	324.09	324.12	13.885	14.109	14.334	22	45	67	90	112
31	+325.41	+325.33	+334.89	+334.92	−14.348	−14.579	−14.812	23	46	70	93	116
32	335.91	335.82	345.69	345.72	14.811	15.050	15.290	24	48	72	96	120
33	346.41	346.32	356.50	356.52	15.274	15.520	15.768	25	49	74	99	124
34	356.90	356.81	7.30	7.33	15.736	15.990	16.245	25	51	76	102	127
35	7.39	7.31	18.10	18.13	16.199	16.460	16.722	26	52	79	105	131
36	+ 17.89	+ 17.80	+ 28.90	+ 28.94	−16.662	−16.931	−17.200	27	54	81	108	134
37	28.39	28.29	39.70	39.74	17.125	17.401	17.678	28	55	83	111	138

Having identified the central eclipse of the series to which the required one belongs, the times and arguments from Table V or VI are to be reduced to the required date, for the number of periods elapsed, by means of Table VII. Here the time is to correspond to the middle of the elapsed interval. If the eclipse examined precedes the central one in time, the signs of all the quantities are to be changed.

TABLE VIII, *Arg. g.—For Reduction to Moment of True New Moon.*

$$\delta T = -0^{d}.4089 \sin g + 0^{d}.0161 \sin 2g - 0^{d}.0004 \sin 3g.$$

g	0°	10°	20°	30°	40°	50°	60°	70°	80°	
°	d.	d.	d.	d.	d.	d.	d.	d.	d.	°
0	−.0000 +	−.0657 +	−.1298 +	−.1910 +	−.2473 +	−.2975 +	−.3402 +	−.3736 +	−.3969 +	10
	66	65	63	59	53	46	38	28	17	
1	.0066	.0722	.1361	.1969	.2526	.3021	.3440	.3764	.3986	9
	66	65	62	58	53	46	37	27	15	
2	.0132	.0787	.1423	.2027	.2579	.3067	.3477	.3791	.4001	8
	66	65	63	58	52	45	36	26	15	
3	.0198	.0852	.1486	.2085	.2631	.3112	.3513	.3817	.4016	7
	65	65	61	57	51	44	35	25	13	
4	−.0263 +	−.0917 +	−.1547 +	−.2142 +	−.2682 +	−.3156 +	−.3548 +	.3842 +	−.4029 +	6
	66	64	62	57	51	43	34	24	12	
5	−.0329 +	−.0981 +	−.1609 +	−.2199 +	−.2733 +	−.3199 +	−.3582 +	−.3866 +	−.4041 +	5
	66	64	61	56	50	42	33	23	11	
6	.0395	.1045	.1670	.2255	.2783	.3241	.3615	.3889	.4052	4
	65	64	61	55	49	42	32	22	10	
7	.0460	.1109	.1731	.2310	.2832	.3283	.3647	.3911	.4062	3
	66	63	60	55	48	40	30	20	9	
8	.0526	.1172	.1791	.2365	.2880	.3323	.3677	.3931	.4071	2
	66	63	60	54	48	40	30	20	8	
9	.0592	.1235	.1851	.2419	.2928	.3363	.3707	.3951	.4079	1
	65	63	59	54	47	39	29	18	6	
10	−.0657 +	−.1298 +	−.1910 +	−.2473 +	−.2975 +	−.3402 +	−.3736 +	−.3969 +	−.4085 +	0
	350°	340°	330°	320°	310°	300°	290°	280°	270°	g

g	90°	100°	110°	120°	130°	140°	150°	160°	170°	
°	d.	d.	d.	d.	d.	d.	d.	d.	d.	°
0	−.4085 +	−.4079 +	−.3944 +	−.3680 +	−.3293 +	−.2791 +	−.2188 +	−.1506 +	−.0767 +	10
	5	8	21	33	45	56	65	72	76	
1	.4090	.4071	.3923	.3647	.3248	.2735	.2123	.1434	.0691	9
	3	9	22	35	47	57	66	72	76	
2	.4093	.4062	.3901	.3612	.3201	.2678	.2057	.1362	.0615	8
	3	10	23	36	47	58	67	72	77	
3	.4096	.4052	.3878	.3576	.3154	.2620	.1990	.1290	.0538	7
	1	11	24	37	49	59	67	73	76	
4	−.4097 +	−.4041 +	−.3854 +	−.3539 +	−.3105 +	− 2561 +	−.1923 +	−.1217 +	−.0462 +	6
	0	13	26	38	50	60	68	74	77	
5	−.4097 +	−.4028 +	−.3828 +	−.3501 +	−.3055 +	−.2501 +	−.1855 +	−.1143 +	−.0385 +	5
	1	14	27	39	51	61	69	74	77	
6	.4096	.4014	.3801	.3462	.3004	.2440	.1786	.1069	.0308	4
	2	16	28	41	52	62	69	75	77	
7	.4094	.3998	.3773	.3421	.2952	.2378	.1717	.0994	.0231	3
	4	16	30	41	52	62	70	75	77	
8	.4090	.3982	.3743	.3380	.2900	.2316	.1647	.0919	.0154	2
	5	19	31	43	54	64	70	76	77	
9	.4085	.3963	.3712	.3337	.2846	.2252	.1577	.0843	.0077	1
	6	19	32	44	55	64	71	76	77	
10	−.4079 +	−.3944 +	−.3680 +	−.3293 +	−.2791 +	−.2188 +	−.1506 +	−.0767 +	−.0000 +	0
	260°	250°	240°	230°	220°	210°	200°	190°	180°	g

Table IX, *Arg. g′.—For Reduction to Moment of True New Moon.*

$$\delta T = +\ 0^{d}.1743 \sin g' + 0^{d}.0021 \sin 2g'.$$

g'	0°	10°	20°	30°	40°	50°	60°	70°	80°	
°	*d.*	*d.*	*d.*	*d.*	*d.*	*d.*	*d.*	*d.*	*d.*	°
0	+.0000 −	+.0310 −	+.0609 −	+.0890 −	+.1140 −	+.1356 −	+.1528 −	+.1651 −	+.1724 −	10
	31	31	30	26	25	19	14	10	4	
1	.0031	.0341	.0639	.0916	.1165	.1375	.1542	.1661	.1728	9
	32	30	29	27	22	19	14	9	4	
2	.0063	.0371	.0668	.0943	.1187	.1394	.1556	.1670	.1732	8
	30	30	28	26	23	19	14	9	3	
3	.0093	.0401	.0696	.0969	.1210	1413	.1570	.1679	.1735	7
	31	31	29	25	22	17	13	8	2	
4	+.0124 −	+.0432 −	+.0725 −	+.0994 −	+.1232 −	+.1430 −	+.1583 −	+.1687 −	+.1737 −	6
	32	30	27	25	21	17	13	7	3	
5	+.0156 −	+.0462 −	+.0752 −	+.1019 −	+.1253 −	+.1447 −	+.1596 −	+.1694 −	+.1740 −	5
	31	30	29	26	22	18	12	6	2	
6	.0187	.0492	.0781	.1045	.1275	.1465	.1608	.1700	.1742	4
	30	29	27	24	21	16	12	7	1	
7	.0217	.0521	.0808	.1069	.1296	.1481	.1620	.1707	.1743	3
	31	30	28	24	20	16	10	6	0	
8	.0248	.0551	.0836	.1093	.1316	.1497	.1630	.1713	.1743	2
	31	29	27	24	20	15	11	6	1	
9	.0279	.0580	.0863	.1117	.1336	.1512	.1641	.1719	.1744	1
	31	29	27	23	20	16	10	5	1	
10	+.0310 −	+.0609 −	+.0890 −	+.1140 −	+.1356 −	+.1528 −	+.1651 −	+.1724 −	+.1743 −	0
	350°	340°	330°	320°	310°	300°	290°	280°	270°	g'

g'	90°	100°	110°	120°	130°	140°	150°	160°	170°	
°	*d.*	*d.*	*d.*	*d.*	*d.*	*d.*	*d.*	*d.*	*d.*	°
0	+.1743 −	+.1710 −	+.1625 −	+.1492 −	+.1314 −	+.1100 −	+.0852 −	+.0583 −	+.0296 −	10
	1	7	12	17	20	23	25	29	31	
1	.1742	.1703	.1613	.1475	.1294	.1077	.0827	.0554	.0265	9
	1	6	11	16	20	24	27	27	29	
2	.1741	.1697	.1602	.1459	.1274	.1053	.0800	.0527	.0236	8
	2	8	12	18	20	24	26	28	29	
3	.1739	.1689	.1590	.1441	.1254	.1029	.0774	.0499	.0207	7
	3	7	12	16	21	24	27	29	30	
4	+.1736 −	+.1682 −	+.1578 −	+.1425 −	+.1233 −	+.1005 −	+.0747 −	+.0470 −	+.0177 −	6
	4	8	14	18	22	24	27	29	29	
5	+.1732 −	+.1674 −	+.1564 −	+.1407 −	+.1211 −	+.0981 −	+.0720 −	+.0441 −	+.0148 −	5
	3	9	13	17	21	25	27	29	30	
6	.1729	.1665	.1551	.1390	.1190	.0956	.0693	.0412	.0118	4
	4	10	15	19	22	25	27	29	29	
7	.1725	.1655	.1536	.1371	.1168	.0931	.0666	.0383	.0089	3
	5	9	14	19	23	26	28	28	30	
8	.1720	.1646	.1522	.1352	.1145	.0905	.0638	.0355	.0059	2
	4	11	14	19	22	27	27	30	30	
9	.1716	.1635	.1508	.1333	.1123	.0878	.0611	.0325	.0029	1
	6	10	16	19	23	26	28	29	29	
10	+.1710 −	+.1625 −	+.1492 −	+.1314 −	+.1100 −	+.0852 −	+.0583 −	+.0296 −	+.0000 −	0
	260°	250°	240°	230°	220°	210°	200°	190°	180°	g'

TABLE X.—*Arg.* $g+g'$.

$\delta T = -0^d.0051 \sin (g+g')$.

$g+g'$.	δT.	
°	d.	°
0	0.0000	360
10	− 09 +	350
20	17	340
30	25	330
40	33	320
50	39	310
60	44	300
70	48	290
80	50	280
90	51	270
100	50	260
110	48	250
120	44	240
130	39	230
140	33	220
150	25	210
160	17	200
170	− 09 +	190
180	0.0000	180
		$g+g'$.

TABLE XI.—*Arg.* $g-g'$.

$\delta T = +0^d.0075 \sin (g-g')$.

$g-g'$.	δT.	
°	d.	°
0	0.0000	360
10	+ 13 −	350
20	26	340
30	37	330
40	48	320
50	57	310
60	65	300
70	70	290
80	74	280
90	75	270
100	74	260
110	70	250
120	65	240
130	57	230
140	48	220
150	37	210
160	26	200
170	+ 13 −	190
180	0.0000	180
		$g+g'$.

TABLE XII.—*Arg.* u.

$\delta T = +0^d.0104 \sin 2u$.

u.	δT.
°	d.
0	0.0000
1	+ 04
2	07
3	11
4	14
5	18
6	22
7	25
8	29
9	32
10	36
11	39
12	42
13	46
14	49
15	52
16	55
17	58
18	61
19	+ 0.0064

TABLE XIII *a*.

Day of the Year to Day of the Month.

		Common Year.	Bissext. Year.
Jan.	0	0	− 1
	10	10	9
	20	20	19
Feb.	0	31	30
	10	41	40
	20	51	50
Mar.	0	59	
	10	69	
	20	79	
April	0	90	
	10	100	
	20	110	
May	0	120	
	10	130	
	20	140	
June	0	151	
	10	161	
	20	171	
July	0	181	
	10	191	
	20	201	
Aug.	0	212	
	10	222	
	20	232	
Sept.	0	243	
	10	253	
	20	263	
Oct.	0	273	
	10	283	
	20	293	
Nov.	0	304	
	10	314	
	20	324	
Dec.	0	334	
	10	344	
	20	354	

The sum of the quantities from Tables VIII to XII, inclusive, being applied to the time T of mean conjunction of the sun and moon in longitude, will give the Greenwich fictitious mean time of true conjunction in longitude.

To reduce the fictitious time thus found to the ordinary calendar, a correction from the table following is to be applied. The correction for bissextile years presupposes that January 1 is counted as the zero day of the year. In order that it may correspond to the civil count of days, it must be increased by 1.

TABLE XIII *b*.—*For Reducing Fictitious Julian Dates to those of the Ordinary Calendar.*

Calendar and Limiting Dates.	Bissextile Years.	Year 1 after Bis.	Year 2 after Bis.	Year 3 after Bis.
	d.	d.	d.	d.
Julian calendar	+ 0.00	+ 0.25	+ 0.50	+ 0.75
Gregorian calendar, 1582 to 1700, February	10.00	10.25	10.50	10.75
Gregorian calendar, 1700, March, to 1800, February	11.00	11.25	11.50	11.75
Gregorian calendar, 1800, March, to 1900, February	12.00	12.25	12.50	12.75
Gregorian calendar, 1900, March, to 2100, February	13.00	13.25	13.50	13.75
Gregorian calendar, 2100, March, to 2200, February	14.00	14.25	14.50	14.75
Gregorian calendar, 2200, March, to 2300, February	+ 15.00	+ 15.25	+ 15.50	+ 15.75

For the further expression of the time in days and hours, Tables XIII *a* and XIV are added.

TABLE XIV.—*For Changing Decimals of a Day to Time and Arc.*

T.	Time.	Arc.	For $\frac{T}{100}$. Time.	For $\frac{T}{100}$. Arc.	For $\frac{T}{10000}$. Time.	For $\frac{T}{10000}$. Arc.	T.	Time.	Arc.	For $\frac{T}{100}$. Time.	For $\frac{T}{100}$. Arc.	For $\frac{T}{10000}$. Time.	For $\frac{T}{10000}$. Arc.
d.	h. m. s.	° ′	m. s.	° ′	s.	′	d.	h. m. s.	° ′	m. s.	° ′	s.	′
0.01	0 14 24	3 36	0 8.64	0 2.16	0.09	0.02	0.51	12 14 24	183 36	7 20.64	1 50.16	4.41	1.10
0.02	0 28 48	7 12	0 17.28	0 4.32	0.17	0.04	0.52	12 28 48	187 12	7 29.28	1 52.32	4.49	1.12
0.03	0 43 12	10 48	0 25.92	0 6.48	0.26	0.06	0.53	12 43 12	190 48	7 37.92	1 54.48	4.58	1.14
0.04	0 57 36	14 24	0 34.56	0 8.64	0.35	0.09	0.54	12 57 36	194 24	7 46.56	1 56.64	4.67	1.17
0.05	1 12 0	18 0	0 43.20	0 10.80	0.43	0.11	0.55	13 12 0	198 0	7 55.20	1 58.80	4.75	1.19
0.06	1 26 24	21 36	0 51.84	0 12.96	0.52	0.13	0.56	13 26 24	201 36	8 3.84	2 0.96	4.84	1.21
0.07	1 40 48	25 12	1 0.48	0 15.12	0.60	0.15	0.57	13 40 48	205 12	8 12.48	2 3.12	4.92	1.23
0.08	1 55 12	28 48	1 9.12	0 17.28	0.69	0.17	0.58	13 55 12	208 48	8 21.12	2 5.28	5.01	1.25
0.09	2 9 36	32 24	1 17.76	0 19.44	0.78	0.19	0.59	14 9 36	212 24	8 29.76	2 7.44	5.10	1.27
0.10	2 24 0	36 0	1 26.40	0 21.60	0.86	0.22	0.60	14 24 0	216 0	8 38.40	2 9.60	5.18	1.30
0.11	2 38 24	39 36	1 35.04	0 23.76	0.95	0.24	0.61	14 38 24	219 36	8 47.04	2 11.76	5.27	1.32
0.12	2 52 48	43 12	1 43.68	0 25.92	1.04	0.26	0.62	14 52 48	223 12	8 55.68	2 13.92	5.36	1.34
0.13	3 7 12	46 48	1 52.32	0 28.08	1.12	0.28	0.63	15 7 12	226 48	9 4.32	2 16.08	5.44	1.36
0.14	3 21 36	50 24	2 0.96	0 30.24	1.21	0.30	0.64	15 21 36	230 24	9 12.96	2 18.24	5.53	1.38
0.15	3 36 0	54 0	2 9.60	0 32.40	1.30	0.32	0.65	15 36 0	234 0	9 21.60	2 20.40	5.62	1.40
0.16	3 50 24	57 36	2 18.24	0 34.56	1.38	0.35	0.66	15 50 24	237 36	9 30.24	2 22.56	5.70	1.43
0.17	4 4 48	61 12	2 26.88	0 36.72	1.47	0.37	0.67	16 4 48	241 12	9 38.88	2 24.72	5.79	1.45
0.18	4 19 12	64 48	2 35.52	0 38.88	1.56	0.39	0.68	16 19 12	244 48	9 47.52	2 26.88	5.88	1.47
0.19	4 33 36	68 24	2 44.16	0 41.04	1.64	0.41	0.69	16 33 36	248 24	9 56.16	2 29.04	5.96	1.49
0.20	4 48 0	72 0	2 52.80	0 43.20	1.73	0.43	0.70	16 48 0	252 0	10 4.80	2 31.20	6.05	1.51
0.21	5 2 24	75 36	3 1.44	0 45.36	1.81	0.45	0.71	17 2 24	255 36	10 13.44	2 33.36	6.13	1.53
0.22	5 16 48	79 12	3 10.08	0 47.52	1.90	0.48	0.72	17 16 48	259 12	10 22.08	2 35.52	6.22	1.56
0.23	5 31 12	82 48	3 18.72	0 49.68	1.99	0.50	0.73	17 31 12	262 48	10 30.72	2 37.68	6.31	1.58
0.24	5 45 36	86 24	3 27.36	0 51.84	2.07	0.52	0.74	17 45 36	266 24	10 39.36	2 39.84	6.39	1.60
0.25	6 0 0	90 0	3 36.00	0 54.00	2.16	0.54	0.75	18 0 0	270 0	10 48.00	2 42.00	6.48	1.62
0.26	6 14 24	93 36	3 44.64	0 56.16	2.25	0.56	0.76	18 14 24	273 36	10 56.64	2 44.16	6.57	1.64
0.27	6 28 48	97 12	3 53.28	0 58.32	2.33	0.58	0.77	18 28 48	277 12	11 5.28	2 46.32	6.65	1.66
0.28	6 43 12	100 48	4 1.92	1 0.48	2.42	0.60	0.78	18 43 12	280 48	11 13.92	2 48.48	6.74	1.68
0.29	6 57 36	104 24	4 10.56	1 2.64	2.51	0.63	0.79	18 57 36	284 24	11 22.56	2 50.64	6.83	1.71
0.30	7 12 0	108 0	4 19.20	1 4.80	2.59	0.65	0.80	19 12 0	288 0	11 31.20	2 52.80	6.91	1.73
0.31	7 26 24	111 36	4 27.84	1 6.96	2.68	0.67	0.81	19 26 24	291 36	11 39.84	2 54.96	7.00	1.75
0.32	7 40 48	115 12	4 36.48	1 9.12	2.76	0.69	0.82	19 40 48	295 12	11 48.48	2 57.12	7.08	1.77
0.33	7 55 12	118 48	4 45.12	1 11.28	2.85	0.71	0.83	19 55 12	298 48	11 57.12	2 59.28	7.17	1.79
0.34	8 9 36	122 24	4 53.76	1 13.44	2.94	0.73	0.84	20 9 36	302 24	12 5.76	3 1.44	7.26	1.81
0.35	8 24 0	126 0	5 2.40	1 15.60	3.02	0.76	0.85	20 24 0	306 0	12 14.40	3 3.60	7.34	1.84
0.36	8 38 24	129 36	5 11.04	1 17.76	3.11	0.78	0.86	20 38 24	309 36	12 23.04	3 5.76	7.43	1.86
0.37	8 52 48	133 12	5 19.68	1 19.92	3.20	0.80	0.87	20 52 48	313 12	12 31.68	3 7.92	7.52	1.88
0.38	9 7 12	136 48	5 28.32	1 22.08	3.28	0.82	0.88	21 7 12	316 48	12 40.32	3 10.08	7.60	1.90
0.39	9 21 36	140 24	5 36.96	1 24.24	3.37	0.84	0.89	21 21 36	320 24	12 48.96	3 12.24	7.69	1.92
0.40	9 36 0	144 0	5 45.60	1 26.40	3.46	0.86	0.90	21 36 0	324 0	12 57.60	3 14.40	7.78	1.94
0.41	9 50 24	147 36	5 54.24	1 28.56	3.54	0.89	0.91	21 50 24	327 36	13 6.24	3 16.56	7.86	1.97
0.42	10 4 48	151 12	6 2.88	1 30.72	3.63	0.91	0.92	22 4 48	331 12	13 14.88	3 18.72	7.95	1.99
0.43	10 19 12	154 48	6 11.52	1 32.88	3.72	0.93	0.93	22 19 12	334 48	13 23.52	3 20.88	8.04	2.01
0.44	10 33 36	158 24	6 20.16	1 35.04	3.80	0.95	0.94	22 33 36	338 24	13 32.16	3 23.04	8.12	2.03
0.45	10 48 0	162 0	6 28.80	1 37.20	3.89	0.97	0.95	22 48 0	342 0	13 40.80	3 25.20	8.21	2.05
0.46	11 2 24	165 36	6 37.44	1 39.36	3.97	0.99	0.96	23 2 24	345 36	13 49.44	3 27.36	8.29	2.07
0.47	11 16 48	169 12	6 46.08	1 41.52	4.06	1.02	0.97	23 16 48	349 12	13 58.08	3 29.52	8.38	2.10
0.48	11 31 12	172 48	6 54.72	1 43.68	4.15	1.04	0.98	23 31 12	352 48	14 6.72	3 31.68	8.47	2.12
0.49	11 45 36	176 24	7 3.36	1 45.84	4.23	1.06	0.99	23 45 36	356 24	14 15.36	3 33.84	8.55	2.14
0.50	12 0 0	180 0	7 12.00	1 48.00	4.32	1.08	1.00	24 0 0	360 0	14 24.00	3 36.00	8.64	2.16

TABLE XV, *Arg. g.—Values of* $u_1 - u_0$.

$$u_1 - u_0 = -0^\circ.403 \sin g + 0^\circ.016 \sin 2g.$$

g	0°	10°	20°	30°	40°	50°	60°	70°	80°	
°	°	°	°	°	°	°	°	°	°	°
0	−.000+	−.064+	−.128+	−.188+	−.243+	−.293+	−.335+	−.368+	−.391+	10
1	.006	.071	.134	.193	.249	.297	.339	.371	.393	9
2	.013	.077	.140	.199	.254	.302	.343	.374	.395	8
3	.019	.084	.146	.205	.259	.306	.346	.376	.396	7
4	−.026+	−.090+	−.152+	−.211+	−.264+	−.311+	−.349+	−.379+	−.397+	6
5	−.032+	−.096+	−.158+	−.216+	−.269+	−.315+	−.353+	−.381+	−.399+	5
6	.039	.103	.164	.222	.274	.319	.356	.383	.400	4
7	.045	.109	.170	.227	.279	.323	.359	.386	.401	3
8	.052	.115	.176	.233	.283	.327	.362	.388	.402	2
9	.058	.121	.182	.238	.288	.331	.365	.390	.402	1
10	−.064+	−.128+	−.188+	−.243+	−.293+	−.335+	−.368+	−.391+	−.403+	0
	350°	340°	330°	320°	310°	300°	290°	280°	270°	g

g	90°	100°	110°	120°	130°	140°	150°	160°	170°	
°	°	°	°	°	°	°	°	°	°	°
0	−.403+	−.402+	−.389+	−.363+	−.324+	−.275+	−.215+	−.148+	−.075+	10
1	.403	.402	.387	.360	.320	.269	.209	.141	.068	9
2	.404	.401	.385	.356	.316	.264	.202	.134	.060	8
3	.404	.400	.383	.352	.311	.258	.196	.127	.053	7
4	−.404+	−.399+	−.380+	−.348+	−.306+	−.252+	−.189+	−.120+	−.045+	6
5	−.404+	−.397+	−.377+	−.344+	−.301+	−.246+	−.182+	−.112+	−.038+	5
6	.404	.396	.375	.340	.296	.240	.176	.105	.030	4
7	.404	.394	.372	.336	.291	.234	.169	.097	.023	3
8	.403	.393	.369	.332	.286	.228	.162	.090	.015	2
9	.403	.391	.366	.328	.281	.222	.155	.083	.008	1
10	−.402+	−.389+	−.363+	−.324+	−.275+	−.215+	−.148+	−.075+	−.000+	0
	260°	250°	240°	230°	220°	210°	200°	190°	180°	g

TABLE XVI.—*Arg. g′*.

$$u_1 - u_0 = +\ 2^\circ.094 \sin g' + 0^\circ.027 \sin 2g'.$$

g'	0°	10°	20°	30°	40°	50°	60°	70°	80°	
°	°	°	°	°	°	°	°	°	°	°
0	0.000	+0.373	+0.734	+1.070	+1.373	+1.631	+1.837	+1.985	+2.072	10
	38	37	35	32	28	23	17	12	5	
1	+0.038	0.410	0.769	1.102	1.401	1.654	1.854	1.997	2.077	9
	37	36	34	32	27	22	17	10	4	
2	0.075	0.446	0.803	1.134	1.428	1.676	1.871	2.007	2.081	8
	37	37	35	31	27	22	17	11	4	
3	0.112	0.483	0.838	1.165	1.455	1.698	1.888	2.018	2.085	7
	38	36	34	31	27	22	15	9	3	
4	+0.150	+0.519	+0.872	+1.196	+1.482	+1.720	+1.903	+2.027	+2.088	6
	37	36	34	30	26	21	15	9	3	
5	+0.187	+0.555	+0.906	+1.226	+1.508	+1.741	+1.918	+2.036	+2.091	5
	37	36	33	30	25	20	15	8	2	
6	0.224	0.591	0.939	1.256	1.533	1.761	1.933	2.044	2.093	4
	38	36	33	30	25	20	14	8	1	
7	0.262	0.627	0.972	1.286	1.558	1.781	1.947	2.052	2.094	3
	37	36	33	29	25	19	13	7	0	
8	0.299	0.663	1.005	1.315	1.583	1.800	1.960	2.059	2.094	2
	37	35	33	29	24	19	13	7	1	
9	0.336	0.698	1.038	1.344	1.607	1.819	1.973	2.066	2.095	1
	37	36	32	29	24	18	12	6	1	
10	+0.373	+0.734	+1.070	+1.373	+1.631	+1.837	+1.985	+2.072	+2.094	0
	350°	340°	330°	320°	310°	300°	290°	280°	270°	g'

g'	90°	100°	110°	120°	130°	140°	150°	160°	170°	
°	°	°	°	°	°	°	°	°	°	°
0	+2.094	+2.053	+1.950	+1.790	+1.578	+1.319	+1.024	+0.699	+0.354	10
	1	8	13	19	24	28	32	34	35	
1	2.0 3	2.045	1.937	1.771	1.554	1.291	0.992	0.665	0.319	9
	2	8	14	20	25	28	32	34	35	
2	2.091	2.037	1.923	1.751	1.529	1.263	0.960	0.631	0.284	8
	3	9	15	20	25	29	31	34	35	
3	2.088	2.028	1.908	1.731	1.504	1.234	0.929	0.597	0.249	7
	3	9	15	20	25	29	32	34	36	
4	+2.085	+2.019	+1.893	+1.711	+1.479	+1.205	+0.897	+0.563	+0.213	6
	4	10	16	21	25	29	33	35	35	
5	+2.081	+2.009	+1.877	+1.690	+1.454	+1.176	+0.864	+0.528	+0.178	5
	4	10	16	22	26	30	32	34	36	
6	2.077	1.999	1.861	1.668	1.428	1.146	0.832	0.494	0.142	4
	5	12	17	22	27	30	33	35	35	
7	2.072	1.987	1.844	1.646	1.401	1.116	0.799	0.459	0.107	3
	6	11	17	22	27	30	33	35	36	
8	2.066	1.976	1.827	1.624	1.374	1.086	0.766	0.424	0.071	2
	6	13	18	23	27	31	34	34	35	
9	2.060	1.963	1.809	1.601	1.347	1.055	0.732	0.390	+ 0.036	1
	7	13	19	23	28	31	33	36	36	
10	+2.053	+1.950	+1.790	+1.578	+1.319	+1.024	+0.699	+0.354	0.000	0
	260°	250°	240°	230°	220°	210°	200°	190°	180°	g'

The sum of the three numbers from Tables XV–XVII is the reduction from the mean argument of latitude at mean conjunction to true argument at true ecliptic conjunction, measured on the ecliptic.

TABLE XVII.—*Arg.* $(g + g')$.

$u_1 - u_0 = - 0^\circ.012 \sin (g + g')$.

$g+g'$			$g+g'$		
°	°	°	°	°	°
0	0.000	360	90	− .012 +	270
10	− .002 +	350	100	.012	260
20	.004	340	110	.011	250
30	.006	330	120	.010	240
40	.008	320	130	.009	230
50	.009	310	140	.008	220
60	.010	300	150	.006	210
70	.011	290	160	.004	200
80	.012	280	170	− .002 +	190
90	− .012 +	270	180	0.000	180
		$g+g'$			$g+g'$

For Values of y_2 at the Moment of Ecliptic Conjunction (y_2°).

TABLE XVIII.—*Arg.* g.

$y_2^\circ = - .0006 \sin g + .0091 \sin 2g$ (near ascending node).
$y_2^\circ = + .0006 \sin g - .0091 \sin 2g$ (near descending node).

g			g		
°		°	°		°
0	.000	360	90	− .001 +	270
10	+ .003 −	350	100	.004	260
20	.006	340	110	.006	250
30	+ .008 −	330	120	− .008 +	240
40	.009	320	130	.010	230
50	.008	310	140	.009	220
60	+ .007 −	300	150	− .008 +	210
70	.005	290	160	.006	200
80	+ .002 −	280	170	− .003 +	190
90	− .001 +	270	180	.000	180
		g			g

TABLE XIX.—*Arg.* g'.

$y_2^\circ = + .0163 \sin g'$ (near ascending node).
$y_2^\circ = - .0163 \sin g'$ (near descending node).

g'			g'	
°	°		°	°
180	0	.000	180	360
170	10	+ .003 −	190	350
160	20	.006	200	340
150	30	+ .008 −	210	330
140	40	.011	220	320
130	50	.012	230	310
120	60	+ .014 −	240	300
110	70	.015	250	290
100	80	.016	260	280
90	90	+ .016 −	270	270

In Tables XVIII and XIX, the numbers have the sign given with them near the ascending node, and the opposite sign near the descending node.

The algebraic sum of the numbers taken from the three tables, XVIII to XX, is the value of y_2, the ordinate of the point in which the axis of the shadow intersects the fundamental plane, at the moment of true ecliptic conjunction. This ordinate is reckoned in a direction perpendicular to the ecliptic.

In Table XX, the algebraic sign of the numbers is the same as that of $\sin u_1$. Near the descending node u_1 differs little from 180°; hence near the ascending node the number from Table XX has the same sign as u_1; near the descending node the opposite sign of $u_1 - 180^\circ$.

TABLE XX.—*Hor. Arg.*, g; *Vertical Arg.*, u_1.

$$y_3^\circ = +(5.245 - 0.330 \cos g) \sin u_1.$$

u_1 (°)	g 0° / 360°	Δ	10° / 350°	Δ	20° / 340°	Δ	30° / 330°	Δ	40° / 320°	Δ	50° / 310°	Δ	60° / 300°	Δ	70° / 290°	Δ	80° / 280°	Δ	90° / 270°	Δ
0	0.000	86	0.000	86	0.000	86	0.000	86	0.000	87	0.000	88	0.000	89	0.000	90	0.000	91	0.000	92
1	0.086	86	0.086	86	0.086	87	0.086	88	0.087	88	0.088	88	0.089	89	0.090	90	0.091	90	0.092	91
2	0.172	85	0.172	85	0.173	85	0.174	86	0.175	86	0.176	87	0.178	88	0.180	89	0.181	91	0.183	92
3	0.257	86	0.257	86	0.258	86	0.260	86	0.261	87	0.263	88	0.266	88	0.269	89	0.272	90	0.275	91
4	0.343	85	0.343	86	0.344	86	0.346	86	0.348	87	0.351	88	0.354	89	0.358	89	0.362	90	0.366	91
5	0.428	85	0.429	85	0.430	86	0.432	86	0.435	87	0.439	87	0.443	88	0.447	89	0.452	90	0.457	91
6	0.513	86	0.514	86	0.516	85	0.518	86	0.522	87	0.526	87	0.531	88	0.536	89	0.542	90	0.548	91
7	0.599	85	0.600	85	0.601	86	0.604	86	0.609	86	0.613	87	0.619	88	0.625	89	0.632	90	0.639	91
8	0.684	85	0.685	85	0.687	85	0.690	86	0.695	86	0.700	87	0.707	88	0.714	89	0.722	90	0.730	91
9	0.769	84	0.770	84	0.772	85	0.776	85	0.781	86	0.787	87	0.795	87	0.803	88	0.812	89	0.821	90
10	0.853	85	0.854	85	0.857	85	0.861	85	0.867	86	0.874	86	0.882	87	0.891	88	0.901	89	0.911	90
11	0.938	84	0.939	84	0.942	84	0.946	85	0.953	85	0.960	86	0.969	87	0.979	88	0.990	89	1.001	89
12	1.022	84	1.023	84	1.026	84	1.031	85	1.038	85	1.046	86	1.056	87	1.067	88	1.079	88	1.090	90
13	1.106	83	1.107	83	1.110	84	1.116	84	1.123	85	1.132	85	1.143	86	1.155	87	1.167	88	1.180	89
14	1.189	83	1.190	83	1.194	83	1.200	84	1.208	84	1.217	85	1.229	86	1.242	86	1.255	88	1.269	89
15	1.272	83	1.273	83	1.277	83	1.284	83	1.292	84	1.302	85	1.315	85	1.328	87	1.343	87	1.358	88
16	1.355	82	1.356	82	1.360	83	1.367	83	1.376	84	1.387	84	1.400	85	1.415	86	1.430	87	1.446	88
17	1.437	82	1.438	82	1.443	82	1.450	83	1.460	83	1.471	84	1.485	85	1.501	85	1.517	86	1.534	87
18	1.519		1.520		1.525		1.533		1.543		1.555		1.570		1.586		1.603		1.621	

u_1 (°)	g 90° / 270°	Δ	100° / 260°	Δ	110° / 250°	Δ	120° / 240°	Δ	130° / 230°	Δ	140° / 220°	Δ	150° / 210°	Δ	160° / 200°	Δ	170° / 190°	Δ	180° / 180°	Δ
0	0.000	92	0.000	92	0.000	93	0.000	94	0.000	95	0.000	96	0.000	96	0.000	97	0.000	97	0.000	97
1	0.092	91	0.092	93	0.093	94	0.094	95	0.095	96	0.096	96	0.096	97	0.097	97	0.097	98	0.097	98
2	0.183	92	0.185	92	0.187	93	0.189	94	0.191	95	0.192	95	0.193	96	0.194	97	0.195	96	0.195	97
3	0.275	91	0.277	93	0.280	94	0.283	94	0.286	95	0.287	96	0.289	97	0.291	97	0.291	98	0.292	97
4	0.366	91	0.370	92	0.374	93	0.377	95	0.381	95	0.383	96	0.386	96	0.388	96	0.389	96	0.389	97
5	0.457	91	0.462	92	0.467	93	0.472	93	0.476	94	0.479	95	0.482	96	0.484	97	0.485	97	0.486	97
6	0.548	91	0.554	92	0.560	93	0.565	94	0.570	95	0.574	96	0.578	96	0.581	96	0.582	97	0.583	96
7	0.639	91	0.646	92	0.653	93	0.659	94	0.665	95	0.670	95	0.674	96	0.677	96	0.679	96	0.679	97
8	0.730	91	0.738	91	0.746	92	0.753	93	0.760	94	0.765	95	0.770	95	0.773	96	0.775	96	0.776	96
9	0.821	90	0.829	92	0.838	92	0.846	94	0.854	94	0.860	94	0.865	95	0.869	96	0.871	96	0.872	96
10	0.911	90	0.921	91	0.930	92	0.940	92	0.948	93	0.954	95	0.960	95	0.965	95	0.967	96	0.968	96
11	1.001	89	1.012	90	1.022	92	1.032	93	1.041	94	1.049	94	1.055	95	1.060	95	1.063	95	1.064	95
12	1.090	90	1.102	91	1.114	91	1.125	92	1.135	93	1.143	94	1.150	94	1.155	95	1.158	95	1.159	95
13	1.180	89	1.193	90	1.205	91	1.217	92	1.228	92	1.237	93	1.244	94	1.250	94	1.253	95	1.254	95
14	1.269	89	1.283	89	1.296	91	1.309	91	1.320	92	1.330	93	1.338	93	1.344	94	1.348	94	1.349	94
15	1.358	88	1.372	89	1.387	90	1.400	91	1.412	92	1.423	92	1.431	92	1.438	93	1.442	93	1.443	94
16	1.446	88	1.461	89	1.477	89	1.491	91	1.504	92	1.515	92	1.523	94	1.531	93	1.535	93	1.537	93
17	1.534	87	1.550	88	1.566	89	1.582	90	1.596	90	1.607	92	1.617	92	1.624	93	1.628	93	1.630	93
18	1.621		1.638		1.655		1.672		1.686		1.699		1.709		1.717		1.721		1.723	

TABLE XXI.—*For Hourly Motion of Axis of Shadow.*

$$x'_2 = +0.5410 + 0.0397 \cos g.$$

g	0°	10°	20°	30°	40°	50°	60°	70°	80°	
°										°
0	.5807	.5801	.5783	.5754	.5714	.5665	.5608	.5546	.5479	10
1	.5807	.5800	.5781	.5750	.5710	.5660	.5602	.5539	.5472	9
2	.5807	.5798	.5778	.5747	.5705	.5654	.5596	.5533	.5465	8
3	.5806	.5797	.5775	.5743	.5700	.5649	.5590	.5526	.5458	7
4	.5806	.5795	.5773	.5739	.5696	.5643	.5584	.5519	.5452	6
5	.5805	.5793	.5770	.5735	.5691	.5638	.5578	.5513	.5445	5
6	.5805	.5792	.5767	.5731	.5686	.5632	.5571	.5506	.5438	4
7	.5804	.5790	.5764	.5727	.5681	.5626	.5565	.5499	.5431	3
8	.5803	.5788	.5761	.5723	.5676	.5620	.5559	.5492	.5424	2
9	.5802	.5785	.5757	.5719	.5670	.5614	.5552	.5486	.5417	1
10	.5801	.5783	.5754	.5714	.5665	.5608	.5546	.5479	.5410	0
	350°	340°	330°	320°	310°	300°	290°	280°	270°	g

g	90°	100°	110°	120°	130°	140°	150°	160°	170°	
°										°
0	.5410	.5341	.5274	.5212	.5155	.5106	.5066	.5037	.5019	10
1	.5403	.5334	.5268	.5206	.5150	.5101	.5063	.5035	.5018	9
2	.5396	.5328	.5261	.5200	.5144	.5097	.5059	.5032	.5017	8
3	.5389	.5321	.5255	.5194	.5139	.5093	.5056	.5030	.5016	7
4	.5382	.5314	.5249	.5188	.5134	.5089	.5053	.5028	.5015	6
5	.5375	.5307	.5242	.5182	.5129	.5085	.5050	.5027	.5015	5
6	.5368	.5301	.5236	.5177	.5124	.5081	.5047	.5025	.5014	4
7	.5362	.5294	.5230	.5171	.5119	.5077	.5045	.5023	.5014	3
8	.5355	.5287	.5224	.5166	.5114	.5073	.5042	.5022	.5013	2
9	.5348	.5281	.5218	.5160	.5110	.5070	.5039	.5020	.5013	1
10	.5341	.5274	.5212	.5155	.5106	.5066	.5037	.5019	.5013	0
	260°	250°	240°	230°	220°	210°	200°	190°	180°	g

TABLE XXII.—*For Hourly Motion of Axis of Shadow.*

$$x'_2 = -0^d.0010 \cos g' + 0^d.0006 \cos (g + g') - 0^d.0004 \cos (g - g').$$

g	0°	10°	20°	30°	40°	50°	60°	70°	80°	90°	100°	110°	120°	130°	140°	150°	160°	170°	180°	
g'																				
0°	− 8	− 8	− 8	− 8	− 8	− 9	− 9	− 9	−10	−10	−10	−11	−11	−11	−11	−12	−12	−12	−12	360°
10°	− 8	− 8	− 8	− 9	− 9	−10	−10	−11	−11	−11	−12	−12	−12	−12	−12	−12	−12	−12	−12	350°
20°	− 7	− 8	− 9	− 9	−10	−11	−11	−12	−12	−13	−13	−13	−13	−13	−13	−13	−12	−12	−11	340°
30°	− 7	− 8	− 9	−10	−11	−11	−12	−13	−13	−13	−14	−14	−14	−14	−13	−13	−12	−11	−10	330°
40°	− 6	− 7	− 8	− 9	−11	−12	−12	−13	−14	−14	−14	−14	−14	−14	−13	−12	−11	−10	− 9	320°
50°	− 5	− 6	− 8	− 9	−10	−11	−12	−13	−14	−14	−14	−14	−14	−13	−12	−11	−10	− 9	− 8	310°
60°	− 4	− 5	− 7	− 8	−10	−11	−12	−13	−13	−14	−14	−14	−13	−12	−11	−10	− 9	− 7	− 6	300°
70°	− 3	− 4	− 6	− 7	− 9	−10	−11	−12	−13	−13	−13	−13	−12	−11	−10	− 9	− 7	− 6	− 4	290°
80°	− 1	− 3	− 5	− 6	− 8	− 9	−10	−11	−11	−11	−11	−11	−10	− 9	− 8	− 7	− 5	− 4	− 2	280°
90°	0	− 2	− 3	− 5	− 7	− 8	− 9	− 9	−10	−10	−10	− 9	− 9	− 8	− 6	− 5	− 3	− 2	0	270°
100°	+ 1	0	− 2	− 3	− 5	− 6	− 7	− 8	− 8	− 8	− 8	− 7	− 7	− 6	− 4	− 3	− 1	0	+ 2	260°
110°	3	+ 1	0	− 2	− 3	− 4	− 5	− 6	− 6	− 6	− 6	− 5	− 4	− 3	− 2	− 1	+ 1	+ 3	4	250°
120°	4	3	+ 1	0	− 1	− 2	− 3	− 3	− 4	− 4	− 3	− 3	− 2	− 1	0	+ 1	3	5	6	240°
130°	5	4	3	+ 1	+ 1	0	− 1	− 1	− 1	− 1	− 1	0	0	+ 1	+ 2	4	5	6	8	230°
140°	6	5	4	3	2	+ 2	+ 1	+ 1	+ 1	+ 1	+ 2	+ 2	+ 3	4	5	6	7	3	9	220°
150°	7	6	5	5	4	4	3	3	4	4	4	5	5	6	7	8	9	10	10	210°
160°	7	7	6	6	6	6	5	6	6	6	6	7	8	8	9	9	10	11	11	200°
170°	8	8	7	7	7	7	7	8	8	8	8	9	9	10	10	11	11	12	12	190°
180°	8	8	8	8	8	9	9	9	10	10	10	11	11	11	11	12	12	12	12	180°
190°	8	8	8	9	9	10	10	11	11	11	12	12	12	12	12	12	12	12	12	170°
200°	7	8	9	9	10	11	11	12	12	13	13	13	13	13	13	13	12	12	11	160°
210°	7	8	9	10	11	11	12	13	13	13	14	14	14	14	13	13	12	11	10	150°
220°	6	7	8	9	11	12	12	13	14	14	14	14	14	14	13	12	11	10	9	140°
230°	5	6	8	9	10	11	12	13	14	14	14	14	14	13	12	11	10	9	8	130°
240°	4	5	7	8	10	11	12	13	13	13	14	14	13	12	11	10	9	7	6	120°
250°	3	4	6	7	9	10	11	12	13	13	13	13	12	11	10	9	7	6	4	110°
260°	+ 1	3	5	6	8	9	10	11	11	11	11	11	10	9	8	7	5	4	+ 2	100°
270°	0	+ 2	3	5	6	8	9	9	10	10	10	9	9	8	6	5	3	+ 2	0	90°
280°	− 1	0	+ 2	3	5	6	7	8	8	8	8	7	7	6	4	3	+ 1	0	− 2	80°
290°	− 3	− 1	0	+ 2	3	4	5	6	6	6	6	5	4	3	+ 2	+ 1	− 1	− 3	− 4	70°
300°	− 4	− 3	− 1	0	+ 1	+ 2	3	3	4	4	3	+ 3	+ 2	+ 1	0	− 2	− 3	− 5	− 6	60°
310°	− 5	− 4	− 3	− 2	− 1	0	+ 1	+ 1	+ 1	+ 1	+ 1	0	0	− 1	− 2	− 4	− 5	− 6	− 8	50°
320°	− 6	− 5	− 4	− 3	− 2	− 2	− 1	− 1	− 1	− 1	− 2	− 3	− 3	− 4	− 5	− 6	− 7	− 8	− 9	40°
330°	− 7	− 6	− 5	− 5	− 4	− 4	− 3	− 3	− 3	− 4	− 4	− 5	− 5	− 6	− 7	− 8	− 9	−10	−10	30°
340°	− 7	− 7	− 6	− 6	− 6	− 6	− 5	− 6	− 6	− 6	− 6	− 7	− 8	− 8	− 9	− 9	−10	−11	−11	20°
350°	− 8	− 8	− 7	− 7	− 7	− 7	− 7	− 8	− 8	− 8	− 8	− 9	− 9	−10	−10	−11	−11	−12	−12	10°
360°	− 8	− 8	− 8	− 8	− 8	− 9	− 9	− 9	−10	−10	−10	−11	−11	−11	−11	−12	−12	−12	−12	0°
																				g'
	360°	350°	340°	330°	320°	310°	300°	290°	280°	270°	260°	250°	240°	230°	220°	210°	200°	190°	180°	g

When the argument g is at the bottom of the page, or is negative, g' is to be sought for at the right.

The algebraic sum of the numbers from Tables XXI and XXII is the hourly variation of the co-ordinate x_2 of the point in which the axis of the shadow intersects the fundamental plane.

Table XXIII.—*For Radius of Shadow on Fundamental Plane.*

$l = .0059 - .0182 \cos g + .0004 \cos 2g.$

g	0°	10°	20°	30°	40°	50°	60°	70°	80°	
°										°
0	−.0119	−.0117	−.0109	−.0097	−.0080	−.0059	−.0034	−.0006	.0024	10
1	119	116	108	95	78	56	31	− 03	27	9
2	119	115	107	94	76	54	29	00	30	8
3	119	115	106	92	74	52	26	+ 03	33	7
4	119	114	105	90	72	49	23	05	36	6
5	−.0118	−.0113	−.0103	−.0089	−.0070	−.0047	−.0020	+.0008	.0039	5
6	118	112	102	87	67	44	18	11	42	4
7	118	112	101	85	65	42	15	1	46	3
8	117	111	100	83	63	39	12	17	49	2
9	117	110	098	82	61	37	09	20	52	1
10	117	109	097	80	59	34	06	24	55	0
	350°	340°	330°	320°	310°	300°	290°	280°	270°	g

g	90°	100°	110°	120°	130°	140°	150°	160°	170°	
°										°
0	+.0055	+.0087	+.0118	+.0148	+.0175	+.0199	+.0219	+.0233	+.0242	10
1	58	90	121	151	178	201	220	234	243	9
2	61	93	124	154	180	203	222	235	243	8
3	64	96	129	157	183	205	224	236	243	7
4	68	99	131	159	185	207	225	237	244	6
5	+.0071	+.0103	+.0135	+.0162	+.0188	+.0209	+.0226	+.0238	+.0244	5
6	76	106	136	165	190	211	228	239	244	4
7	79	109	139	167	192	213	229	240	245	3
8	81	112	142	170	195	215	231	241	245	2
9	84	115	145	173	197	217	252	241	245	1
10	87	118	148	175	199	219	233	242	245	0
	260°	250°	240°	230°	220°	210°	200°	190°	180°	g

For radius of penumbra add 0.5460.

TABLE XXIV.—*For Radius of Shadow on Fundamental Plane.*

$l = +0^d.0046 \cos g' - 0^d.0005 \cos (g + g')$.

g / g'	0°	10°	20°	30°	40°	50°	60°	70°	80°	90°	100°	110°	120°	130°	140°	150°	160°	170°	180°	
0°	+41	+41	+41	+42	+42	+43	+43	+44	+45	+46	+47	+48	+48	+49	+50	+50	+51	+51	+51	360°
10°	40	41	41	42	42	43	44	44	45	46	47	48	49	49	49	50	50	50	50	350°
20°	39	39	39	40	41	42	42	43	44	45	46	46	47	48	48	48	48	48	48	340°
30°	36	36	37	37	38	39	40	41	41	42	43	44	44	45	45	45	45	45	44	330°
40°	31	32	33	34	35	35	36	37	38	38	39	40	40	40	40	40	40	40	39	320°
50°	26	27	28	29	30	30	31	32	33	33	34	35	34	35	35	34	34	33	33	310°
60°	21	21	22	23	24	25	25	26	27	27	28	28	28	28	28	27	27	26	25	300°
70°	14	15	16	17	17	18	19	20	20	20	21	21	21	20	20	19	19	18	17	290°
80°	+ 7	8	9	10	10	11	12	12	13	13	13	13	13	12	12	11	11	10	+ 9	280°
90°	0	+ 1	+ 2	+ 3	+ 3	+ 4	+ 4	+ 5	+ 5	+ 5	+ 5	+ 5	+ 4	+ 4	+ 3	+ 2	+ 2	+ 1	0	270°
100°	− 7	− 6	− 5	− 5	− 4	− 4	− 3	− 3	− 3	− 3	− 3	− 4	− 4	− 5	− 5	− 6	− 7	− 8	− 9	260°
110°	−14	−13	−12	−12	−11	−11	−11	−11	−11	−11	−11	−12	−13	−13	−14	−15	−16	−17	−17	250°
120°	−21	−20	−19	−19	−18	−18	−18	−18	−18	−19	−19	−20	−21	−21	−22	−23	−24	−25	−25	240°
130°	−26	−26	−25	−25	−25	−25	−24	−25	−25	−26	−26	−27	−28	−29	−30	−30	−31	−32	−33	230°
140°	−31	−31	−30	−30	−30	−30	−31	−31	−31	−32	−33	−34	−34	−35	−36	−37	−38	−38	−39	220°
150°	−36	−35	−35	−35	−35	−35	−36	−36	−37	−37	−38	−39	−40	−41	−42	−42	−43	−44	−44	210°
160°	−38	−38	−38	−38	−38	−39	−39	−40	−41	−41	−42	−43	−44	−45	−46	−46	−47	−48	−48	200°
170°	−40	−41	−40	−41	−41	−41	−42	−43	−43	−44	−45	−46	−47	−48	−49	−49	−50	−50	−50	190°
180°	−41	−41	−41	−42	−42	−43	−43	−44	−45	−46	−47	−48	−49	−49	−50	−50	−51	−51	−51	180°
190°	−40	−41	−41	−42	−42	−43	−44	−44	−45	−46	−47	−48	−49	−49	−50	−50	−50	−50	−50	170°
200°	−38	−39	−39	−40	−41	−42	−42	−43	−44	−45	−46	−46	−47	−47	−48	−48	−48	−48	−48	160°
210°	−36	−36	−37	−37	−38	−39	−40	−41	−41	−42	−43	−44	−44	−45	−45	−45	−45	−45	−44	150°
220°	−31	−32	−33	−34	−34	−35	−36	−37	−38	−38	−39	−39	−40	−40	−40	−40	−40	−40	−39	140°
230°	−26	−27	−28	−29	−30	−30	−31	−32	−33	−33	−34	−34	−35	−35	−35	−34	−34	−33	−33	130°
240°	−21	−21	−22	−23	−24	−25	−25	−26	−27	−27	−28	−28	−28	−28	−28	−27	−27	−26	−25	120°
250°	−14	−15	−16	−17	−17	−18	−19	−20	−20	−20	−21	−21	−21	−20	−20	−19	−19	−18	−17	110°
260°	− 7	− 8	− 9	−10	−10	−11	−12	−12	−13	−13	−13	−13	−13	−12	−12	−11	−11	−10	− 9	100°
270°	0	− 1	− 2	− 3	− 4	− 4	− 4	− 5	− 5	− 5	− 5	− 5	− 4	− 4	− 3	− 2	− 2	− 1	0	90°
280°	+ 7	+ 6	+ 5	+ 5	+ 4	+ 4	+ 3	+ 3	+ 3	+ 3	+ 3	+ 4	+ 4	+ 5	+ 5	+ 6	+ 7	+ 8	+ 9	80°
290°	14	13	12	12	11	11	11	11	11	11	11	12	13	13	14	15	16	17	17	70°
300°	21	20	19	19	18	18	18	18	18	19	19	20	21	21	22	23	24	25	25	60°
310°	26	26	25	25	25	25	25	25	25	26	26	27	28	29	30	30	31	32	33	50°
320°	31	31	30	30	30	30	31	31	31	32	33	33	34	35	36	37	38	38	39	40°
330°	36	35	35	35	35	35	36	36	36	37	38	39	40	41	42	42	43	44	44	30°
340°	39	38	38	38	38	39	39	40	41	41	42	43	44	45	46	46	47	48	48	20°
350°	40	40	40	41	41	41	42	43	43	44	45	46	47	48	48	49	50	50	50	10°
360°	+41	+41	+41	+42	+42	+43	+43	+44	+45	+46	+47	+48	+48	+49	+50	+50	+51	+51	+51	0°
																				g'
	360°	350°	340°	330°	320°	310°	300°	290°	280°	270°	260°	250°	240°	230°	220°	210°	200°	190°	180°	g

The algebraic sum of the numbers from Tables XXIII and XXIV is the radius of the shadow-cone on the fundamental plane. If this radius is negative, it indicates a total eclipse; if positive, an annular one.

To find the radius of the penumbra, the sum of the numbers is to be increased by 0.5460.

Table XXV.—*Angle of Shadow Cone.*

$\sin f = 0.004653 + 0.000078 \cos g'$.

g'		$\sin f$	$\log \sin f$	g'		$\sin f$	$\log \sin f$
°	°			°	°		
0	360	0.004731	7.6750	90	270	0.004653	7.6677
10	350	.004730	7.6749	100	260	.004640	7.6665
20	340	.004726	7.6745	110	250	.004626	7.6652
30	330	.004720	7.6739	120	240	.004614	7.6641
40	320	.004713	7.6733	130	230	.004603	7.6630
50	310	.004703	7.6724	140	220	.004593	7.6621
60	300	.004692	7.6714	150	210	.004586	7.6614
70	290	.004680	7.6702	160	200	.004580	7.6609
80	280	.004666	7.6689	170	190	.004576	7.6605
90	270	.004653	7.6677	180	180	0.004575	7.6604

Table XXVI, *Arg. g'.—Sun's Equation of the Centre, or Reduction from Mean to True Longitude.*

g'	Year 0.	2000.		g'	Year 0.	2000.	
°	°	°	°	°	°	°	°
0	+ 0.00 −	+ 0.00 −	360	90	+ 2.01 −	+ 1.91 −	270
5	0.18	0.17	355	95	1.99	1.89	265
10	0.36	0.34	350	100	1.97	1.88	260
15	0.53	0.50	345	105	1.93	1.84	255
20	0.70	0.67	340	110	1.87	1.79	250
25	0.86	0.82	335	115	1.80	1.72	245
30	+ 1.02 −	+ 0.97 −	330	120	+ 1.72 −	+ 1.64 −	240
35	1.17	1.12	325	125	1.62	1.55	235
40	1.31	1.25	320	130	1.52	1.45	230
45	1.44	1.37	315	135	1.40	1.33	225
50	1.56	1.49	310	140	1.27	1.21	220
55	1.66	1.59	305	145	1.13	1.08	215
60	+ 1.76 −	+ 1.68 −	300	150	+ 0.98 −	+ 0.94 −	210
65	1.84	1.75	295	155	0.83	0.80	205
70	1.90	1.81	290	160	0.67	0.64	200
75	1.95	1.86	285	165	0.51	0.49	195
80	1.98	1.89	280	170	0.34	0.33	190
85	2.00	1.90	275	175	0.17	0.16	185
90	+ 2.01 −	+ 1.91 −	270	180	+ 0.00 −	+ 0.00 −	180
	Year 0.	2000.	g'		Year 0.	2000.	g'

Table XXV gives the angle of the shadow cone and its logarithm.

Table XXVI gives the sun's equation of the centre. By applying this quantity to L, the sun's mean longitude, we obtain ⊙, its true longitude.

Table XXVII.—*Reduction from ⊙'s Longitude to ⊙'s Right Ascension.*

⊙		Year 0.	2000.			⊙		Year 0.	2000.		
°	°	°	°	°	°	°	°	°	°	°	°
0	180	− 0.00	− 0.00	180	360	45	225	− 2.52 +	− 2.46 +	135	315
1	181	0.08	0.08	179	359	46	226	2.52	2.47	134	314
2	182	0.17	0.17	178	358	47	227	2.52	2.47	133	313
3	183	0.25	0.25	177	357	48	228	2.52	2.46	132	312
4	184	0.34	0.33	176	356	49	229	2.51	2.46	131	311
5	185	− 0.42 +	− 0.41 +	175	355	50	230	− 2.50 +	− 2.45 +	130	310
6	186	0.50	0.49	174	354	51	231	2.49	2.43	129	309
7	187	0.58	0.57	173	353	52	232	2.47	2.42	128	308
8	188	0.66	0.65	172	352	53	233	2.45	2.40	127	307
9	189	0.75	0.73	171	351	54	234	2.43	2.38	126	306
10	190	− 0.83 +	− 0.81 +	170	350	55	235	− 2.40 +	− 2.35 +	125	305
11	191	0.91	0.89	169	349	56	236	2.37	2.32	124	304
12	192	0.99	0.96	168	348	57	237	2.34	2.29	123	303
13	193	1.06	1.04	167	347	58	238	2.31	2.26	122	302
14	194	1.14	1.12	166	346	59	239	2.27	2.22	121	301
15	195	− 1.21 +	− 1.19 +	165	345	60	240	− 2.23 +	− 2.18 +	120	300
16	196	1.29	1.26	164	344	61	241	2.19	2.14	119	299
17	197	1.36	1.33	163	343	62	242	2.14	2.09	118	298
18	198	1.43	1.40	162	342	63	243	2.09	2.05	117	297
19	199	1.50	1.47	161	341	64	244	2.04	2.00	116	296
20	200	− 1.57 +	− 1.53 +	160	340	65	245	− 1.99 +	− 1.94 +	115	295
21	201	1.63	1.60	159	339	66	246	1.93	1.89	114	294
22	202	1.70	1.66	158	338	67	247	1.87	1.83	113	293
23	203	1.76	1.72	157	337	68	248	1.81	1.77	112	292
24	204	1.82	1.78	156	336	69	249	1.74	1.70	111	291
25	205	− 1.88 +	− 1.84 +	155	335	70	250	− 1.68 +	− 1.64 +	110	290
26	206	1.93	1.89	154	334	71	251	1.61	1.57	109	289
27	207	1.99	1.94	153	333	72	252	1.54	1.50	108	288
28	208	2.04	1.99	152	332	73	253	1.46	1.43	107	287
29	209	2.09	2.04	151	331	74	254	1.39	1.35	106	286
30	210	− 2.14 +	− 2.09 +	150	330	75	255	− 1.31 +	− 1.28 +	105	285
31	211	2.18	2.13	149	329	76	256	1.23	1.20	104	284
32	212	2.22	2.17	148	328	77	257	1.15	1.12	103	283
33	213	2.26	2.21	147	327	78	258	1.07	1.04	102	282
34	214	2.30	2.25	146	326	79	259	0.99	0.96	101	281
35	215	− 2.33 +	− 2.28 +	145	325	80	260	− 0.90 +	− 0.88 +	100	280
36	216	2.37	2.31	144	324	81	261	0.81	0.79	99	279
37	217	2.40	2.34	143	323	82	262	0.72	0.71	98	278
38	218	2.42	2.36	142	322	83	263	0.64	0.62	97	277
39	219	2.44	2.39	141	321	84	264	0.55	0.53	96	276
40	220	− 2.46 +	− 2.41 +	140	320	85	265	− 0.46 +	− 0.45 +	95	275
41	221	2.48	2.43	139	319	86	266	0.37	0.36	94	274
42	222	2.50	2.44	138	318	87	267	0.28	0.27	93	273
43	223	2.51	2.45	137	317	88	268	0.18	0.18	92	272
44	224	2.51	2.46	136	316	89	269	− 0.09	− 0.09	91	271
45	225	− 2.52 +	− 2.46 +	135	315	90	270	0.00	0.00	90	270
		Year 0.	2000.	⊙				Year 0.	2000.	⊙	

Table XXVII gives, with argument ⊙, a quantity which, when added to the equation of the centre (Table XXVI), will be the equation of time, E, expressed in degrees and hundredths.

Table XXVIII.—*Coefficients for Besselian Co-ordinates of Shadow Axis.*

☉ Sun's True Longitude.		a	log a	log a'	At Ascending Node.			At Descending Node.				
					log b	log b'	b'	log b	log b'	b'		
°	°										°	°
0	360	− .3981 +	−9.6000 +	9.9625	9.9440	+9.6871 −	+ .4865 −	9.9803	+9.4909 −	+ .3097 −	180	180
1	359	.3980	9.5999	25	40	9.6871	.4865	03	9.4908	.3096	181	179
2	358	.3979	9.5998	26	40	9.6869	.4863	04	9.4906	.3095	182	178
3	357	.3976	9.5995	26	41	9.6867	.4861	04	9.4902	.3092	183	177
4	356	.3973	9.5991	27	42	9.6864	.4858	04	9.4897	.3088	184	176
5	355	− .3968 +	−9.5986 +	9.9628	9.9443	+9.6860 −	+ .4853 −	9.9805	+9.4891 −	+ .3083 −	185	175
6	354	.3963	9.5980	29	44	9.6855	.4848	06	9.4882	.3078	186	174
7	353	.3956	9.5973	30	46	9.6850	.4841	07	9.4872	.3071	187	173
8	352	.3948	9.5964	32	48	9.6843	.4834	08	9.4861	.3063	188	172
9	351	.3940	9.5955	34	50	9.6836	.4826	09	9.4848	.3054	189	171
10	350	− .3930 +	−9.5944 +	9.9636	9.9453	+9.6827 −	+ .4816 −	9.9811	+9.4834 −	+ .3044 −	190	170
11	349	.3919	9.5932	38	56	9.6818	.4806	13	9.4818	.3032	191	169
12	348	.3907	9.5919	40	59	9.6808	.4795	14	9.4800	.3020	192	168
13	347	.3895	9.5905	43	62	9.6797	.4783	16	9.4781	.3007	193	167
14	346	.3881	9.5889	46	65	9.6785	.4769	18	9.4760	.2992	194	166
15	345	− .3866 +	−9.5873 +	9.9649	9.9469	+9.6772 −	+ .4755 −	9.9820	+9.4738 −	+ .2977 −	195	165
16	344	.3850	9.5855	52	73	9.6758	.4740	23	9.4713	.2960	196	164
17	343	.3833	9.5836	55	77	9.6743	.4724	25	9.4688	.2943	197	163
18	342	.3815	9.5815	58	82	9.6727	.4706	28	9.4660	.2924	198	162
19	341	.3796	9.5795	62	87	9.6710	.4688	31	9.4631	.2904	199	161
20	340	− .3776 +	−9.5771 +	9.9666	9.9492	+9.6692 −	+ .4669 −	9.9833	+9.4599 −	+ .2884 −	200	160
21	339	.3755	9.5746	70	97	9.6673	.4648	36	9.4566	.2862	201	159
22	338	.3733	9.5720	74	9.9502	9.6653	.4627	39	9.4531	.2839	202	158
23	337	.3710	9.5693	78	08	9.6632	.4605	43	9.4494	.2815	203	157
24	336	.3685	9.5665	83	14	9.6610	.4582	46	9.4455	.2790	204	156
25	335	− .3660 +	−9.5635 +	9.9688	9.9520	+9.6587 −	+ .4557 −	9.9849	+9.4414 −	+ .2763 −	205	155
26	334	.3634	9.5604	92	26	9.6563	.4532	53	9.4371	.2736	206	154
27	333	.3606	9.5571	97	32	9.6538	.4506	56	9.4326	.2707	207	153
28	332	.3578	9.5537	9.9702	39	9.6511	.4478	60	9.4278	.2678	208	152
29	331	.3549	9.5501	08	46	9.6483	.4450	64	9.4228	.2647	209	151
30	330	− .3518 +	−9.5463 +	9.9713	9.9553	+9.6455 −	+ .4420 −	9.9868	+9.4176 −	+ .2616 −	210	150
31	329	.3486	9.5424	19	60	9.6425	.4390	72	9.4121	.2583	211	149
32	328	.3454	9.5383	24	67	9.6393	.4358	76	9.4064	.2549	212	148
33	327	.3420	9.5341	30	75	9.6361	.4326	80	9.4004	.2514	213	147
34	326	.3385	9.5296	36	82	9.6327	.4292	84	9.3941	.2478	214	146
35	325	− .3349 +	−9.5250 +	9.9742	9.9590	+9.6292 −	+ .4258 −	9.9888	+9.3876 −	+ .2441 −	215	145
36	324	.3313	9.5202	48	98	9.6258	.4222	92	9.3808	.2403	216	144
37	323	.3275	9.5153	54	9.9606	9.6218	.4186	96	9.3736	.2364	217	143
38	322	.3236	9.5100	60	14	9.6178	.4148	9.9901	9.3662	.2324	218	142
39	321	.3196	9.5046	66	22	9.6138	.4109	05	9.3584	.2282	219	141
40	320	− .3155 +	−9.4990 +	9.9772	9.9631	+9.6095 −	+ .4070 −	9.9909	+9.3502 −	+ .2240 −	220	140
41	319	.3113	9.4931	79	39	9.6052	.4029	14	9.3417	.2196	221	139
42	318	.3069	9.4871	85	48	9.6006	.3987	18	9.3328	.2152	222	138
43	317	.3025	9.4808	92	57	9.5960	.3944	23	9.3232	.2106	223	137
44	316	.2980	9.4742	98	65	9.5911	.3900	27	9.3138	.2060	224	136
45	315	− .2934 +	−9.4674 +	9.9805	9.9674	+9.5861 −	+ .3855 −	9.9931	+9.3037 −	+ .2012 −	225	135
		a	log a	log a'	log b	log b'	b'	log b	log b'	b'	Sun's True Longitude.	
					At Descending Node.			At Ascending Node.				

Table XXVIII gives the coefficients by which to express the co-ordinates, x_1 and y_1, of the shadow axis on the fundamental plane. These correspond to the co-ordinates x and y of the Besselian theory of eclipses and of the American Ephemeris. The expressions are:—

$$x_1 = a\, y^\circ_2 + b\, x'_2\, t,$$
$$y_1 = a'\, y^\circ_2 + b'\, x'_2\, t,$$

y°_2 having been obtained from Tables XVIII to XX, and x'_2 from Tables XXI and XXII.

TABLE XXVIII.—*Coefficients for Besselian Co-ordinates of Shadow Axis*—Continued.

☉ Sun's True Longitude.		a	log a	log a'	At Ascending Node.			At Descending Node.				
					log b	log b'	b'	log b	log b'	b'		
°	°										°	°
45	315	− .2934+	−9.4674+	9.9805	9.9674	+9.5861−	+ .3855−	9.9931	+9.3037−	+ .2012−	225	135
46	314	.2886	9.4604	11	83	9.5809	.3809	36	9.2930	.1963	226	134
47	313	.2838	9.4530	18	92	9.5755	.3762	40	9.2819	.1914	227	133
48	312	.2789	9.4454	24	9.9701	9.5699	.3714	44	9.2702	.1863	228	132
49	311	.2738	9.4375	31	10	9.5641	.3665	48	9.2580	.1811	229	131
50	310	− .2687+	−9.4292+	9.9837	9.9719	+9.5581−	+ .3615−	9.9952	+9.2452−	+ .1759−	230	130
51	309	.2634	9.4207	44	28	9.5520	.3564	56	9.2317	.1705	231	129
52	308	.2581	9.4118	50	37	9.5456	.3512	61	9.2175	.1650	232	128
53	307	.2527	9.4026	57	46	9.5390	.3459	65	9.2026	.1594	233	127
54	306	.2472	9.3930	63	55	9.5322	.3406	68	9.1869	.1538	234	126
55	305	− .2415+	−9.3830+	9.9869	9.9764	+9.5252−	+ .3351−	9.9972	+9.1703−	+ .1480−	235	125
56	304	.2358	9.3726	76	73	9.5179	.3295	76	9.1528	.1422	236	124
57	303	.2300	9.3618	82	82	9.5103	.3238	80	9.1342	.1362	237	123
58	302	.2241	9.3505	88	91	9.5025	.3181	83	9.1145	.1302	238	122
59	301	.2181	9.3387	.9894	99	9.4944	.3122	87	9.0936	.1241	239	121
60	300	− .2120+	−9.3264+	9.9900	9.9808	+9.4861−	+ .3063−	9.9990	+9.0713−	+ .1178−	240	120
61	299	.2059	9.3136	06	17	9.4774	.3002	93	9.0475	.1115	241	119
62	298	.1996	9.3002	12	26	9.4685	.2941	96	9.0219	.1052	242	118
63	297	.1933	9.2862	17	34	9.4591	.2879	99	8.9944	.0987	243	117
64	296	.1869	9.2716	23	43	9.4496	.2816	0.0002	8.9647	.0922	244	116
65	295	− .1804+	−9.2562+	9.9928	9.9851	+9.4397−	+ .2752−	0.0004	+8.9324−	+ .0856−	245	115
66	294	.1738	9.2401	33	59	9.4293	.2688	07	8.8970	.0789	246	114
67	293	.1672	9.2232	38	67	9.4187	.2622	09	8.8581	.0721	247	113
68	292	.1605	9.2054	43	75	9.4076	.2556	11	8.8150	.0653	248	112
69	291	.1537	9.1866	48	82	9.3961	.2489	13	8.7666	.0584	249	111
70	290	− .1468+	−9.1668+	9.9953	9.9890	+9.3840−	+ .2422−	0.0014	+8.7115−	+ .0515−	250	110
71	289	.1399	9.1458	57	98	9.3717	.2353	16	8.6478	.0444	251	109
72	288	.1329	9.1236	61	9.9905	9.3588	.2285	17	8.5724	.0374	252	108
73	287	.1259	9.0999	65	12	9.3454	.2215	18	8.4804	.0302	253	107
74	286	.1188	9.0747	69	19	9.3314	.2145	19	8.3627	.0230	254	106
75	285	− .1116+	−9.0477+	9.9973	9.9925	+9.3168−	+ .2074−	0.0019	+8.1992−	+ .0158−	255	105
76	284	.1044	9.0187	76	32	9.3016	.2003	20	7.9313	.0085	256	104
77	283	.0971	8.9874	80	38	9.2858	.1931	20	+7.0828−	+ .0012−	257	103
78	282	.0898	8.9535	83	45	9.2692	.1859	20	−7.7889+	− .0062+	258	102
79	281	.0825	8.9165	85	50	9.2519	.1786	20	−8.1316	− .0135	259	101
80	280	− .0751+	−8.8759+	9.9988	9.9956	+9.2337−	+ .1713−	0.0019	−8.3220	− .0210+	260	100
81	279	.0677	8.8307	90	62	9.2146	.1639	18	−8.4542+	− .0285	261	99
82	278	.0603	8.7802	92	67	9.1946	.1565	17	−8.5557	− .0360	262	98
83	277	.0528	8.7228	94	72	9.1735	.1491	16	−8.6381	− .0435	263	97
84	276	.0453	8.6563	96	77	9.1512	.1416	14	−8.7075	− .0510	264	96
85	275	− .0378+	−8.5775+	9.9997	9.9981	+9.1276−	+ .1341−	0.0013	−8.7674+	− .0585+	265	95
86	274	.0303	8.4809	98	85	9.1025	.1266	11	−8.8202	− .0661	266	94
87	273	.0227	8.3562	99	89	9.0758	.1191	08	−8.8673	− .0737	267	93
88	272	.0151	8.1804	99	93	9.0474	.1115	06	−8.9098	− .0812	268	92
89	271	.0076	7.8797	0.0000	97	9.0169	.1040	03	−8.9485	− .0888	269	91
90	270	− .0000+	− ∞ +	0.0000	0.0000	+8.9841−	+ .0964−	0.0000	−8.9841+	− .0964+	270	90
					log b	log b'	b'	log b	log b'	b'	Sun's True Longitude. ☉	
		a	log a	log a'	At Descending Node.			At Ascending Node.				

When the argument ☉ is found on the right, the headings of the columns are to be sought at the bottom of the page.

Table XXIX.—*Sun's Declination, etc.*

⊙		d	d_1	$\frac{1}{\rho_1}$			⊙		d	d_1	$\frac{1}{\rho_1}$		
°	°	°	°		°	°	°	°	°	°		°	°
0	180	+ 0.00−	+ 0.00−	1.0033	180	360	45	135	+16.35−	+16.40−	1.0031	225	315
1	179	0.40	0.40	.0033	181	359	46	134	16.64	16.69	.0031	226	314
2	178	0.80	0.80	.0033	182	358	47	133	16.93	16.98	.0031	227	313
3	177	1.19	1.19	.0033	183	357	48	132	17.21	17.26	.0030	228	312
4	176	1.59	1.60	.0033	184	356	49	131	17.49	17.54	.0030	229	311
5	175	+ 1.99−	+ 2.00−	1.0033	185	355	50	130	+17.76−	+17.81−	1.0030	230	310
6	174	2.38	2.39	.0033	186	354	51	129	18.02	18.08	.0030	231	309
7	173	2.78	2.79	.0033	187	353	52	128	18.28	18.34	.0030	232	308
8	172	3.17	3.18	.0033	188	352	53	127	18.54	18.60	.0030	233	307
9	171	3.57	3.58	.0033	189	351	54	126	18.79	18.85	.0030	234	306
10	170	+ 3.96−	+ 3.97−	1.0033	190	350	55	125	+19.03−	+19.09−	1.0030	235	305
11	169	4.35	4.36	.0033	191	349	56	124	19.27	19.33	.0030	236	304
12	168	4.75	4.77	.0033	192	348	57	123	19.50	19.56	.0030	237	303
13	167	5.14	5.16	.0033	193	347	58	122	19.73	19.79	.0030	233	302
14	166	5.52	5.54	.0033	194	346	59	121	19.95	20.01	.0030	239	301
15	165	+ 5.91−	+ 5.93−	1.0033	195	345	60	120	+20.17−	+20.23−	1.0029	240	300
16	164	6.30	6.32	.0033	196	344	61	119	20.38	20.44	.0029	241	299
17	163	6.68	6.70	.0033	197	343	62	118	20.58	20.64	.0029	242	298
18	162	7.07	7.09	.0033	198	342	63	117	20.78	20.84	.0029	243	297
19	161	7.45	7.48	.0033	199	341	64	116	20.97	21.03	.0029	244	296
20	160	+ 7.83−	+ 7.86−	1.0033	200	340	65	115	+21.15−	+21.21−	1.0029	245	295
21	159	8.20	8.23	.0033	201	339	66	114	21.33	21.40	.0029	246	294
22	158	8.58	8.61	.0033	202	338	67	113	21.50	21.57	.0029	247	293
23	157	8.95	8.98	.0033	203	337	68	112	21.66	21.73	.0029	248	292
24	156	9.32	9.35	.0033	204	336	69	111	21.82	21.89	.0029	249	291
25	155	+ 9.69−	+ 9.72−	1.0033	205	335	70	110	+21.97−	+22.04−	1.0028	250	290
26	154	10.05	10.08	.0033	206	334	71	109	22.11	22.18	.0028	251	289
27	153	10.41	10.44	.0032	207	333	72	108	22.25	22.32	.0028	252	288
28	152	10.77	10.80	.0032	208	332	73	107	22.38	22.45	.0028	253	287
29	151	11.13	11.17	.0032	209	331	74	106	22.50	22.57	.0028	254	286
30	150	+11.48−	+11.52−	1.0032	210	330	75	105	+22.61−	+22.68−	1.0028	255	285
31	149	11.83	11.87	.0032	211	329	76	104	22.72	22.79	.0028	256	284
32	148	12.18	12.22	.0032	212	328	77	103	22.82	22.89	.0028	257	283
33	147	12.52	12.56	.0032	213	327	78	102	22.92	22.99	.0028	258	282
34	146	12.86	12.90	.0032	214	326	79	101	23.00	23.07	.0028	259	281
35	145	+13.20−	+13.24−	1.0032	215	325	80	100	+23.08−	+23.15−	1.0028	260	280
36	144	13.53	13.57	.0032	216	324	81	99	23.15	23.22	.0028	261	279
37	143	13.86	13.90	.0032	217	323	82	98	23.22	23.29	.0028	262	278
38	142	14.19	14.23	.0032	218	322	83	97	23.27	23.34	.0028	263	277
39	141	14.51	14.56	.0031	219	321	84	96	23.32	23.39	.0028	264	276
40	140	+14.83−	+14.88−	1.0031	220	320	85	95	+23.36−	+23.43−	1.0028	265	275
41	139	15.14	15.19	.0031	221	319	86	94	23.40	23.47	.0028	266	274
42	138	15.45	15.50	.0031	222	318	87	93	23.43	23.50	.0028	267	273
43	137	15.75	15.80	.0031	223	317	88	92	23.44	23.51	.0028	268	272
44	136	16.05	16.10	.0031	224	316	89	91	23.45	23.52	.0028	269	271
45	135	+16.35−	+16.40−	1.0031	225	315	90	90	+23.46−	+23.53−	1.0028	270	270
		d	d_1	$\frac{1}{\rho_1}$	⊙				d	d_1	$\frac{1}{\rho_1}$	⊙	

Table XXIX gives, with argument ⊙, the value of the sun's declination, d, that of d_1, the reduced declination, and that of $\frac{1}{\rho_1}$ for computing the central line on the earth's surface.

As an example of the use of the Tables, we shall examine what eclipses of the sun were visible during the year B. C. 584. From Table I, we find the argument of Table II to be $7^y.772$. From Table II, the times of conjunction of the mean sun with the node are found to be $144^d.8$ for ascending and $318^d.1$ for descending node. The values of D show that there were two central eclipses, of which the second was central only in the southern hemisphere. We therefore consider only the first one, which is the celebrated eclipse of Thales.

		Asc.	Desc.
Table III,	− 600,	27.6	
	+ 16,	1.5	29.1
		144.8	318.1
		173.9	347.2
Multiple,		177.2	354.4
D,		− 3.3	− 7.2
P,		7 ±	
T,		+ 126 ± 18	
Year of central eclipse,		− 458 ± 18	

		T
Table V, c. p. 4,	-440^y	$+ 234^d.6227$
Table VII, -512^y,	-144^y	$- 86^d.5816$
Arguments for date,	-584^y	$+ 148^d.0411$
Table VIII,		$+ {}^d.0786$
Table IX,		$- {}^d.0015$
Table X,		$- {}^d.0010$
Table XI,		$+ {}^d.0016$
Table XII,		$+ {}^d.0012$
		$148^d.1200$
Red. for calendar,		$0^d.00$
H_0, True conj., May 28,		$2^h\ 52^m.8$
T_1 in arc,		$43°.20$
− E,		$+2°.28$
H_1 at conj.,		$45°.48$

By Table XXVIII:—

$$x_1 = -\ .0624 + .5550\,t$$
$$y_1 = +\ .2832 + .1796\,t$$

	g	g'	L
	$-35°.27$	$264°.52$	$145°.93$
	$+23°.29$	$-83°.99$	$-86°.42$
	$-11°.98$	$+180°.53$	$+59°.51$
Table XXVI,			$- °.02$
		$\odot =$	$59°.49$
Table XXVII,			$- 2°.26$
Eq. Cent. = E,			$- 2°.28$

	u
	$-0°.294$
	$+3°.673$
	$+3°.379$
Table XV,	$+ °.077$
Table XVI,	$- °.020$
Table XVII,	$- °.002$
$u_1 =$	$+3°.431$

	y°_2
Table XVIII,	− .004
Table XIX,	0
Table XX,	+ 0.294
$y^\circ_2 =$	+ 0.290
Table XXIII,	− 0.0115
Table XXIV,	− 41
$l' =$	− .0156

	x'_2
Table XXI,	.5798
Table XXII,	+ .0008
$x'_2 =$	0.5806
Table XXV,	
$\sin f =$	.004575
log	7.6604

Track of Central Eclipse.

t	x_1	y'_1	H	H_1	Long.	Lat.
$1^h.35$	+ .6868	+ .5270	67°.4	65°.8	1°.6 E.	+ 41°.1
$1^h.40$	.7146	.5360	71°.8	66°.5	5°.3	41°.3
$1^h.45$	.7423	.5450	76°.7	67°.3	9°.4	40°.8
$1^h.50$	.7701	.5540	82°.4	68°.0	14°.4	39°.1
$1^h.55$	.7978	.5631	89°.8	68°.8	21°.0	37°.2
$1^h.57$	.8089	.5668	93°.5	69°.1	24°.4	36°.0
$1^h.59$	.8200	.5704	100°.6	69°.4	30°.6	33°.6
$1^h.5918$	.8211	.5708	103°.6	69.°4	34°.2	32°.5

The last point of the shadow-path is between 4° and 6° south of the region within which the celebrated battle must have been fought, which was supposed to have been stopped by this eclipse. This large deviation is due to the corrections which have been applied to Hansen's mean longitude of the moon. If these corrections are well founded, the sun set upon the combatants about nine tenths eclipsed.

A TRANSFORMATION

OF

HANSEN'S LUNAR THEORY

COMPARED WITH THE

THEORY OF DELAUNAY.

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TRANSFORMATION OF HANSEN'S LUNAR THEORY.

The numerical computation of the inequalities in the moon's motion executed by HANSEN was probably the greatest step taken in recent times toward placing the theory of the lunar perturbations on an accurate numerical basis. It was the step which first rendered it certain that any discrepancy between the theoretical and observed values of the inequalities produced by the sun arose from some other cause than errors in theory. The theoretical values to which it led must be considered the most accurate which astronomy now possesses.

The only theory which can compete with HANSEN'S is that of DELAUNAY. Here the coefficients are developed in series converging so slowly that some of the results are still a little doubtful, notwithstanding the great extent to which the approximation was carried. It may be expected that the numerical theory on which Sir GEORGE AIRY is now engaged will form yet another step in advance, in which nothing will be wanting for the purposes of accurate astronomy, so that three theories of the highest order of accuracy will ultimately be available for the construction of lunar tables. The work in question being still unfinished, the results of HANSEN and DELAUNAY are the only ones now available.

Unfortunately, the theory of HANSEN cannot be directly compared with those which have preceded it, owing to the peculiar form of the variables in which the co-ordinates of the moon are expressed. In saying this, I do not contest the proposition that this form has advantages. But, apart from the question of its merits in form, it becomes important to have the means of making a direct comparison of HANSEN'S theory with that of his predecessors and colaborers, who have expressed the co-ordinates of the moon directly in terms of the time. This has twice been partially done: by the writer in the *Comptes Rendus* for 1868, I (Tome LXVI, p. 1197), and, independently, by SCHJELLERUP, in a paper published in 1874 by the Danish Academy of Sciences. Both depend on data for the transformation given by HANSEN himself, which, though they may be accurate enough to give an idea of the agreement between the theories of HANSEN and DELAUNAY, cannot be regarded as sufficiently precise for a satisfactory transformed theory. The object of the present paper is to make a transformation which shall faithfully represent HANSEN'S latest theory, and be expressed in arguments depending directly on the time.

§ 1.

EXPRESSION OF THE MOON'S LONGITUDE.

In HANSEN's theory the moon's longitude is represented in the following form. Put

g, the moon's mean anomaly;
g', the sun's mean anomaly;
ω, the distance from the node to the perigee;
ω', the distance from the node to the solar perigee;
π, the longitude of the perigee;
e, the eccentricity of the moon's orbit, as used by HANSEN;
$n\delta z$, the Hansenian perturbations of mean anomaly;
s, the Hansenian perturbations of latitude;
I, the inclination of the moon's orbit.

Then put, as auxiliary quantities,

$$f = \text{elta}\ (e, g + n\delta z), \text{ the true anomaly};$$

$$\mathrm{R} = -\tan^2 \tfrac{1}{2}\mathrm{I} \sin 2\,(f+\omega) + \tfrac{1}{2}\tan^4 \tfrac{1}{2}\mathrm{I} \sin 4\,(f+\omega) - \text{etc.},$$

the reduction to the ecliptic;

$$\begin{aligned}\mathrm{R}' = &- s\,\frac{\tan \mathrm{I} \cos (f+\omega)}{1 - \sin^2 \mathrm{I} \sin^2 (f+\omega)} \\ &- 0''.397 \sin 2\omega \\ &- 1''.198 \sin (2g' + 2\omega') \\ &- 0''.285 \sin (2g - 4g' + 2\omega - 4\omega'),\end{aligned}$$

the inequalities of this reduction.

Then, for the moon's longitude,

$$\mathrm{L} = f + \pi + \mathrm{R} + \mathrm{R}'.$$

The latitude, β, is given by the equation

$$\sin \beta = \sin \mathrm{I} \sin (f+\omega) + s.$$

In presenting HANSEN's results in the form of a complete and exact numerical theory, several precautions have to be taken. In the first place, all the results must, so far as possible, depend upon or be reduced to one and the same homogeneous set

of elements. In the next place, those inequalities which express the solution of the problem of three bodies, considered as material points, must be separated from inequalities arising from other sources, such, for instance, as the distance between the moon's centres of gravity and figure, and the ellipticity of the earth.

Three values of the eccentricity appear in HANSEN's theory and tables:

(1) A provisional or ideal eccentricity, with which the inequalities were originally computed.

(2) An apparent eccentricity, which he found to represent the observed motion of the moon's centre of figure, and used in his tables.

(3) A theoretical eccentricity of the true orbit described by the moon's centre of gravity.

These three values of the element are:—

$$(1) \quad e = .05490079$$
$$(2) \quad e = .05490807$$
$$(3) \quad e = .05489959$$

According to HANSEN's view it is the third value which should be used in computing the moon's perturbations; but as he actually used the first value, it is the one which we should employ in the transformation.

In the case of the inclination there are three corresponding values, with an additional complication arising from the question whether we shall add to the inclination a term in the perturbations, $2''.705 \sin (g + \omega)$, having the mean argument of latitude as its argument.

Omitting this term, the values of the inclination will be:—

$$(1) \quad \mathrm{I} = 5^\circ \; 8' \; 48''$$
$$(2) \quad \mathrm{I} = 5^\circ \; 8' \; 43''.66$$
$$(3) \quad \mathrm{I} = 5^\circ \; 8' \; 39''.96$$

Here, again, it is only the first value with which we are concerned in the transformation, because it is the one employed by HANSEN in computing the perturbations.

The Hansenian perturbation $n\delta z$ is an explicit function of g, g', ω and ω'. So far as the longitude is concerned, our present problem is to express f, R and R$'$, and thence L, as explicit functions of the above four quantities. If we put:—

$$z = g + n\delta z$$

e_1, e_2, e_3, etc., the coefficients of $\sin z$, $\sin 2z$, etc. in the development of elta (e, z), we shall have,

$$f = z + e_1 \sin z + e_2 \sin 2z + \text{etc.}$$

If, then, we put $g + n\delta z$ for z, develop in powers of $n\delta z$, call $(e, g)_0$ the part of f independent of $n\delta z$, and $(e, g)_i$ the coefficient of $(n\delta z)^i$ in f, we shall have,

$$(e, g)_0 = g + e_1 \sin g + e_2 \sin 2g + e_3 \sin 3g + e_4 \sin 4g + \text{etc.}$$
$$(e, g)_1 = 1 + e_1 \cos g + 2e_2 \cos 2g + 3e_3 \cos 3g + \text{etc.}$$
$$(e, g)_2 = -\frac{1}{2} e_1 \sin g - \frac{2^2}{2} e_2 \sin 2g - \frac{3^2}{2} e_3 \sin 3g - \text{etc.}$$
$$(e, g)_3 = -\frac{1}{2.3} e_1 \cos g - \frac{2^3}{2.3} e_2 \cos 2g - \frac{3^3}{2.3} e_3 \cos 3g - \text{etc.}$$
$$(e, g)_4 = \frac{1}{2.3.4} e_1 \sin g + \text{etc.}$$
$$\text{etc.} \qquad \text{etc.}$$

The coefficients e_1, e_2, etc., are dependent on the eccentricity. The well-known analytical values, and the numerical values obtained by putting $e = .05490079$, are:

$$e_1 = 2e - \frac{1}{4}e^3 + \frac{5}{96}e^5 = .10976024 = 22639''.676$$
$$e_2 = \frac{5}{4}e^2 - \frac{11}{24}e^4 + \frac{17}{192}e^6 = .00376346 = 776''.269$$
$$e_3 = \frac{13}{12}e^3 - \frac{43}{64}e^5 = .00017893 = 36''.907$$
$$e_4 = \frac{103}{96}e^4 - \frac{451}{480}e^6 = .00000972 = 2''.005$$
$$e_5 = \frac{1097}{960}e^5 = .00000057 = 0''.118$$
$$e_6 = \frac{1223}{960}e^6 = .00000004 = 0''.007$$

The value of $n\delta z$ is taken, not from HANSEN's tables, but from his revised results given in the *Darlegung**. They are found in Part I, pp. 409–411, and Part II, pp. 224, 242, 258, and 268, and, for convenience of reference, are all collected in Table I of the present paper. In this table are given also the powers of $n\delta z$, the computations of which were all made in duplicate, that of the square being executed by two independent computers.

We thus have all the data for the numerical value of f, the formula for which is,

$$f = (e, g)_0 + (e, g)_1 n\delta z + (e, g)_2 (n\delta z)^2 + \text{etc.} \qquad (1)$$

Consider next the first term of R, which we may call R_1. We have

$$R_1 = -\tan^2 \frac{1}{2} I \sin (2f + 2\omega),$$

which is also to be developed in powers of $n\delta z$.

* Under this title reference is made to Hansen's two papers, *Darlegung der theoretischen Berechnung der in den Mondtafeln angewandten Störungen*, in the *Abhandlungen der königlich-sächsischen Gesellschaft der Wissenschaften*. *Band IX, XI.*

If we substitute for f its value in terms of e and z, and develop in powers of e, we find *:—

$$R_1 = -\tan^2 \frac{1}{2} I \times \left\{ \begin{array}{ll} \frac{1}{24} e^4 & \sin(-2z + 2\omega) \\ +\frac{1}{12} e^3 & \sin(-z + 2\omega) \\ +\left(\frac{3}{4}e^2 + \frac{1}{8}e^4\right) & \sin 2\omega \\ +\left(-2e + \frac{7}{4}e^3\right) & \sin(z + 2\omega) \\ +\left(1 - 4e^2 + \frac{55}{16}e^4\right) & \sin(2z + 2\omega) \\ +\left(2e - \frac{27}{4}e^3\right) & \sin(3z + 2\omega) \\ +\left(\frac{13}{4}e^2 - \frac{259}{24}e^4\right) & \sin(4z + 2\omega) \\ +\frac{59}{12} e^3 & \sin(5z + 2\omega) \\ +\frac{115}{16} e^4 & \sin(6z + 2\omega) \end{array} \right.$$

If, in this equation, we substitute for e and I their numerical values and then differentiate with respect to z, so as to obtain the coefficients of the powers of $n\delta z$, putting

$$R_1 = R_{1,0} + R_{1,1}\, n\delta z + R_{1,2}\, (n\delta z)^2 + \text{etc.},$$

we have

$$\begin{array}{rrl} R_{1,0} = - & 0''.006 & \sin(-g + 2\omega) \\ - & 0''.942 & \sin(2\omega) \\ + & 45''.627 & \sin(g + 2\omega) \\ - & 411''.626 & \sin(2g + 2\omega) \\ - & 45''.281 & \sin(3g + 2\omega) \\ - & 4''.040 & \sin(4g + 2\omega) \\ - & 0''.338 & \sin(5g + 2\omega) \\ - & 0''.027 & \sin(6g + 2\omega) \end{array}$$

$$\begin{array}{rl} R_{1,1} = +.000\,221{,}2 & \cos(g + 2\omega) \\ -.003\,991{,}2 & \cos(2g + 2\omega) \\ -.000\,658{,}6 & \cos(3g + 2\omega) \\ -.000\,078{,}3 & \cos(4g + 2\omega) \\ -.000\,008{,}2 & \cos(5g + 2\omega) \\ -.000\,000{,}8 & \cos(6g + 2\omega) \end{array}$$

* Tables of this and the other developments in the elliptic motion have been given by Professor CAYLEY in the *Memoirs of the Royal Astronomical Society*, Vol. XXIX, but the above development was executed independently before the applicability of Professor CAYLEY's formulæ was remarked.

$$
\begin{aligned}
R_{1,2} = &- .00011 \sin(g + 2\omega) \\
&+ .00399 \sin(2g + 2\omega) \\
&+ .00099 \sin(3g + 2\omega) \\
&+ .00016 \sin(4g + 2\omega)
\end{aligned}
$$

$$
\begin{aligned}
R_{1,3} = &+ .0027 \cos(2g + 2\omega) \\
&+ .0010 \cos(3g + 2\omega)
\end{aligned}
$$

In the same way, putting

$$R_2 = \frac{1}{2} \tan^4 \frac{1}{2} I \sin(4f + 4\omega)$$

we have by substituting for f its value in z, and developing in powers of e,

$$
\begin{aligned}
\sin(4f + 4\omega) = \quad & \frac{11}{2} e^2 \sin(2z + 4\omega) \\
&- 4e \sin(3z + 4\omega) \\
&+ (1 - 16e^2) \sin(4z + 4\omega) \\
&+ 4e \sin(5z + 4\omega) \\
&+ \frac{21}{2} e^2 \sin(6z + 4\omega)
\end{aligned}
$$

Putting as before,

$$R_2 = R_{2,0} + R_{2,1}\, n\delta z + R_{2,2}\, (n\delta z)^2 + \text{etc.},$$

we find by substituting the numerical values of I and e

$$
\begin{aligned}
R_{2,0} = &+ 0''.007 \sin(2g + 4\omega) \\
&- 0''.092 \sin(3g + 4\omega) \\
&+ 0''.400 \sin(4g + 4\omega) \\
&+ 0''.092 \sin(5g + 4\omega) \\
&+ 0''.013 \sin(6g + 4\omega)
\end{aligned}
$$

$$
\begin{aligned}
R_{2,1} = &- .000\,001{,}3 \cos(3g + 4\omega) \\
&+ .000\,007{,}8 \cos(4g + 4\omega) \\
&+ .000\,002{,}2 \cos(5g + 4\omega)
\end{aligned}
$$

The terms of $R_{2,2}\, (n\delta z)^2$ are less than $0''.001$.

The coefficient of $-s \tan I$ in R' is, with sufficient accuracy,

$$\cos(f + \omega)\, [1 + \sin^2 I \sin^2 (f + \omega)]$$

or

$$\left(1 + \frac{1}{4} \sin^2 I\right) \cos(f + \omega) - \frac{1}{4} \sin^2 I \cos(3f + 3\omega).$$

By the developments of the elliptic motion we have,

$$
\begin{aligned}
\cos(f+\omega) = & -\frac{1}{12}e^3\cos(-2z+\omega) \\
& -\frac{1}{8}e^2\cos(-z+\omega) \\
& -e\cos\omega \\
& +(1-e^2)\cos(z+\omega) \\
& +\left(e-\frac{5}{4}e^3\right)\cos(2z+\omega) \\
& +\frac{9}{8}e^2\cos(3z+\omega) \\
& +\frac{4}{3}e^3\cos(4z+\omega)
\end{aligned}
$$

$$
\begin{aligned}
\cos(3f+3\omega) = & \ \frac{21}{8}e^2\cos(z+3\omega) \\
& -3e\cos(2z+3\omega) \\
& +(1-9e^2)\cos(3z+3\omega) \\
& +3e\cos(4z+3\omega) \\
& +\frac{51}{8}e^2\cos(5z+3\omega)
\end{aligned}
$$

If we represent by S the coefficient of s in R′, that is,

$$S = -\tan I\cos(f+\omega)\{1+\sin^2 I\sin^2(f+\omega)\},$$

and suppose

$$S = S_0 + S_1\, n\delta z + S_2\,(n\delta z)^2,$$

we shall have,

$$
\begin{aligned}
S_0 = & +.000034\cos(-g+\omega) \\
& +.004955\cos\omega \\
& -.089978\cos(g+\omega) \\
& -.004936\cos(2g+\omega) \\
& -.000306\cos(3g+\omega) \\
& -.000020\cos(4g+\omega) \\
& -.000030\cos(2g+3\omega) \\
& +.000176\cos(3g+3\omega) \\
& +.000030\cos(4g+3\omega) \\
& +.000004\cos(5g+3\omega)
\end{aligned}
$$

$$
\begin{aligned}
S_1 = & +.0900\sin(g+\omega) \\
& +.0099\sin(2g+\omega) \\
& +.0009\sin(3g+\omega)
\end{aligned}
$$

$$
\begin{aligned}
S_2 = & +.045\cos(g+\omega) \\
& +.010\cos(2g+\omega)
\end{aligned}
$$

Multiplying these several expressions by HANSEN's s, we find the value of $s\,S_0$, etc., given in Table II.

2

Collecting all the coefficients of the powers of $n\delta z$, we find the following expressions for the moon's true ecliptic longitude, as a function of $n\delta z$:—

$$L = L_0 + L_1 n\delta z + L_2 (n\delta z)^2 + \text{etc.}$$

Terms independent of $n\delta z$.

$$\begin{aligned}
L_0 = g + \pi &\\
&+ 22639''.676 \sin g\\
&+ 776''.269 \sin 2g\\
&+ 36''.907 \sin 3g\\
&+ 2''.005 \sin 4g\\
&+ 0''.118 \sin 5g\\
&+ 0''.007 \sin 6g\\
&- 0''.006 \sin (-g + 2\omega)\\
&\left.\begin{matrix} - 0''.942 \\ - 0''.397 \end{matrix}\right\} \sin 2\omega\\
&+ 45''.627 \sin (g + 2\omega)\\
&- 411''.626 \sin (2g + 2\omega)\\
&- 45''.281 \sin (3g + 2\omega)\\
&- 4''.040 \sin (4g + 2\omega)\\
&- 0''.338 \sin (5g + 2\omega)\\
&- 0''.027 \sin (6g + 2\omega)\\
&+ 0''.007 \sin (2g + 4\omega)\\
&- 0''.092 \sin (3g + 4\omega)\\
&+ 0''.400 \sin (4g + 4\omega)\\
&+ 0''.092 \sin (5g + 4\omega)\\
&+ 0''.013 \sin (6g + 4\omega)\\
&- 1''.198 \sin (2g' + 2\omega')\\
&- 0''.285 \sin (2g - 4g' + 2\omega - 4\omega)\\
&+ s S_0.
\end{aligned}$$

Coefficient of $n\delta z$.

[The comma points off six places of decimals.]

$$\begin{aligned}
L_1 = &\ 1\\
&+ .109760,2 \cos g\\
&+ .007526,9 \cos 2g\\
&+ .000536,8 \cos 3g\\
&+ .000038,9 \cos 4g\\
&+ .000002,8 \cos 5g\\
&+ .000221,2 \cos (g + 2\omega)\\
&- .003991,2 \cos (2g + 2\omega)\\
&- .000658,6 \cos (3g + 2\omega)\\
&- .000078,3 \cos (4g + 2\omega)\\
&- .000008,2 \cos (5g + 2\omega)\\
&- .000000,8 \cos (6g + 2\omega)
\end{aligned}$$

$$
\begin{aligned}
& - .000001{,}3 \cos (3g + 4\omega) \\
& + .000007{,}8 \cos (4g + 4\omega) \\
& + .000002{,}2 \cos (5g + 4\omega) \\
& + s\,S_1
\end{aligned}
$$

$$
\begin{aligned}
L_2 = & - .05488 \sin g \\
& - .00753 \sin 2g \\
& - .00080 \sin 3g \\
& - .00008 \sin 4g \\
& - .00011 \sin (g + 2\omega) \\
& + .00399 \sin (2g + 2\omega) \\
& + .00099 \sin (3g + 2\omega) \\
& + .00016 \sin (4g + 2\omega) \\
& + s\,S_2
\end{aligned}
$$

$$
\begin{aligned}
L_3 = & - .0183 \cos g \\
& - .0050 \cos 2g \\
& - .0008 \cos 3g \\
& + .0027 \cos (2g + 2\omega) \\
& + .0010 \cos (3g + 2\omega)
\end{aligned}
$$

The several parts of this expression for L are given in Table II, omitting the following terms, which are, however, all included in the column giving the concluded coefficients in L:—

1. The terms of L_0, explicitly given in the first of the preceding equations.
2. The expressions for $n\delta z$, $(n\delta z)^2 \times s\,S_2$, $(n\delta z)^3 \times R_{1,3}$, and $(n\delta z) \times R_{2,1}$.

The values of the last three expressions are as follows, the numbers within the parentheses being coefficients of g, g', ω, and ω', respectively:—

$n\delta z \times R_{2,1}$ =	$(n\delta z)^2 \times s\,S_2$ =	$(n\delta z)^3 \times R_{1,3}$ =
$''$	$''$	$''$
$-.001 \sin (3, 3, 2, 2)$	$-.002 \sin (0, -2, 2, -2)$	$-.001 \sin (2, 1, 2, 0)$
$+.001 \sin (1, 2, 2, 2)$	$+.003 \sin (2, -2, 2, -2)$	$+.001 \sin (2, -1, 2, 0)$
$-.005 \sin (2, 2, 2, 2)$	$+.002 \sin (3, -2, 2, -2)$	$-.002 \sin (0, 2, 0, 2)$
$-.020 \sin (3, 2, 2, 2)$	$+.002 \sin (-1, 2, 0, 2)$	$-.004 \sin (1, 2, 0, 2)$
$-.005 \sin (4, 2, 2, 2)$	$+.004 \sin (0, 2, 0, 2)$	$-.002 \sin (2, 2, 0, 2)$
$-.002 \sin (4, 1, 4, 0)$	$-.002 \sin (2, -2, 4, -2)$	$+.002 \sin (2, -2, 4, -2)$
$+.002 \sin (4, -1, 4, 0)$	$-.002 \sin (3, -2, 4, -2)$	$+.003 \sin (3, -2, 4, -2)$
$+.001 \sin (5, -1, 4, 0)$	$-.002 \sin (4, -6, 6, -6)$	$+.003 \sin (4, -2, 4, -2)$
$-.003 \sin (4, -2, 6, -2)$	$-.002 \sin (5, -6, 6, -6)$	$+.002 \sin (5, -2, 4, -2)$
$+.016 \sin (5, -2, 6, -2)$	$-.002 \sin (2, -6, 4, -6)$	$-.001 \sin (1, -6, 4, -6)$
$+.013 \sin (6, -2, 6, -2)$	$-.002 \sin (3, -6, 4, -6)$	$-.001 \sin (2, -6, 4, -6)$
$+.002 \sin (7, -2, 6, -2)$		$-.001 \sin (6, -6, 8, -6)$
		$-.001 \sin (7, -6, 8, -6)$

In Table II the column "Sum" contains the sums of the terms actually given in the preceding columns of the table.

The next column gives the complete coefficient of each term in the ecliptic longitude, and is formed by adding to the column "Sum" the omitted terms just referred to.

The last column gives, for the larger terms, the elements which they principally contain as factors. If these elements be changed, the coefficients must be changed by corresponding quantities.

§ 2.

REDUCTION OF THE PRECEDING EXPRESSIONS TO UNIFORM ELEMENTS, AND COMPARISON WITH DELAUNAY.

The coefficients of the preceding inequalities contain as factors certain elements for which different investigators adopt different values. It is essential to a clear presentation of results that they should be reduced to a uniform and well-defined set of elements having given values We therefore commence by reducing the theories of both HANSEN and DELAUNAY to such a system. The elements principally referred to are—

(α) The ratio of the mean motions of the sun and moon.
(β) The lunar eccentricity.
(γ) The solar parallax.
(δ) The solar eccentricity.
(ε) The inclination of the moon's orbit.

Really, all these elements are contained in all the inequalities in a very complex manner. But there is so little doubt about their true numerical values that it is only necessary to take account of their changes when they appear as factors in coefficients of considerable magnitude. The extent to which each term is affected can be roughly seen from its analytic expression given by DELAUNAY at the end of his *Theorie du Mouvement de la Lune*, Tome II. We take up the several elements in order.

(α) *Ratio of mean motions.* This element is so certain that no reduction need be made on account of it. It is true that theoretical motions of the lunar node and perigee must implicitly enter in connection with this element. But, from a rough examination of HANSEN'S integration coefficients on pp. 350–352 of his *Darlegung*, I do not think any of the larger coefficients will be affected by as much as $\frac{1}{100000}$ of their entire amount by any admissible change of these motions.

(β) *Eccentricity of moon's orbit.* The eccentricities used by the two investigators are not directly comparable, but may be most conveniently compared by reducing each to the coefficient of g in the expression for the moon's ecliptic longitude. DELAUNAY uses AIRY'S value, given in his last paper on the elements of the moon's orbit.* HANSEN corrected his eccentricity for use in his tables, as already mentioned. The writer obtained a small but well-marked correction to HANSEN'S value from the Green-

* Memoirs Royal Astronomical Society, Vol. XXIX.

wich observations 1846–'74, and the Washington observations 1862–'74. The four values of the coefficient in question are:—

Airy, used by Delaunay, ...	22639''.06
Hansen, used in Theory, ...	22637''.15
Hansen, used in Tables, ...	22640''.15
Corrected value found in 1876,* . .	22639''.58

Although there is no reasonable doubt that the eccentricity of Hansen's tables requires a negative correction, it will be adopted for the purposes of comparison because it is now the standard of the ephemerides with which subsequent comparisons must be made. All the terms having e as a coefficient, must therefore be increased by the factor

$$\frac{.00000728}{.05490} = .0001326,$$

and those having e^2 by double this factor. The coefficients in e must, in Delaunay's theory, be increased by the factor

$$\frac{1''.09}{22639''} = .0000482.$$

(γ) *Solar parallax.* Hansen's theory does not set out with a definite solar parallax, but with a ratio of the mean distances of the sun and moon, which ratio again is not the usual one, because Hansen's a and a' are the same functions of the motion of mean anomaly that the usual a and a' are of the sidereal motions. We must therefore adopt an indirect process for finding the relation of solar parallax and parallactic equation on his theory. He finds that his theoretical coefficient has to be multiplied by the factor 1.03573 to make it agree with observation; and then, in § 266 of his *Darlegung*, he deduces the solar parallax 8''.9159. Dividing this parallax by the preceding factor, we conclude that the parallax of his theory is:—

$$8''.6085.$$

In turning his theory into numbers Delaunay used 8''.75. The parallax to which both theories will be actually reduced is:—

$$8''.848.$$

Hence, Hansen's terms having the parallax as a factor must be increased by the factor

$$.02785,$$

and Delaunay's by the factor

$$.01120.$$

(δ) *The solar eccentricity.* The solar eccentricity of Hansen's theory is:—

$$e' = 0.01679226 \qquad \text{(Epoch 1800).}$$

* Papers published by the Commission on the Transit of Venus. Part III.

DELAUNAY uses LE VERRIER'S value:—

$$e' = 0.01677106 \qquad \text{(Epoch 1850).}$$

In strictness these two values are not comparable, owing to the different form of HANSEN'S solar theory; but since HANSEN neglects perturbations of the earth's motion in his lunar theory, it may be assumed that there will be no difference between the form in which the eccentricity enters into the two theories. If we carry LE VERRIER'S eccentricity back to 1800 with his secular variation, we shall have:—

$$e' = 0.01679228 \qquad \text{(Epoch 1800).}$$

This may be regarded as absolutely identical with HANSEN'S value for the same epoch. So, adopting 1800 as the epoch, we have only to increase DELAUNAY'S coefficients in e' by the factor

$$\frac{.00002122}{.01677} = .001265.$$

Or, we may reduce HANSEN'S values to 1850 by dividing them by 1.001265, when they will be comparable with DELAUNAY'S.

The theories of HANSEN and DELAUNAY, thus reduced to a uniform and consistent set of elements, are given and compared in Table III. DELAUNAY'S results are frequently doubtful by a small fraction of a second, owing to the slow convergence of the series in powers of m, and the table has been arranged so as to show the extent of the uncertainty thus arising.

Following the indices expressing the arguments are given, first, HANSEN'S coefficients formed from the values in Table II by multiplying by the appropriate factors for reduction already given. They are only given to $0''.01$, but should the thousandth of seconds be required they are readily obtainable.

The corresponding coefficients of DELAUNAY are derived principally from his presentation of numerical results in the additions to the *Connaissance des Temps* for 1869. On pages 11 to 21 of that paper are given the sums of the terms in each coefficient which were actually computed by him. The parallactic terms, as given by DELAUNAY, are still to be multiplied by $\frac{1-\lambda}{1+\lambda}$, λ being the ratio of the mass of the moon to that of the earth. Putting, with HANSEN, $\lambda = \frac{1}{80}$, the coefficient will be $\frac{79}{81}$. The sums, corrected for this coefficient and for difference of elements, are given in the column *Delaunay* (1). Had all the appreciable terms been actually computed, these coefficients would have been the definitive ones of DELAUNAY'S theory. But it was frequently found that the terms, even of the ninth order, where the development ceased, were still appreciable; it was, therefore, necessary to estimate the probable sum of the omitted terms of higher orders from the law of the series as observed in the terms actually computed. These estimates can have no true mathematical foundation,

because there is no proof of the actual law of the series.* Still, there is a high degree of probability in favor of each one being at least a rude approximation to the truth. A rigorous computation would probably show that a majority differed less than $\frac{1}{4}$ of their amount from the true values, though here and there one might be found entirely illusory. The coefficients of longitude, modified by these estimated additions, are given by DELAUNAY on pages 38–40 of the paper referred to, and are reproduced, with the necessary corrections for changes of elements, in the column *Delaunay* (2).

The difference of these results, given in the next column, is the correction apparently applied by DELAUNAY for the uncomputed terms. It will be noted that we have no independent statement of these terms to refer to, and can only infer their values from the differences between the printed results (1) and (2)

Finally, we have the difference, *Hansen* minus *Delaunay* (2) showing the discrepancies still outstanding between the two theories Each one can judge for himself how far these discrepancies arise from the uncertainty of DELAUNAY's semi-empirical corrections, and how far from errors in the two theories.

One or two terms are worthy of a special examination, and among these the parallactic equation takes the first rank, as upon it depends the value of the solar parallax to be derived from a given observed value of this equation Arranging DELAUNAY's terms according to the power of m, which enters as a factor, the result will be that given below under the head P_1. DELAUNAY omits terms in γ^2 after m^3, and terms in e^2 after m^5. Correcting the result for an estimated value of these terms, derived by induction, we shall have those given under the head P_2. It will be seen that the terms follow a nearly regular law up to m^6, but that m^7 deviates from this law. Assuming this term to be in error, and estimating the value of it and the higher terms as those of a geometrical progression with the ratio $\frac{4}{10}$ we have the results P_3.

	P_1	P_2	P_3
Terms in m	$-$ 73″.1760	$-$ 73″.18	$-$ 73″.18
m^2	$-$ 34 .3021	$-$ 34 .30	$-$ 34 .30
m^3	$-$ 12 .0082	$-$ 12 .01	$-$ 12 .01
m^4	$-$ 4 .6812	$-$ 4 .50	$-$ 4 .50
m^5	$-$ 1 .9815	$-$ 1 .89	$-$ 1 .89
m^6	$-$ 0 .7122	$-$ 0 .72	$-$ 0 .72
m^7	$-$ 0 .3811	$-$ 0 .38	$-$ 0 .48
Sum	$-$ 127″.2423	$-$ 126″.98	$-$ 127″.08

Our choice must lie between the results P_2 and P_3. If we adopt the former we may add 0″.26 as an estimate of omitting terms giving :—

$$P = - 127''.24 ; \; P' = \frac{79}{81} P = - 124''.10.$$

* It may be remarked that in the series for the secular acceleration DELAUNAY found the terms of a higher order actually to change their sign, directly contrary to the estimate which would have been formed from those of a lower order.

If we adopt the latter we have

$$P = -127''.08; \qquad P' = \frac{79}{81} P = -123''.94.$$

Multiplying by the coefficient 1.0112 to reduce to the parallax 8''.848 the result will be:—

$$\begin{array}{ll} (2) & -125''.49 \\ (3) & -125''.33. \end{array}$$

HANSEN's coefficient, $-125''.43$, falls between these results and may be regarded as certainly correct within less than 0''.1.

The other term referred to is that depending on the argument:—

$$g - g' + 2\omega - 2\omega',$$

of which the principal parts of the coefficient are, in DELAUNAY's theory,—

3d order, ...	$-53''.10$
4th order, ...	$-5''.80$
5th order, ...	$+10''.15$
6th order, ...	$+9''.34$
7th order, ...	$+5''.62$
8th order, ...	$+2''.90$
9th order, ...	$+1''.43$

DELAUNAY seems to have taken 1''.18 as the probable sum of the omitted terms, whereas they should have been taken as 0''.94 to agree with HANSEN.

§ 3.

LATITUDE.

Taking HANSEN's expression for the moon's latitude:—

$$\sin \beta = \sin I \sin (f + \omega) + s;$$

the first step is to form the expression $\sin (f + \omega)$ in terms of g, ω, etc. This may be done in two ways. By the first we express the required quantity as a function of z, and

then put $g + n\delta z$ for g and develop in powers of $n\delta z$. By the theory of elliptic motion the expression of $\sin(f + \omega)$ in terms of z will be

$$
\begin{aligned}
\sin(f+\omega) = & - \frac{625}{9216} e^6 \sin(-5z+\omega) \\
& - \frac{1}{15} e^5 \sin(-4z+\omega) \\
& + \left(-\frac{9}{128} e^4 + \frac{9}{320} e^6\right) \sin(-3z+\omega) \\
& + \left(-\frac{1}{12} e^3 + \frac{1}{48} e^5\right) \sin(-2z+\omega) \\
& + \left(-\frac{1}{8} e^2 + \frac{1}{48} e^4 + \frac{37}{3072} e^6\right) \sin(-z+\omega) \\
& - e \sin\omega \\
& + \left(1 - e^2 + \frac{7}{64} e^4 - \frac{5}{288} e^6\right) \sin(z+\omega) \\
& + \left(e - \frac{5}{4} e^3 + \frac{17}{48} e^5\right) \sin(2z+\omega) \\
& + \left(\frac{9}{8} e^2 - \frac{27}{16} e^4 + \frac{765}{1024} e^6\right) \sin(3z+\omega) \\
& + \left(\frac{4}{3} e^3 - \frac{7}{3} e^5\right) \sin(4z+\omega) \\
& + \left(\frac{625}{384} e^4 - \frac{625}{192} e^6\right) \sin(5z+\omega) \\
& + \frac{81}{40} e^5 \sin(6z+\omega) \\
& + \frac{117649}{46080} e^6 \sin(7z+\omega).
\end{aligned}
$$

If we now substitute for z, $g + n\delta z$, for e its numerical value, and develop, putting:—

$$\sin I \sin(f+\omega) = F_0 + F_1 n\delta z + F_2 (n\delta z)^2 + F_3 (n\delta z)^3$$

we shall have

$$
\begin{aligned}
F_0 \text{ (in arc)} = & - 0''.012 \sin(-3g+\omega) \\
& - 0''.255 \sin(-2g+\omega) \\
& - 6''.968 \sin(-g+\omega) \\
& - 1015''.834 \sin\omega \\
& + 18447''.342 \sin(g+\omega) \\
& + 1012''.011 \sin(2g+\omega) \\
& + 62''.458 \sin(3g+\omega) \\
& + 4''.061 \sin(4g+\omega)
\end{aligned}
$$

$$
\begin{aligned}
F_0 \text{ (in arc) (cont'd)} = &+ 0''.272 \sin (5g+\omega)\\
&+ 0''.019 \sin (6g+\omega)\\
&+ 0''.001 \sin (7g+\omega)
\end{aligned}
$$

$$
\begin{aligned}
F_0 \text{ (in radius)} = &-.000\,0001 \sin (-3g+\omega)\\
&-.000\,0012 \sin (-2g+\omega)\\
&-.000\,0338 \sin (-g+\omega)\\
&-.004\,9249 \sin \omega\\
&+.089\,4352 \sin (g+\omega)\\
&+.004\,9064 \sin (2g+\omega)\\
&+.000\,3028 \sin (3g+\omega)\\
&+.000\,0197 \sin (4g+\omega)\\
&+.000\,0013 \sin (5g+\omega)\\
&+.000\,0001 \sin (6g+\omega)
\end{aligned}
$$

$$
\begin{aligned}
F_1 = &+.000\,0002 \cos (-3g+\omega)\\
&+.000\,0025 \cos (-2g+\omega)\\
&+.000\,0338 \cos (-g+\omega)\\
&+.089\,4352 \cos (g+\omega)\\
&+.009\,8127 \cos (2g+\omega)\\
&+.000\,9084 \cos (3g+\omega)\\
&+.000\,0788 \cos (4g+\omega)\\
&+.000\,0066 \cos (5g+\omega)\\
&+.000\,0005 \cos (6g+\omega)
\end{aligned}
$$

$$
\begin{aligned}
F_2 = &+.000\,02 \sin (-g+\omega)\\
&-.044\,72 \sin (g+\omega)\\
&-.009\,81 \sin (2g+\omega)\\
&-.001\,36 \sin (3g+\omega)\\
&-.000\,16 \sin (4g+\omega)\\
&-.000\,02 \sin (5g+\omega)
\end{aligned}
$$

$$
\begin{aligned}
F_3 = &-.0149 \cos (g+\omega)\\
&-.0065 \cos (2g+\omega)\\
&-.0014 \cos (3g+\omega)
\end{aligned}
$$

As a check upon the value of $\sin I \sin (f+\omega)$ a second method of computing it was adopted, as follows. Let us put:—

$$\delta f = f - g.$$

Then

$$
\begin{aligned}
\sin (f+\omega) &= \sin (g+\omega+\delta f)\\
&= \cos \delta f \sin (g+\omega)\\
&\quad + \sin \delta f \cos (g+\omega).
\end{aligned}
$$

From the numerical value of δf already given the powers of this quantity were formed, and thence its cosine and sine from the formulæ:—

$$\cos \delta f = 1 - \frac{\delta f^2}{1.2} + \text{etc.}$$

$$\sin \delta f = \delta f - \frac{\delta f^3}{1.2.3} + \text{etc.}$$

These expressions were then multiplied by the sine and cosine of $(g + \omega)$.

The mean difference between the coefficients in $\sin I \sin (f + \omega)$ found by the two methods was less than $0''.003$, the largest one being $''.010$.

Adding HANSEN'S s to this expression we have the value of $\sin \beta$. Then β itself is obtained by the formula

$$\beta = \sin \beta + \frac{1}{6} \sin^3 \beta + \frac{3}{40} \sin^5 \beta.$$

The principal parts of β are given in Table IV, of which the columns referring to HANSEN'S theory seem to need no explanation.

§ 4.

REDUCTION OF THE LATITUDE AND COMPARISON WITH DELAUNAY.

All the terms of the latitude contain the inclination of the moon's orbit as a factor, and are therefore to be multiplied by such a constant coefficient that the principal term of the latitude shall agree with observation. The transformed expressions of HANSEN, given in Table IV, lead to a consistent theory in which the coefficient of the principal term of the latitude is $18463''.248$. The expressions of DELAUNAY also lead to a theory, in which this coefficient is $18461''.26$. Each of these is to be multiplied by such a factor as shall reduce it to the value implicitly adopted in HANSEN'S tables. There HANSEN adopts:—

$$I = 5^\circ \ 8' \ 39''.96,$$

which is less by $8''.04$ than that of the theory. Hence, from this alone would follow the correction:—

$$-8''.04 \sin (f + \omega).$$

But, the tables contain, among the perturbations, two terms which depend mainly on the same argument, namely:—

$$2''.705 \sin (f + \omega),$$

which, developed by putting $g + 2e \sin g$ for f, appears as a perturbation, and

$$3''.70 \sin (g + \omega),$$

which is attributed to the separation of the centers of figure and of gravity of the moon. The sum of the first two expressions being developed, become

$$\begin{aligned}&-5''.319 \sin (g+\omega)\\&+0''.293 \sin \omega\\&-0''.292 \sin (2g+\omega).\end{aligned}$$

Adding the third, the term in $g+\omega$ will become

$$-\ 1''.619 \sin (g+\omega).$$

We are not concerned with the terms in ω and $2g+\omega$. The greater part of their amount may be considered as a *quasi* perturbation, due to the figure of the moon, and implicitly contained in the tables, but not belonging to the problem of three bodies.

With the last correction the term in $g+\omega$ becomes

$$18461''.629 \sin (g+\omega),$$

which is the coefficient implicitly contained in HANSEN's tables.

To this the writer found a correction of $-0''.15$ from Greenwich and Washington observations 1862–'74, but it will be retained without change. Hence all the coefficients in HANSEN's β, as given in Table IV, are to be diminished by the factor

$$.000088,$$

and those of DELAUNAY are to be increased by the factor

$$.000020.$$

The terms in e and e' are to be modified by the same coefficients as in the case of the longitude. The only terms which will be appreciably affected by the change of e are those depending on ω and $2g+\omega$.

The modifications here indicated have not been made in the results, because they are so slight, and affect so few terms, that each one can make them for himself.

The column *Delaunay* (1) contains, as before, the sum of the terms actually computed by DELAUNAY, and given by him in the *Connaissances des Temps* for 1869.

In column *Delaunay* (2) his coefficients are corrected by the higher terms, of which the value has been estimated by induction. DELAUNAY himself did not give these additions, so that they had to be estimated by the writer.

§ 5.

PARALLAX.

HANSEN's theory gives the perturbations of the natural logarithm of the moon's radius vector, which are the negative of the perturbations of the logarithm sine parallax. The value of w, in seconds of arc, is found in the *Darlegung*, Part I, pages 409–411, and Part II, pages 224–226, 258, and 268. The moon's parallax p is given by HANSEN under the form

$$\log \sin p = \log \frac{D\,(1+e\cos f)}{a\,(1-e^2)} - w,$$

in which D is the radius of the earth at the latitude of which the sine is $\sqrt{\frac{1}{3}}$, and a the moon's mean distance in the HANSENIAN theory, which is different in definition from the mean distance of the ordinary theories. It is not, however, necessary to reduce the one to the other directly, because they may be most satisfactorily compared by the values of the constant of parallax to which they lead.

Changing the logarithms to natural quantities and developing in powers of w, the above expression gives:—

$$\sin p = \frac{D}{a} \cdot \frac{1 + e\cos f}{1 - e^2}\left(1 - w + \frac{w^2}{2} - \text{etc.}\right)$$

and then

$$p = \sin p + \frac{\sin^3 p}{6} + \text{etc.}$$

In developing $e \cos f$ two methods of computation were used, as in the computation of the principal term of the latitude.

1. From CAYLEY'S tables we have

$$\begin{aligned}\cos f = -e &+ \left(1 - \frac{9}{8}e^2 + \frac{25}{192}e^4\right)\cos z \\ &+ \left(e - \frac{4}{3}e^3\right)\cos 2z \\ &+ \left(\frac{9}{8}e^2 - \frac{225}{128}e^4\right)\cos 3z \\ &+ \frac{4}{3}e^3 \cos 4z \\ &+ \frac{625}{384}e^4 \cos 5z\end{aligned}$$

and then by substituting $g + n\delta z$ for z we have $\cos f$ developed in multiples of g, etc

2. Putting

$$f = g + \delta f$$

we have

$$\cos f = \cos \delta f \cos g - \sin \delta f \sin g.$$

The value of $\frac{D}{a}$ was derived by HANSEN from the length of the seconds pendulum and the dimensions of the earth as found by BESSEL. The derivation is given in the *Astronomische Nachrichten*, Volume XVII, page 300. The data made use of are:—

D, radius of earth under the parallel arc sin $\sqrt{\frac{1}{3}}$ . .	6370063 metres
P, length of seconds pendulum under same parallel .	$0^{m}.992666$
m, mass of the moon	$\frac{1}{80}$.

The result is

$$\log \frac{D}{a} = 8.2170139.$$

He gives as the resulting constant part of the sine of the parallax

$$3422''.06,$$

and the changes in the constant produced by small changes in the data:—

Increase of $0^{mm}.1$ in P varies the constant by	$-0''.11$
Increase of 1000^{m} in D varies the constant by	$+0''.18$
Increase of unity in denominator of m	$+0''.17$

The development subsequently given leads to a constant of

$$3422''.09,$$

a result $0''.03$ greater than that stated by HANSEN.

In comparing the parallaxes of HANSEN and DELAUNAY the only element which will materially affect the result is the constant of parallax: a comparison of the different values of this constant, which have been recently obtained, will therefore be of interest. Three distinct methods of obtaining this important element have been applied.

(α). The theoretical method founded on KEPLER'S third law as expressed in the theory of gravitation, and derived fundamentally from the equation

$$a^3 n^2 = m + M,$$

a being the mean distance of the moon, which is immediately connected with the parallax; n the mean motion, of the value of which there is no doubt, and m and M the masses of the moon and earth, expressed in appropriate units, the determination of which is the most doubtful part of the problem.

(β). Measures of the moon's position made at two distant stations, and reduced to a common moment.

(γ). Meridian declinations of the moon made at the same station, and reduced on the hypothesis that the undisturbed geocentric orbit is a great circle.

The last method is not well adapted to give a certain result, owing to the constant errors with which measures of absolute declinations are affected. We shall therefore confine our consideration to the first two.

Two determinations by method (α), that of HANSEN, just quoted, and that of ADAMS in the *Monthly Notices*, Vol. XIII, and the British *Nautical Almanac* for 1856, are available.

The data used by Mr. ADAMS are:—

D, from BESSEL, and therefore the same as HANSEN.

P,* 3.256 89 English feet, or . . . $0^{m}.992712$.

m, mass of moon, ... $\frac{1}{81.5}$.

* This value in English feet was kindly communicated by Mr. ADAMS himself, not being explicitly quoted in his published paper.

The resulting value of the constant of the sine is given as $3422''.325$. To compare it with HANSEN we have:—

$$\begin{array}{llll} \text{Change in } D = 0, & - - - & \text{change of } \pi_0 = 0 \\ \text{“} \quad \text{“} \quad P = +0^{m\,m}.046, & \text{“} & \text{“} & -0''.05 \\ \text{“} \quad \text{“} \quad \frac{1}{m} = +1.5, & \text{“} & \text{“} & +0''.26 \end{array}$$

Applying the correction $+0''.21$ to HANSEN's constant, the result would be either $3422''.27$ or $3422''.30$, according as we accept HANSEN's original constant or that deduced from the data of his lunar tables. The latter is probably the value to be preferred.

If we reduce the values both of HANSEN and ADAMS to HANSEN's data, according to the system already adopted, the results will be:—

Constant of sine parallax,	HANSEN,	$3422''.09$.
“ “ “	ADAMS,	$3422''.12$.
Constant of parallax itself,	HANSEN,	$3422''.25$.
“ “ “	ADAMS,	$3422''.28$.

The constant of reduction from the sine to the parallax itself is $+0''.157$.

β. The most recent determinations of the moon's parallax by measurement are those of Mr. BREEN (Memoirs R. A. S. XXXII) and of Mr. STONE (Ibid. XXXIV). Both are founded on Cape observations and both lead to a constant of

$$3422''.70.$$

It is not distinctly stated whether this is the constant of the parallax itself or of its sine. Mr. BREEN's introduction (l. c. pp. 116, 117) seems to imply that he used Mr. ADAMS's expression for sine parallax as the parallax itself in reducing the Cape observations. But, in the reduction of the Greenwich observations, he applies ADAMS's correction to the parallax of AIRY's lunar reductions, which gives the parallax itself. To put the matter into another shape: On p. 116 Mr. BREEN has $3422''.32$ as the constant of parallax. On p. 132 he has a constant correction of $0''.68$ to the AIRY-PLANA parallax, of which the constant is $3421''.80$, which gives $3422''.48$ as the constant of parallax.

We shall probably make a near approximation to the truth by assuming that Mr. BREEN's mean provisional constant was $3422''.40$, and as he deduced a correction of $+0''.38$ this would give us his result:—

Constant of parallax, ...	$3422''.78$
Constant of sine, ...	$3422''.62$

Mr. STONE also finds a correction of $+0''.38$ to Mr. ADAMS's parallax. This would give:—

Constant of parallax, ...	$3422''.86$
Constant of sine, ...	$3422''.70$

The evidence is therefore in favor of a positive correction to HANSEN's constant; but, in accordance with the practice in other parts of this paper, the results as printed are all founded on HANSEN's fundamental data.

In the Table V the columns contain—

(1). The value of $\frac{D}{a} \cdot \frac{1}{1-e^2}(1 + e \cos f)$, expressed in seconds of arc.

(2). The product of this quantity by $-w + \frac{w^2}{2}$.

(3). The coefficients for HANSEN's sine parallax, formed by adding (1) and (2).

If the parallax itself is required, it may be found by adding the reduction from the sine to the parallax itself, namely:—

$$\begin{aligned} +0''.157 \quad & +0''.025 \cos g \\ & +0''.004 \cos (g - 2g^1 + 2\omega - 2\omega^1) \\ & +0''.004 \cos (2g - 2g^1 + 2\omega\ 2\omega^1). \end{aligned}$$

(4). The coefficients of DELAUNAY's sine parallax, so far as actually computed by him. As he stopped at the terms of the fifth order, the hundredths of seconds are not always definitive.

(5). The same, with the addition of quantities estimated by induction to repre sent the omitted terms of higher orders.

(6) The corrections applied in the preceding column to obtain the most probable values of the coefficients.

(7). The deviation of HANSEN's coefficients from the second set of DELAUNAY's.

As some of DELAUNAY's terms are doubtful from the insufficient convergence of his series, the coefficients of ADAMS's parallax, found in the *Monthly Notices R. A. S.*, Vol. XIII, p. 263, have been added for comparison. It will be seen that they agree closely with the coefficients of HANSEN, though derived independently of them.

TABLE I.—*Value of $n\delta z$, from* HANSEN, *together with its powers.*

g	g'	$n\delta z$ sin.	$(n\delta z)^2$ cos.	$(n\delta z)^3$ sin.	$(n\delta z)^4$ cos.
		″	″	″	″
0	0	0.000	+60.860	0.000	+ 0.035
1	0	0.000	+46.934	− 0.008	+ 0.043
2	0	− 4.604	+ 0.899	− 0.004	+ 0.009
3	0	− 0.176	+ 0.015	− 0.002	. .
4	0	− 0.009	+ 0.002	. .	. .
−3	−1	+ 0.029	− 0.003	. .	. .
−2	−1	+ 1.097	− 0.005	+ 0.029	. .
−1	−1	+ 73.234	+ 1.557	+ 0.288	+ 0.002
0	−1	+ 657.468	+ 5.093	+ 0.638	+ 0.005
1	−1	+ 111.681	+ 3.177	+ 0.320	+ 0.004
2	−1	+ 1.215	+ 0.125	+ 0.042	+ 0.001
3	−1	+ 0.026	+ 0.005	+ 0.001	. .
−2	−2	+ 0.002	− 0.017	+ 0.001	. .
−1	−2	+ 0.800	− 0.185	+ 0.014	. .
0	−2	+ 7.319	− 0.953	+ 0.034	. .
1	−2	+ 2.159	− 0.233	+ 0.023	. .
2	−2	+ 0.035	− 0.030	+ 0.003	. .
3	−2	. .	− 0.001	. .	. .
−1	−3	+ 0.011	− 0.005	. .	. .
0	−3	+ 0.075	− 0.019	. .	. .
1	−3	+ 0.044	− 0.007	. .	. .
$2\omega-2\omega'$					
−1	0	. .	+ 0.003	− 0.001	. .
0	0	− 0.094	+ 0.043	− 0.010	. .
1	0	− 2.524	+ 0.082	− 0.039	. .
2	0	− 0.052	+ 0.010	− 0.024	. .
3	0	. .	− 0.002	− 0.004	. .
−1	−1	− 0.040	. .	+ 0.001	. .
0	−1	+ 3.665	+ 2.324	+ 0.046	+ 0.007
1	−1	− 27.620	+15.444	+ 0.085	+ 0.015
2	−1	− 23.006	+ 8.499	+ 0.045	+ 0.012
3	−1	− 1.337	+ 0.984	+ 0.007	+ 0.003
4	−1	− 0.066	+ 0.041	. .	. .
−2	−2	− 0.026	− 0.003	. .	. .
−1	−2	− 1.893	− 0.085	+ 0.007	. .
0	−2	− 41.648	+ 0.395	+ 0.762	. .
1	−2	+ 4466.992	+ 1.867	+ 2.381	+ 0.002
2	−2	+ 2144.995	+ 1.494	+ 1.750	. .
3	−2	+ 60.020	+ 0.420	+ 0.414	. .
4	−2	+ 2.083	+ 0.071	+ 0.018	. .
5	−2	+ 0.084	+ 0.004	. .	. .
6	−2	+ 0.004	. .	. .	. .
−1	−3	− 0.082	− 0.006	. .	. .
0	−3	− 2.352	− 1.437	+ 0.061	− 0.005
1	−3	+ 198.103	−14.857	+ 0.216	− 0.015
2	−3	+ 155.047	− 9.149	+ 0.202	− 0.011
3	−3	+ 5.166	− 1.341	+ 0.059	− 0.003
4	−3	+ 0.198	− 0.047	+ 0.003	. .
5	−3	+ 0.009	− 0.001	. .	. .

g	g'	$n\delta z$ sin.	$(n\delta z)^2$ cos.	$(n\delta z)^3$ sin.	$(n\delta z)^4$ cos.
$2\omega-2\omega'$		″	″	″	″
0	−4	− 0.109	− 0.077	− 0.004	. .
1	−4	+ 7.035	− 0.846	− 0.028	. .
2	−4	+ 7.738	− 0.718	− 0.014	. .
3	−4	+ 0.287	− 0.122	− 0.003	. .
4	−4	+ 0.011	− 0.006	. .	. .
0	−5	. .	− 0.003	. .	. .
1	−5	+ 0.240	− 0.035	− 0.002	. .
2	−5	+ 0.329	− 0.037	− 0.002	. .
3	−5	+ 0.012	− 0.007	. .	. .
2ω					
1	2	. .	− 0.003	. .	. .
2	2	+ 0.002	0.000	. .	. .
−1	1	. .	+ 0.003	. .	. .
0	1	+ 0.037	− 0.039	− 0.001	. .
1	1	− 0.351	− 0.210	− 0.012	. .
2	1	+ 0.127	+ 0.012	− 0.006	. .
3	1	+ 0.001	+ 0.003	. .	. .
−1	0	+ 0.070	. .	. .	. .
0	0	+ 5.846	− 0.266	− 0.024	. .
1	0	− 85.224	+ 1.633	− 0.072	. .
2	0	+ 4.303	+ 0.941	− 0.025	. .
3	0	+ 0.094	+ 0.073	+ 0.001	. .
4	0	+ 0.001	+ 0.003	. .	. .
0	−1	+ 0.046	+ 0.004	− 0.001	. .
1	−1	+ 0.279	+ 0.295	+ 0.005	. .
2	−1	+ 0.119	+ 0.077	+ 0.005	. .
3	−1	+ 0.003	+ 0.003	+ 0.001	. .
1	−2	. .	+ 0.003	. .	. .
2	−2	+ 0.004	+ 0.002	. .	. .
$2\omega'$					
−1	4	. .	− 0.004	. .	. .
0	4	− 0.114	− 0.017	. .	. .
−3	3	. .	− 0.002	. .	. .
−2	3	+ 0.012	+ 0.010	. .	. .
−1	3	+ 0.604	− 0.069	+ 0.002	. .
0	3	− 3.449	− 0.346	+ 0.004	. .
1	3	− 0.152	− 0.036	− 0.001	. .
−3	2	+ 0.005	+ 0.002	. .	. .
−2	2	+ 0.162	+ 0.038	+ 0.003	. .
−1	2	+ 10.938	− 0.756	− 0.021	. .
0	2	− 81.905	− 1.779	− 0.071	. .
1	2	− 4.722	+ 0.116	− 0.022	. .
2	2	− 0.075	. .	− 0.002	. .
−2	1	− 0.006	− 0.005	. .	. .
−1	1	− 0.195	+ 0.006	− 0.004	. .
0	1	+ 2.064	+ 0.274	− 0.007	. .
1	1	− 0.043	+ 0.058	− 0.002	. .
2	1	. .	+ 0.003	. .	. .
0	0	+ 0.012	− 0.002	. .	. .
0	−1	. .	. .	+ 0.003	. .

Table I.—*Value of* $n\delta z$, *&c.*—Continued.

g	g'	$n\delta z$ sin.	$(n\delta z)^2$ cos.	$(n\delta z)^3$ sin.	$(n\delta z)^4$ cos.
$2\omega+2\omega'$		″	″	″	″
0	2	. .	+ 0.007	. .	. .
1	2	− 0.014	− 0.033	. .	. .
2	2	+ 0.006	. .	. .	. .
$\omega-\omega'$					
0	1	+ 0.007	+ 0.010	. .	. .
1	1	− 0.031	+ 0.052	+ 0.001	. .
2	1	. .	+ 0.007	. .	. .
−1	0	+ 0.290	− 0.013	+ 0.002	. .
0	0	+ 0.316	− 0.099	+ 0.017	. .
1	0	+ 17.566	− 0.382	+ 0.019	. .
2	0	+ 0.259	− 0.044	+ 0.005	. .
−1	−1	− 0.564	− 0.021	− 0.005	− 0.001
0	−1	− 11.400	− 2.709	− 0.052	− 0.002
1	−1	− 121.335	− 1.631	− 0.113	− 0.002
2	−1	− 1.619	− 0.163	− 0.042	. .
3	−1	− 0.037	− 0.012	− 0.001	. .
−1	−2	− 0.009	+ 0.006	− 0.001	. .
0	−2	− 0.147	+ 0.343	− 0.016	. .
1	−2	− 0.562	+ 0.476	− 0.014	. .
2	−2	− 0.081	+ 0.074	− 0.004	. .
3	−2	− 0.006	+ 0.004	. .	. .
0	−3	− 0.007	+ 0.012	+ 0.001	. .
1	−3	+ 0.041	+ 0.015	+ 0.002	. .
$3\omega-3\omega'$					
2	−1	. .	+ 0.003	+ 0.001	. .
3	−1	. .	+ 0.002	+ 0.001	. .
0	−2	. .	− 0.007	. .	. .
1	−2	− 0.038	− 0.011	− 0.003	. .
2	−2	+ 0.272	− 0.410	− 0.014	. .
3	−2	+ 0.123	− 0.201	− 0.007	. .
4	−2	+ 0.009	− 0.009	. .	. .
0	−3	− 0.002	+ 0.008	+ 0.001	. .
1	−3	− 1.092	+ 0.210	− 0.044	. .
2	−3	− 3.154	+ 2.708	− 0.049	+ 0.003
3	−3	+ 0.621	+ 1.279	− 0.017	+ 0.001
4	−3	+ 0.018	+ 0.053	− 0.002	. .
5	−3	+ 0.001	. .	. .	. .
1	−4	− 0.066	+ 0.018	+ 0.005	. .
2	−4	− 0.229	+ 0.150	+ 0.014	. .
3	−4	+ 0.078	+ 0.100	+ 0.009	. .
4	−4	+ 0.003	+ 0.006	+ 0.001	. .
2	−5	− 0.012	+ 0.004	. .	. .
3	−5	+ 0.007	+ 0.004	. .	. .
$\omega+\omega'$					
1	2	. .	+ 0.007	. .	. .
−1	1	. .	− 0.001	. .	. .
0	1	+ 0.050	+ 0.045	+ 0.002	. .
1	1	+ 0.757	− 0.047	+ 0.002	. .
2	1	. .	− 0.004	. .	. .

g	g'	$n\delta z$ sin.	$(n\delta z)^2$ cos.	$(n\delta z)^3$ sin.	$(n\delta z)^4$ cos.
$\omega+\omega'$		″	″	″	″
0	0	+ 0.049	− 0.007	. .	. .
1	0	+ 0.024	− 0.001	. .	. .
$3\omega-\omega'$					
2	0	. .	+ 0.007	. .	. .
1	−1	. .	− 0.002	. .	. .
2	−1	+ 0.037	− 0.069	− 0.001	. .
3	−1	+ 0.010	− 0.007	. .	. .
1	−2	. .	− 0.001	. .	. .
2	−2	. .	− 0.002	. .	. .
$\omega-3\omega'$					
1	−2	. .	− 0.008	. .	. .
2	−2	. .	+ 0.002	. .	. .
0	−3	. .	+ 0.024	− 0.001	. .
1	−3	− 0.324	+ 0.057	+ 0.001	. .
2	−3	+ 0.005	− 0.004	. .	. .
1	−4	. .	+ 0.004	. .	. .
$4\omega-4\omega'$					
1	−2	. .	+ 0.002	. .	. .
2	−2	− 0.033	+ 0.054	. .	. .
3	−2	− 0.018	+ 0.023	− 0.002	. .
0	−3	. .	+ 0.002	. .	. .
1	−3	+ 0.042	− 0.066	+ 0.035	. .
2	−3	− 0.350	+ 0.672	+ 0.269	− 0.002
3	−3	− 0.608	+ 0.917	+ 0.260	− 0.001
4	−3	− 0.236	+ 0.323	+ 0.085	. .
5	−3	− 0.023	+ 0.028	+ 0.010	. .
0	−4	− 0.026	+ 0.036	− 0.001	. .
1	−4	+ 0.886	+ 0.924	+ 0.013	− 0.010
2	−4	+ 30.040	− 17.890	+ 0.066	− 0.039
3	−4	+ 35.723	− 46.370	+ 0.077	− 0.040
4	−4	+ 10.683	− 12.420	+ 0.035	− 0.018
5	−4	+ 0.775	− 0.665	+ 0.006	− 0.003
6	−4	+ 0.048	− 0.033	. .	. .
7	−4	+ 0.003	− 0.002	. .	. .
0	−5	. .	+ 0.004	. .	. .
1	−5	+ 0.047	+ 0.079	− 0.020	− 0.001
2	−5	+ 2.666	− 4.341	− 0.246	− 0.004
3	−5	+ 4.140	− 5.544	− 0.259	− 0.007
4	−5	+ 1.508	− 1.834	− 0.094	− 0.004
5	−5	+ 0.118	− 0.112	− 0.013	. .
6	−5	+ 0.006	− 0.006	. .	. .
1	−6	. .	+ 0.004	− 0.002	. .
2	−6	[illegible]	− 0.254	− 0.026	. .
3	−6	+ 0.296	− 0.406	− 0.034	+ 0.001
4	−6	+ 0.125	− 0.158	− 0.013	. .
5	−6	+ 0.010	− 0.010	− 0.002	. .
2	−7	. .	− 0.012	− 0.001	. .
3	−7	+ 0.016	− 0.022	− 0.002	. .
4	−7	+ 0.008	− 0.009	. .	. .

TABLE I.—*Value of* $n\delta z$, *&c.*—Continued.

g	g'	$n\delta z$ sin.	$(n\delta z)^2$ cos.	$(n\delta z)^3$ sin	$(n\delta z)^4$ cos.
$4\omega-2\omega'$		″	″	″	″
2	−1	+ 0.002	− 0.005	− 0.008	. .
3	−1	+ 0.008	− 0.008	− 0.004	. .
4	−1	. .	− 0.002	. .	. .
1	−2	+ 0.015	− 0.146	− 0.004	. .
2	−2	− 1.092	+ 1.796	+ 0.024	. .
3	−2	− 0.602	+ 0.805	+ 0.026	. .
4	−2	+ 0.010	− 0.017	+ 0.008	. .
1	−3	+ 0.002	− 0.008	+ 0.001	. .
2	−3	− 0.044	+ 0.076	+ 0.009	. .
3	−3	− 0.041	+ 0.058	+ 0.008	. .
4	−3	+ 0.002	. .	+ 0.002	. .
2	−4	. .	+ 0.003	. .	. .
3	−4	. .	+ 0.003	. .	. .
$2\omega-4\omega'$					
0	−3	. .	− 0.001	+ 0.001	. .
1	−3	− 0.023	+ 0.058	+ 0.009	. .
2	−3	− 0.012	+ 0.027	+ 0.004	. .
0	−4	+ 0.020	− 0.086	+ 0.001	. .
1	−4	+ 0.214	− 1.834	− 0.028	− 0.002
2	−4	+ 0.228	− 0.628	− 0.025	. .
3	−4	− 0.066	+ 0.090	− 0.006	. .
4	−4	− 0.005	+ 0.004	. .	. .
0	−5	. .	− 0.006	− 0.001	. .
1	−5	+ 0.021	− 0.162	− 0.011	. .
2	−5	+ 0.030	− 0.078	− 0.007	. .
3	−5	− 0.008	+ 0.010	− 0.002	. .
1	−6	. .	− 0.008	. .	. .
2	−6	. .	− 0.006	. .	. .
$5\omega-5\omega'$					
3	−4	. .	− 0.009	− 0.006	. .
4	−4	. .	− 0.010	− 0.006	. .
5	−4	. .	− 0.002	− 0.001	. .
2	−5	. .	+ 0.026	+ 0.003	+ 0.001
3	−5	− 0.056	+ 0.099	+ 0.045	+ 0.001
4	−5	− 0.005	+ 0.042	+ 0.042	. .
5	−5	+ 0.007	− 0.001	+ 0.011	. .
2	−6	. .	+ 0.002	. .	. .
3	−6	− 0.006	+ 0.012	+ 0.004	. .
4	−6	− 0.001	+ 0.003	+ 0.005	. .
5	−6	+ 0.002	. .	+ 0.001	. .

g	g'	$n\delta z$ sin.	$(n\delta z)^2$ cos.	$(n\delta z)^3$ sin.	$(n\delta z)^4$ cos.
$6\omega-6\omega'$		″	″	″	″
2	−5	. .	− 0.002	− 0.001	. .
3	−5	− 0.004	+ 0.011	+ 0.011	− 0.003
4	−5	− 0.011	+ 0.028	+ 0.022	− 0.005
5	−5	− 0.008	+ 0.018	+ 0.014	− 0.003
6	−5	. .	+ 0.004	+ 0.002	. .
2	−6	+ 0.009	− 0.013	+ 0.015	. .
3	−6	+ 0.285	− 0.652	− 0.508	. .
4	−6	+ 0.538	− 1.082	− 0.749	. .
5	−6	+ 0.334	− 0.610	− 0.381	. .
6	−6	+ 0.084	− 0.138	− 0.079	. .
7	−6	+ 0.009	− 0.012	− 0.006	. .
2	−7	. .	− 0.001	+ 0.002	. .
3	−7	+ 0.037	− 0.088	− 0.070	+ 0.003
4	−7	+ 0.085	− 0.176	− 0.126	+ 0.005
5	−7	+ 0.061	− 0.116	− 0.076	+ 0.003
6	−7	+ 0.016	− 0.029	− 0.017	. .
7	−7	. .	− 0.002	− 0.001	. .
3	−8	. .	− 0.007	− 0.005	. .
4	−8	. .	− 0.016	− 0.012	. .
5	−8	. .	− 0.011	− 0.007	. .
6	−8	. .	− 0.002	− 0.001	. .
$6\omega-4\omega'$					
2	−4	. .	− 0.001	− 0.002	. .
3	−4	− 0.011	+ 0.035	+ 0.028	. .
4	−4	− 0.016	+ 0.038	+ 0.027	. .
5	−4	− 0.005	+ 0.009	+ 0.006	. .
3	−5	. .	+ 0.003	− 0.003	. .
4	−5	. .	+ 0.005	+ 0.003	. .
$4\omega-6\omega'$					
1	−6	. .	− 0.001	− 0.001	. .
2	−6	+ 0.003	− 0.017	− 0.030	. .
3	−6	+ 0.005	− 0.019	− 0.025	. .
4	−6	+ 0.001	− 0.004	− 0.004	. .
5	−6	. .	+ 0.002	+ 0.001	. .
2	−7	. .	− 0.002	− 0.004	. .
3	−7	. .	− 0.004	− 0.004	. .

TABLE II.—*Principal parts of* HANSEN'S *Ecliptic Longitude, with the Coefficients of the Concluded Longitude.*

g	g'	sS_0	$n\delta z \times$ $(e,g)_1 - 1$	$R_{1,1}$	sS_1	$(n\delta z)^2 \times$ $(e,g)_2$	$R_{1,2}$	$(n\delta z)^3 \times$ $(e,g)_3$	Sum.	Terms in Ecliptic Longitude.	Principal Coefficient.
		"	"	"	"	"	"	"	"	"	
1	0	+ 1.103	− .253	− .169	+ .283	− 3.493	+ .003		− 2.526	+ 22637.150	e
2	0	− 1.058	− .010	− .017	+ .044	− 1.766			− 2.807	+ 768.858	e^2
3	0	− .117	− .253	− .002	+ .005	− .252			− .619	+ 36.112	e^3
4	0	− .010	− .027			− .028			− .065	+ 1.931	e^4
5	0		− .002			− .003			− .005	+ .113	.
6	0									+ .007	.
−5	−1		+ .003						+ .003	+ .003	.
−4	−1		+ .038			+ .001			+ .039	+ .039	.
−3	−1	− .001	+ .515			+ .008			+ .522	+ .551	$e^3 e'$
−2	−1	− .012	+ 6.525		− .003	+ .063		− .004	+ 6.569	+ 7.666	$e e'$
−1	−1	− .022	+ 36.563		− .012	+ .152		− .007	+ 36.674	+ 109.908	$e e'$
0	−1	+ 2.187	+ 10.157		+ .003	+ .045		− .006	+ 12.386	+ 669.852	e'
1	−1	+ .034	+ 36.424		+ .011	− .143		− .007	+ 36.319	+ 148.000	$e e'$
2	−1	− .009	+ 8.625			− .107		− .005	+ 8.504	+ 9.719	$e^2 e'$
3	−1	− .002	+ .665			− .018		− .001	+ .644	+ 670	$e^3 e'$
4	−1		+ .049			− .002			+ .047	+ .047	.
5	−1		+ .003						+ .003	+ .003	.
−3	−2		+ .005			− .002			+ .003	+ .003	.
−2	−2		+ .072			− .009			+ .063	+ .065	.
−1	−2		+ .410			− .026			+ .384	+ 1.184	$e e'^2$
0	−2	+ .027	+ .163			− .002			+ .188	+ 7.507	e'^2
1	−2		+ .407		+ .001	+ .026			+ .434	+ 2.593	$e e'^2$
2	−2		+ .146			+ .011			+ .157	+ .192	.
3	−2		+ .012			+ .002			+ .014	+ .014	.
−2	−3		+ .001						+ .001	+ .001	.
−1	−3		+ .004						+ .004	+ .015	.
0	−3		+ .003						+ .003	+ .078	.
1	−3		+ .004						+ .004	+ .048	.
2	−3		+ .003						+ .003	+ .003	.
$2\omega - 2\omega'$											
−1	0		− .015						− .015	− .015	.
0	0		− .139			+ .003			− .136	− .230	.
1	0	− .001	− .008			− .002			− .011	− 2.535	$e e'^2$
2	0	+ .006	− .139			− .003			− .136	− .188	.
3	0		− .013						− .013	− .013	.
−2	−1		+ .004			+ .014			+ .018	+ .018	.
−1	−1		+ .091			+ .126			+ .217	+ .177	.
0	−1	+ .004	− 1.605			+ .457			− 1.144	+ 2.521	$e^2 e'$
1	−1	− .046	− 1.067		+ .002	+ .173		− .001	− .939	− 28.559	$e e'$
2	−1	+ .494	− 1.576	+ .004	+ .037	− .405			− 1.446	− 24.452	e'
3	−1	+ .062	− 1.369		+ .010	− .292			− 1.589	− 2.926	$e e'$
4	−1	+ .006	− .168		+ .001	− .065			− .226	− .292	.
5	−1		− .015			− .009			− .024	− .024	.
6	−1		− .001			− .001			− .002	− .002	.
−4	−2		+ .005						+ .005	+ .005	.
−3	−2		+ .071						+ .071	+ .071	.
−2	−2	− .001	+ .980		− .001			− .003	+ .975	+ .949	.

TABLE II.—*The Moon's Longitude*—Continued.

g	g'	sS_0	$n\delta z\times$ $(e,g)_1-1$	$R_{1,1}$	sS_1	$(n\delta z)^2\times$ $(e,g)_2$	$R_{1,2}$	$(n\delta z)^3\times$ $(e,g)_3$	Sum.	Terms in Ecliptic Longitude.	Principal Coefficient.
$2\omega-2\omega'$		″	″	″	″	″	″	″	″	″	
−1	−2	− .013	+ 15.102	. .	−.011	+ .019	−.001	−.014	+ 15.082	+ 13.189	e^3
0	−2	+ .137	+ 253.133	+ .003	+.003	+ .059	−.004	−.026	+ 253.305	+ 211.655	e^2
1	−2	+ 3.304	+ 115.652	. .	. .	+ .032	−.002	−.024	+ 118.962	+ 4585.954	e
2	−2	−23.322	+ 248.293	− .166	+.016	− .041	−.004	−.028	+ 224.748	+ 2369.746	m^2
3	−2	− 2.653	+ 134.633	− .006	−.003	− .046	−.003	−.023	+ 131.899	+ 191.921	e
4	−2	− .246	+ 12.569	+ .001	−.006	− .018	−.001	−.008	+ 12.291	+ 14.374	e^2
5	−2	− .021	+ 1.003	. .	−.001	− .004	. .	−.001	+ .976	+ 1.060	e^3
6	−2	− .002	+ .077	. .	. .	. .	. .	. .	+ .075	+ .079	.
7	−2	. .	+ .005	. .	. .	. .	. .	. .	+ .005	+ .005	.
−3	−3	. .	+ .003	. .	. .	− .001	. .	. .	+ .002	+ .002	.
−2	−3	. .	+ .043	. .	. .	− .012	. .	. .	+ .031	+ .031	.
−1	−3	− .001	+ .558	. .	. .	− .099	. .	−.001	+ .557	+ .475	.
0	−3	+ .004	+ 11.453	. .	. .	− .443	. .	−.002	+ 11.012	+ 8.660	e^2e'
1	−3	+ .150	+ 8.399	. .	. .	− .217	. .	−.003	+ 8.329	+ 206.432	ee'
2	−3	− 1.009	+ 11.147	− .007	−.035	+ .377	−.001	−.002	+ 10.470	+ 165.517	e'
3	−3	− .126	+ 9.265	. .	−.011	+ .306	. .	−.003	+ 9.431	+ 14.597	ee'
4	−3	− .012	+ .921	. .	−.002	+ .078	. .	−.001	+ .984	+ 1.182	e^2e'
5	−3	. .	+ .076	. .	. .	+ .011	. .	. .	+ .087	+ .096	.
6	−3	. .	+ .006	. .	. .	+ .001	. .	. .	+ .007	+ .007	.
−2	−4	. .	+ .002	. .	. .	. .	. .	. .	+ .002	+ .002	.
−1	−4	. .	+ .023	. .	. .	− .005	. .	. .	+ .018	+ .018	.
0	−4	. .	+ .415	. .	. .	− .026	. .	. .	+ .389	+ .280	.
1	−4	+ .004	+ .419	. .	. .	− .013	. .	. .	+ .405	+ 7.440	ee'^2
2	−4	− .033	+ .402	. .	−.002	+ .020	. .	. .	+ .387	+ 8.125	e'^2
3	−4	− .004	+ .452	. .	. .	+ .023	. .	. .	+ .471	+ .758	.
4	−4	. .	+ .047	. .	. .	+ .006	. .	. .	+ .053	+ .064	.
5	−4	. .	+ .004	. .	. .	. .	. .	. .	+ .004	+ .004	.
0	−5	. .	+ .013	. .	. .	− .001	. .	. .	+ .012	+ .012	.
1	−5	. .	+ .018	. .	. .	− .001	. .	. .	+ .017	+ .257	.
2	−5	. .	+ .014	. .	. .	+ .001	. .	. .	+ .015	+ .344	.
3	−5	. .	+ .019	. .	. .	+ .001	. .	. .	+ .020	+ .032	.
4	−5	. .	+ .002	. .	. .	. .	. .	. .	+ .002	+ .002	.
2ω											
1	2	+ .002	. .	+ .004	. .	. .	−.001	. .	+ .005	+ .005	.
2	2	− .011	. .	+ .015	+.002	. .	−.002	. .	+ .004	+ .006	.
3	2	− .001	. .	+ .004	. .	. .	−.001	. .	+ .002	+ .002	.
0	1	+ .010	− .019	− .010	−.002	− .005	. .	. .	− .027	+ .010	.
1	1	+ .084	+ .009	+ .152	+.011	+ .002	+.006	. .	+ .264	− .087	.
2	1	− 1.054	− .019	+1.341	+.004	+ .006	+.012	. .	+ .290	+ .416	.
3	1	− .115	+ .006	+ .366	+ 001	+ .001	+.005	. .	+ .264	+ .265	.
4	1	− .010	. .	+ .052	. .	. .	+.001	. .	+ .043	+ .043	.
5	1	. .	. .	+ .006	. .	. .	. .	. .	+ .006	+ .006	.
−2	0	. .	+ .003	. .	. .	− .001	. .	. .	+ .002	+ .002	.
−1	0	. .	+ .001	. .	+.001	− .001	. .	. .	+ .001	+ .065	.
0	0	+ 1.180	− 4.657	− .009	+.015	+ .049	−.002	. .	− 3.424	+ 1.083	I^2e^2
1	0	− .926	+ .558	− .002	+ 263	+ .034	+.087	. .	+ .014	− 39.583	I^2e
2	0	− .053	− 4.650	. .	+.130	− .042	+.264	. .	− 4.351	− 411.674	I^2
3	0	+ .049	− .084	. .	+.005	− .032	+.158	. .	+ .096	− 45.091	I^2e
4	0	+ .007	− .002	+ .009	−.001	− .006	+.035	. .	+ .042	− 3.997	I^2e^2
5	0	. .	. .	+ .002	. .	− .001	+.008	. .	+ .009	− .329	.
6	0	. .	. .	. .	. .	. .	+.001	. .	+ .001	− .026	.

TABLE II.—*The Moon's Longitude*—Continued.

g	g'	$s\,S_0$	$n\,\delta z \times$ $(e,g)_1 - 1$	$R_{1,1}$	$s\,S_1$	$(n\,\delta z)^2 \times$ $(e,g)_2$	$R_{1,2}$	$(n\,\delta z)^3 \times$ $(e,g)_3$	Sum.	Terms in Ecliptic Longitude.	Principal Coefficient.
2ω		"	"	"	"	"	"	"	"	"	
−1	−1	. .	+.004	. .	. .	+.001	. .	. .	+.005	+.005	.
0	−1	−.007	+.016	+.006	+.002	+.008	. .	. .	+.025	+.071	.
1	−1	−.144	+.009	−.074	+.005	+.002	+.003	. .	−.199	+.080	.
2	−1	+1.104	+.016	−1.324	+.005	−.009	+.010	. .	−.198	−.078	.
3	−1	+.123	+.008	−.444	−.001	−.002	+.009	. .	−.307	−.304	.
4	−1	+.010	. .	−.065	. .	. .	+.002	. .	−.053	−.053	.
5	−1	. .	. .	−.007	. .	. .	. .	. .	−.007	−.007	.
1	−2	−.002	. .	−.001	. .	. .	. .	. .	−.003	−.003	.
2	−2	+.016	. .	−.015	+.002	. .	−.002	. .	+.001	+.005	.
3	−2	+.003	. .	−.007	. .	. .	−.001	. .	−.005	−.005	.
$2\omega'$											
−1	4	. .	−.007	. .	. .	. .	. .	. .	−.007	−.007	.
0	4	+.034	. .	+.015	−.001	. .	−.002	. .	+.046	−.068	.
1	4	. .	−.007	+.017	. .	. .	−.001	. .	+.009	+.009	.
2	4	. .	. .	+.002	. .	. .	. .	. .	+.002	+.002	.
−3	3	. .	+.002	. .	. .	. .	. .	. .	+.002	+.002	.
−2	3	. .	+.020	. .	. .	−.004	. .	. .	+.016	+.028	.
−1	3	+.014	−.190	−.006	−.008	−.010	−.002	. .	−.202	+.402	.
0	3	+1.032	+.025	+.288	−.032	+.001	−.018	. .	+1.296	−2.153	e'
1	3	−.024	−.187	+.446	+.006	+.010	−.035	. .	+.216	+.064	.
2	3	−.008	−.022	+.066	+.001	+.003	−.011	. .	+.029	+.029	.
3	3	. .	−.002	+.008	. .	. .	−.002	. .	+.004	+.004	.
−4	2	. .	+.002	. .	. .	. .	. .	. .	+.002	+.002	.
−3	2	+.001	+.028	. .	. .	−.003	. .	. .	+.026	+.031	.
−2	2	+.008	+.291	−.003	−.005	−.028	. .	. .	+.263	+.425	.
−1	2	+.094	−4.504	−.117	−.001	−.049	. .	. .	−4.577	+6.363	$I^2 e$
0	2	+23.660	+.342	+3.794	+.016	+.024	+.003	. .	+27.839	−55.262	I^2
1	2	−.676	−4.458	+9.628	. .	+.052	+.005	. .	+4.551	−.175	.
2	2	−.275	−.564	+1.472	. .	+.003	+.002	. .	+.638	+.561	.
3	2	−.028	−.044	+.167	. .	. .	. .	. .	+.095	+.095	.
4	2	−.003	−.003	+.018	. .	. .	. .	. .	+.012	+.012	.
5	2	. .	. .	+.002	. .	. .	. .	. .	+.002	+.002	.
−2	1	. .	−.003	. .	. .	+.001	. .	. .	−.002	−.008	.
−1	1	−.008	+.113	. .	+.006	+.007	+.002	. .	+.120	−.075	.
0	1	−.505	−.013	−.044	+.034	+.002	+.016	. .	−.510	+1.554	e'
1	1	−.020	+.113	−.063	−.005	−.008	+.035	. .	+.052	+.009	.
2	1	−.003	+.006	−.003	−.002	−.003	+.013	. .	+.008	+.008	.
3	1	. .	+.001	. .	. .	. .	+.002	. .	+.003	+.003	.
0	0	−.006	. .	. .	. .	. .	. .	. .	−.006	+.006	.
1	0	. .	. .	−.005	. .	. .	. .	. .	−.005	−.005	.
$2\omega + 2\omega'$											
2	3	−.003	. .	+.007	. .	. .	. .	. .	+.004	+.004	.
3	3	. .	. .	+.002	. .	. .	. .	. .	+.002	+.001	.
0	2	. .	+.001	−.001	+.001	. .	. .	. .	+.001	+.001	.
1	2	+.016	. .	−.031	−.004	. .	−.002	. .	−.021	−.034	.
2	2	−.081	−.001	+.160	. .	. .	−.004	. .	+.074	+.075	.
3	2	−.008	. .	+.036	. .	. .	−.001	. .	+.027	+.007	.
4	2	. .	. .	+.005	. .	. .	. .	. .	+.005	000	.
2	1	+.001	. .	−.004	. .	. .	. .	. .	−.003	−.003	.

TABLE II.—*The Moon's Longitude*—Continued.

g	g'	$s\,S_0$	$n\,\delta z\times$ $(e,g)_1-1$	$n\,\delta z\times$ $R_{1,1}$	$n\,\delta z\times$ $s\,S_1$	$(n\,\delta z)_2\times$ $(e,g)_2$	$(n\,\delta z)_2\times$ $(R_{1,2})$	$(n\,\delta z)^3\times$ $(e,g)_3$	Sum.	Terms in Ecliptic Longitude.	Principal Co-efficient.
$\omega-\omega'$		"	"	"	"	"	"	"	"	"	
0	1	. .	− .002	. .	. .	+ .002	. .	. .	.000	+ .007	.
1	1	. .	. .	. .	. .	. .	. .	. .	. .	− .031	.
2	1	. .	− .002	. .	. .	− .002	. .	. .	− .004	− .004	.
−2	0	. .	+ .022	. .	. .	. .	. .	. .	+ .022	+ .022	.
−1	0	. .	+ .083	. .	. .	− .004	. .	. .	+ .079	+ .369	π
0	0	+ .006	+ .981	. .	. .	− .010	. .	. .	+ .977	+ 1.293	$\pi\,ee'$
1	0	. .	+ .033	. .	. .	+ .002	. .	. .	+ .035	+ 17.601	$\pi\,e'$
2	0	. .	+ .965	. .	. .	+ .011	. .	. .	+ .976	+ 1.235	$\pi\,ee'$
3	0	. .	+ .080	. .	. .	+ .003	. .	. .	+ .083	+ .083	.
4	0	. .	+ .006	. .	. .	. .	. .	. .	+ .006	+ .006	.
−3	−1	. .	− .008	. .	. .	− .001	. .	. .	− .009	− .009	.
−2	−1	. .	− .107	. .	. .	− .011	. .	. .	− .118	− .118	.
−1	−1	. .	− 1.083	. .	. .	− .082	. .	. .	− 1.165	− 1.729	$\pi\,e^2$
0	−1	− .058	− 6.696	. .	. .	− .045	. .	+ .001	− 6.798	− 18.198	$\pi\,e$
1	−1	− .047	− .717	+ .002	− .007	+ .071	. .	+ .001	− .697	− 122.032	π
2	−1	+ .020	− 6.704	. .	− .002	+ .055	. .	+ .001	− 6.630	− 8.249	$\pi\,e$
3	−1	+ .002	− .549	. .	. .	+ .012	. .	. .	− .535	− .572	$\pi\,e^2$
4	−1	. .	− .041	. .	. .	+ .002	. .	. .	− .039	− .039	.
5	−1	. .	− .003	. .	. .	. .	. .	. .	− .003	− .003	.
−2	−2	. .	− .001	. .	. .	+ .002	. .	. .	+ .001	+ .001	.
−1	−2	. .	− .010	. .	. .	+ .007	. .	. .	− .003	− .012	.
0	−2	− .001	− .032	. .	. .	+ .013	. .	. .	− .020	− .167	.
1	−2	− .001	− .013	. .	. .	− .008	. .	. .	− .022	− .584	$\pi\,e'$
2	−2	. .	− .032	. .	+ .001	− .015	. .	. .	− .046	− .127	.
3	−2	. .	− .007	. .	. .	− .004	. .	. .	− .011	− .017	.
0	−3	. .	+ .002	. .	. .	+ .001	. .	. .	+ .003	− .004	.
1	−3	. .	. .	. .	. .	− .001	. .	. .	− .001	+ .040	.
2	−3	. .	+ .002	. .	. .	− .001	. .	. .	+ .001	+ .001	.
$3\omega-3\omega'$											
0	−2	. .	− .001	. .	. .	− .002	. .	. .	− .003	− .003	.
1	−2	. .	+ .016	. .	. .	− .013	. .	. .	+ .003	− .035	.
2	−2	. .	+ .005	. .	− .001	− .006	. .	. .	− .002	+ .270	.
3	−2	− .001	+ .016	. .	. .	+ .012	. .	. .	+ .027	+ .150	.
4	−2	. .	+ .008	. .	. .	+ .007	. .	. .	+ .015	+ .024	.
5	−2	. .	+ .001	. .	. .	+ .001	. .	. .	+ .002	+ .002	.
−1	−3	. .	− .005	. .	. .	+ .002	. .	. .	− .003	− .003	.
0	−3	. .	− .072	. .	. .	+ .017	. .	. .	− .055	− .057	.
1	−3	− .001	− .171	. .	. .	+ .080	. .	. .	− .092	− 1.184	π
2	−3	+ .009	− .026	. .	− .001	+ .029	. .	. .	+ .011	− 3.143	$\pi\,e$
3	−3	+ .018	− .176	. .	+ .006	− .074	. .	. .	− .226	+ .395	π
4	−3	+ .003	+ .022	. .	+ .001	− .045	. .	. .	− .019	− .001	.
5	−3	. .	+ .003	. .	. .	− .008	. .	. .	− .005	− .004	.
0	−4	. .	− .005	. .	. .	+ .001	. .	. .	− .004	− .004	.
1	−4	. .	− .013	. .	. .	+ .005	. .	. .	− .008	− .074	.
2	−4	. .	+ .001	. .	. .	+ .002	. .	. .	+ .003	− .226	.
3	−4	. .	− .013	. .	. .	− .004	. .	. .	− .017	+ .061	.
4	−4	. .	+ .004	. .	. .	− .003	. .	. .	+ .001	+ .004	.
2	−5	. .	. .	. .	. .	. .	. .	. .	. .	− .012	.
3	−5	. .	− .001	. .	. .	. .	. .	. .	− .001	+ .006	.

TABLE II.—*The Moon's Longitude*—Continued.

g	g'	sS_0	$n\delta z \times$			$(n\delta z)^2 \times$		$(n\delta z^3) \times$	Sum.	Terms in Ecliptic Longitude.	Principal Coefficient.
			$(e, g)_s - 1$	$R_{1,1}$	sS_1	(e, g)	$R_{1,2}$	$(e, g)_3$			
$\omega + \omega'$		″	″	″	″	″	″	″	″	″	
2	2	+ .001	. .	. .	. .	. .	+.001	. .	+ .002	+ .002	.
−1	1	. .	+ .006	. .	. .	+ .001	. .	. .	+ .007	+ .007	.
0	1	− .024	+ .042	+ .010	−.001	− .002	. .	. .	+ .025	+ .075	.
1	1	+ .034	+ .003	− .243	−.006	− .001	−.003	. .	− .216	+ .541	.
2	1	+ .035	+ .042	− .063	. .	+ .002	−.006	. .	+ .010	+ .010	.
3	1	+ .004	+ .003	− .010	. .	. .	−.002	. .	− .005	− .005	.
−1	0	. .	+ .003	. .	. .	. .	. .	. .	+ .003	+ .003	.
0	0	. .	+ .002	− .002	. .	. .	. .	. .	.000	+ .049	.
1	0	. .	+ .003	+ .035	. .	. .	−.001	. .	+ .037	+ .061	.
2	0	− .003	+ .002	+ .006	. .	. .	. .	. .	+ .005	+ .005	.
$3\omega - \omega'$											
2	0	+ .003	. .	− .002	. .	. .	. .	. .	+ .001	+ .001	.
3	0	. .	. .	− .035	. .	. .	−.001	. .	− .036	− .036	.
4	0	. .	. .	− .007	. .	. .	. .	. .	− .007	− .007	.
1	−1	+ .003	+ .002	. .	. .	− .002	. .	. .	+ .003	+ .003	.
2	−1	− .025	. .	+ .009	. .	. .	−.006	. .	− .022	+ .015	.
3	−1	− .010	+ .002	+ .246	. .	+ .002	−.005	. .	+ .235	+ .245	.
4	−1	− .001	+ .001	+ .043	. .	. .	−.001	. .	+ .042	+ .042	.
5	−1	. .	. .	+ .005	. .	. .	. .	. .	+ .005	+ .005	.
3	−2	− .001	. .	+ .001	. .	. .	+.001	. .	+ .001	+ .001	.
$\omega - 3\omega'$											
−1	−3	+ .001	− .001	+ .003	. .	. .	+.001	. .	+ .004	+ .004	.
0	−3	+ .011	− .018	+ .007	. .	+ .002	−.005	. .	− .003	− .003	.
1	−3	+ .017	. .	− .002	−.006	. .	−.003	. .	+ .006	− .318	.
2	−3	. .	− .018	+ .001	. .	− .002	. .	. .	− .019	− .014	.
3	−3	. .	− .001	. .	. .	. .	. .	. .	− .001	− .001	.
$4\omega - 4\omega'$											
1	−2	. .	− .002	. .	. .	+ .002	. .	. .	.000	.000	.
2	−2	. .	− .001	. .	. .	+ .001	. .	. .	.000	− .033	.
3	−2	. .	− .002	. .	. .	− .002	. .	. .	− .004	− .022	.
4	−2	. .	− .001	. .	. .	− .001	. .	. .	− .002	− .002	.
0	−3	. .	+ .001	. .	. .	+ .001	. .	. .	+ .002	+ .002	.
1	−3	. .	− .022	. .	. .	+ .022	. .	−.003	− .003	+ .039	.
2	−3	. .	− .032	. .	. .	+ .028	. .	−.002	− .006	− .356	.
3	−3	+ .007	− .032	. .	+.007	− .010	. .	−.004	− .032	− .640	ee'
4	−3	+ .005	− .037	. .	+.005	− .027	. .	−.003	− .057	− .293	.
5	−3	+ .001	− .016	. .	. .	− .013	. .	−.001	− .029	− .052	.
6	−3	. .	− .003	. .	. .	− .002	. .	. .	− .005	− .005	.
−2	−4	. .	+ .001	. .	. .	− .002	. .	. .	− .001	− .001	.
−1	−4	. .	+ .011	. .	. .	− .017	. .	. .	− .006	− .006	.
0	−4	. .	+ .172	. .	. .	− .174	. .	. .	− .002	− .028	.
1	−4	− .001	+ 1.785	. .	+.002	− 1.495	. .	. .	+ .291	+ 1.177	e^3
2	−4	− .003	+ 2.050	. .	+.025	− 1.344	. .	. .	+ .728	+ 30.768	e^2
3	−4	− .261	+ 2.242	. .	−.242	+ .968	−.004	. .	+ 2.703	+ 38.426	e
4	−4	− .168	+ 2.117	. .	−.164	+ 1.434	−.002	. .	+ 3.217	+ 13.900	m^4
5	−4	− .030	+ .732	. .	−.029	+ .533	. .	. .	+ 1.206	+ 1.981	e
6	−4	− .003	+ .093	. .	−.003	+ .086	. .	. .	+ .173	+ .221	.
7	−4	. .	+ .009	. .	. .	+ .011	. .	. .	+ .020	+ .023	.

TABLE II.—*The Moon's Longitude*—Continued.

g	g'	$s\,S_0$	$n\,\delta z \times$ $(e,g)_1 - 1$	$n\,\delta z \times$ $R_{1,1}$	$n\,\delta z \times$ $s\,S_1$	$(n\,\delta z)^2 \times$ $(e,g)_2$	$(n\,\delta z)^2 \times$ $R_{1,2}$	$(n\,\delta z)^3 \times$ $(e,g)_3$	Sum.	Terms in Ecliptic Longitude.	Principal Coefficient.
$4\omega - 4\omega'$		"	"	"	"	"	"	"	"	"	
−1	−5	. .	+ .001	. .	. .	− .002	. .	. .	− .001	− .001	.
0	−5	. .	+ .014	. .	. .	− .017	. .	. .	− .003	− .003	.
1	−5	. .	+ .163	. .	. .	− .141	. .	+.003	+ .025	+ .072	.
2	−5	+ .001	+ .235	. .	+.002	− .161	. .	+.003	+ .080	+ 2.746	$e^2 e'$
3	−5	− .019	+ .230	. .	−.021	+ .068	. .	+.004	+ .262	+ 4.402	$e e'$
4	−5	− .018	+ .244	. .	−.017	+ .166	. .	+.003	+ .378	+ 1.886	e'
5	−5	− .003	+ .099	. .	−.003	+ .073	. .	+.002	+ .168	+ .286	.
6	−5	. .	+ .013	. .	. .	+ .012	. .	. .	+ .025	+ .031	.
7	−5	. .	+ .001	. .	. .	+ .001	. .	. .	+ .002	+ .002	.
1	−6	. .	+ .010	. .	. .	− .009	. .	. .	+ .001	+ .001	.
2	−6	. .	+ .017	. .	. .	− .012	. .	. .	+ .005	+ .156	.
3	−6	− .001	+ .016	. .	−.001	+ .003	. .	. .	+ .017	+ .313	.
4	−6	− 001	+ .017	. .	. .	+ .012	. .	. .	+ .028	+ .153	.
5	−6	. .	+ .008	. .	. .	+ .006	. .	. .	+ .014	+ .024	.
2	−7	. .	+ .001	. .	. .	− .001	. .	. .	.000	.000	.
3	−7	. .	+ .001	. .	. .	. .	. .	. .	+ .001	+ .017	.
4	−7	. .	+ .001	. .	. .	+ .001	. .	. .	+ .002	+ .010	.
$4\omega - 2\omega'$											
3	0	. .	. .	+ .005	. .	. .	. .	. .	+ .005	+ .005	.
4	0	. .	. .	+ .001	. .	. .	. .	. .	+ .001	+ .001	.
2	−1	+ .002	+ .001	− .011	. .	. .	+.004	. .	− .004	− .002	.
3	−1	− .011	. .	+ .052	−.011	. .	+.032	. .	+ .062	+ .070	.
4	−1	− .009	. .	+ .055	−.007	. .	+.025	. .	+ .064	+ .064	.
5	−1	− .002	. .	+ .011	−.001	. .	+.007	. .	+ .015	+ .015	.
6	−1	. .	. .	+ .001	. .	. .	+.002	. .	+ .003	+ .003	.
−1	−2	. .	+ .001	. .	. .	. .	. .	. .	+ .001	+ .001	.
0	−2	− .001	− .003	. .	. .	+ .003	. .	. .	− .001	− .001	.
1	−2	− .012	− .063	. .	+.013	+ .053	. .	. .	− .009	+ .006	.
2	−2	− .012	− .032	+ .578	−.003	+ .026	+.001	. .	+ .558	− .534	.
3	−2	+ .001	− .060	−8.664	. .	− .050	+.004	. .	− 8.769	− 9.370	$I^2 e$
4	−2	+ .950	− .037	−5.744	. .	− .029	+.004	. .	− 5.756	− 5.743	I^2
5	−2	+ .011	− .002	−1.001	. .	− .003	+.001	. .	− .994	− .991	.
6	−2	+ .002	. .	− .126	. .	. .	. .	. .	− .124	− .124	.
7	−2	. .	. .	− .014	. .	. .	. .	. .	− .014	− .014	.
8	−2	. .	. .	− .001	. .	. .	. .	. .	− .001	− .001	.
1	−3	. .	− .003	. .	. .	+ .002	. .	. .	− .001	+ .001	.
2	−3	− .001	− .003	+ .027	−.001	+ .001	−.002	. .	+ .021	− .023	.
3	−3	+ .012	− .003	− .377	+.011	− .002	−.030	. .	− .389	− .430	.
4	−3	+ .011	− .003	− .374	+.007	− .002	−.025	. .	− .386	− .384	.
5	−3	+ .002	. .	− .070	+.001	. .	−.008	. .	− .075	− .075	.
6	−3	. .	. .	− .009	. .	. .	−.001	. .	− .010	− .010	.
7	−3	. .	. .	− .001	. .	. .	. .	. .	− .001	− .001	.
2	−4	. .	. .	+ .001	. .	. .	. .	. .	+ .001	+ .001	.
3	−4	. .	. .	− .013	. .	. .	−.002	. .	− .015	− .015	.
4	−4	+ .001	. .	− .016	. .	. .	−.002	. .	− .017	− .017	.
$2\omega - 4\omega'$											
0	−3	. .	− .002	+ .001	. .	+ .002	−.002	. .	− .001	− .001	.
1	−3	+ .006	. .	+ .001	−.008	+ .001	−.002	. .	− .002	− .025	.
2	−3	+ .004	− .001	. .	−.004	− .002	. .	. .	− .003	− .015	.

TABLE II.—*The Moon's Longitude*—Continued.

g	g'	$s\,S_0$	$n\delta z\times$ $(e,g)_1-1$	$n\delta z\times$ $R_{1,1}$	$n\delta z\times$ S_1	$(n\delta z)^2\times$ $(e,g)_2$	$(n\delta z)^2\times$ $R_{1,2}$	$(n\delta z)^3\times$ $(e,g)_3$	Sum.	Terms in Ecliptic Longitude.	Principal Co-efficient.
$2\omega-4\omega'$		″	″	″	″	″	″	″	″	″	
−2	−4	. .	. .	−.001	. .	. .	+.004	. . .	+.003	+.003	.
−1	−4	−.004	. .	−.014	−.002	−.010	+.026	. .	−.004	−.004	.
0	−4	−.033	+.013	−.072	+.003	−.053	+.120	. .	−.022	−.002	.
1	−4	−.275	+.014	−.071	+.260	−.015	+.096	. .	+.009	+.223	.
2	−4	−.143	+.008	−.018	+.133	+.053	+.023	. .	+.056	−.001	.
3	−4	−.011	+.014	−.001	+.011	+.024	. .	. .	+.037	−.029	.
4	−4	−.001	−.003	−.002	. .	. .	. .	. .	−.006	−.011	.
5	−4	. .	. .	−.002	. .	. .	. .	. .	−.002	−.002	.
−1	−5	. .	. .	−.001	. .	. .	+.002	. .	+.001	+.001	.
0	−5	−.001	+.001	−.007	. .	−.005	+.011	. .	−.001	−.001	.
1	−5	−.021	+.002	−.008	+.023	−.002	+.012	. .	+.006	+.027	.
2	−5	−.016	+.001	−.003	+.015	+.005	+.003	. .	+.005	+.035	.
3	−5	−.001	+.002	. .	. .	+.003	. .	. .	+.004	−.004	.
1	−6	−.001	. .	. .	+.001	. .	. .	. .	.000	.000	.
$5\omega-5\omega'$											
2	−5	. .	−.003	. .	. .	+.003	. .	. .	.000	.000	.
3	−5	. .	. .	. .	. .	+.001	. .	. .	+.001	−.055	.
4	−5	. .	−.003	. .	. .	−.003	. .	. .	−.006	−.011	.
5	−5	. .	. .	. .	. .	−.001	. .	. .	−.001	+.006	.
3	−6	. .	. .	. .	. .	. .	. .	. .	. .	−.006	.
4	−6	. .	. .	. .	. .	. .	. .	. .	. .	−.001	.
5	−6	. .	. .	. .	. .	. .	. .	. .	. .	+.002	.
$6\omega-6\omega'$											
3	−5	. .	. .	. .	. .	+.001	. .	. .	+.001	−.003	.
4	−5	. .	. .	. .	. .	+.001	. .	. .	+.001	−.010	.
5	−5	. .	. .	. .	. .	−.001	. .	. .	−.001	−.009	.
1	−6	. .	+.002	. .	. .	−.004	. .	+.002	.000	.000	.
2	−6	. .	+.018	. .	. .	−.022	. .	+.006	+.002	+.011	.
3	−6	. .	+.031	. .	. .	−.032	. .	+.008	+.007	+.292	.
4	−6	−.003	+.034	. .	−.004	+.001	. .	+.008	+.036	+.572	.
5	−6	−.005	+.035	. .	−.004	+.028	. .	+.009	+.063	+.395	.
6	−6	−.002	+.021	. .	−.002	+.020	. .	+.005	+.042	+.126	.
7	−6	. .	+.006	. .	. .	+.007	. .	+.001	+.014	+.023	.
2	−7	. .	+.002	. .	. .	−.003	. .	+.001	.000	.000	.
3	−7	. .	+.005	. .	. .	−.006	. .	+.001	.000	+.037	.
4	−7	. .	+.006	. .	. .	−.003	. .	+.001	+.004	+.089	.
5	−7	. .	+.006	. .	. .	−.001	. .	+.001	+.006	+.067	.
6	−7	. .	+.004	. .	. .	. .	. .	. .	+.004	+.020	.
7	−7	. .	+.001	. .	. .	. .	. .	. .	+.001	+.001	.
$6\omega-4\omega'$											
4	−3	. .	. .	+.001	. .	. .	+.002	. .	+.003	+.003	.
5	−3	. .	. .	+.001	. .	. .	+.002	. .	+.003	+.003	.
3	−4	. .	−.001	+.002	. .	+.001	+.004	. .	+.006	−.005	.
4	−4	. .	. .	−.056	. .	−.001	−.093	. .	−.150	−.166	.
5	−4	. .	−.001	−.080	. .	−.001	−.116	. .	−.198	−.203	.
6	−4	. .	. .	−.034	. .	. .	−.052	. .	−.086	−.086	.
7	−4	. .	. .	−.007	. .	. .	−.012	. .	−.019	−.019	.
8	−4	. .	. .	. .	. .	. .	−.002	. .	−.002	−.002	.

TABLE II.—*The Moon's Longitude*—Continued.

g g'	$s\,S_0$	$n\,\delta z\times$			$(n\,\delta z)^2\times$		$(n\,\delta z)^3\times$	Sum.	Terms in Ecliptic Longitude.	Principal Co-efficient.
		$(e,g)_1-1$	$R_{1,1}$	$s\,S_1$	$(e,g)_2$	$R_{1,2}$	$(e\,g)_3$			
6 ω−4 ω′		″	″	″	″	″	″	″	″	
4 −5	. .	. .	− .005	. .	. .	−.009	. .	− .014	− .014	.
5 −5	. .	. .	− .009	. .	. .	−.013	. .	− .022	− .022	.
6 −5	. .	. .	− .005	. .	. .	−.007	. .	− .012	− .012	.
7 −5	. .	. .	. .	. .	. .	−.002	. .	− .002	− .002	.
4 ω−6 ω′										
1 −6	. .	. .	. .	. .	. .	+.002	. .	+ .002	+ .001	.
2 −6	− .003	. .	− .001	+.005	. .	+.002	. .	+ .003	+ .003	.
3 −6	− .004	. .	− .001	+.005	. .	+.002	. .	+ .002	+ .005	.
4 −6	− .002	. .	. .	+.002	. .	. .	. .	.000	+ .001	.
4 ω										
4 1	+ .002	. .	. .	. .	. .	. .	. .	+ .002	.000	.
2 0	+ .004	. .	− .021	. .	. .	. .	. .	− .017	− .010	.
3 0	+ .002	. .	+ .169	. .	. .	+.003	. .	+ .174	+ .082	.
4 0	. .	. .	+ .020	. .	. .	+.002	. .	+ .022	+ .422	.
5 0	. .	. .	+ .002	. .	. .	. .	. .	+ .002	+ .094	.
6 0	. .	. .	. .	. .	. .	. .	. .	. .	+ .013	.
4 1	− .002	. .	. .	. .	. .	. .	. .	− .002	.000	.
5 1	. .	. .	. .	. .	. .	. .	. .	. .	+ .001	.
4 ω′										
0 4	+ .002	. .	. .	−.004	. .	−.002	. .	− .004	− .004	.
1 4	. .	. .	. .	. .	. .	−.003	. .	− .003	− .003	.
6 ω−2 ω′										
4 −2	. .	. .	+ .002	. .	. .	+.003	. .	+ .005	+ .002	.
5 −2	. .	. .	+ .002	. .	. .	+.002	. .	+ .004	+ .020	.
6 −2	. .	. .	. .	. .	. .	. .	. .	. .	+ .013	.
7 −2	. .	. .	. .	. .	. .	. .	. .	. .	+ .002	.
8 ω−6 ω′										
5 −6	. .	. .	. .	. .	. .	−.002	. .	− .002	− .002	.
6 −6	. .	. .	− .001	. .	. .	−.002	. .	− .003	− .004	.
7 −6	. .	. .	. .	. .	. .	−.002	. .	− .002	− .003	.
5 ω−3 ω′										
3 −3	. .	. .	+ .002	. .	. .	. .	. .	+ .002	+ .002	.
4 −3	. .	. .	+ .007	. .	. .	+.006	. .	+ .013	+ .013	.
5 −3	. .	. .	. .	. .	. .	+.004	. .	+ .004	+ .004	.
6 −3	. .	. .	. .	. .	. .	+.001	. .	+ .001	+ .001	.

TABLE III.—*Reduced Coefficients of Longitude, according to* HANSEN *and* DELAUNAY

g	*g'*	*Hansen.*	*Delaunay* (1).	*Delaunay* (2).	$D_2 - D_1$	$H - D_2$
		"	"	"		
1	0	22640.15	22640.15	22640.15	.	.
2	0	+ 769.06	+ 769.12	+ 769.06	− 6	0
3	0	+ 36.13	+ 36.16	+ 36.12	− 4	+ 1
4	0	+ 1.94	+ 1.96	+ 1.94	− 2	0
5	0	+ 0.11	+ 0.12	+ 0.11	− 1	0
6	0	+ 0.01	+ 0.01	+ 0.01	0	0
−4	−1	+ 0.04	+ 0.04	. .	.	.
−3	−1	+ 0.55	+ 0.52	+ 0.56	+ 4	− 1
−2	−1	+ 7.67	+ 7.62	+ 7.69	+ 7	− 2
−1	−1	+ 109.92	+ 109.79	+ 109.85	+ 6	+ 7
0	−1	+ 669.85	+ 669.57	+ 669.76	+ 19	+ 9
1	−1	+ 148.02	+ 147.46	+ 148.43	+ 97	− 41
2	−1	+ 9.72	+ 9.59	+ 9.71	+ 12	+ 1
3	−1	+ 0.67	+ 0.63	+ 0.66	+ 3	+ 1
4	−1	+ 0.05	+ 0.04	. .	.	.
−2	−2	+ 0.06	+ 0.07	. .	.	.
−1	−2	+ 1.18	+ 1.16	+ 1.16	0	+ 2
0	−2	+ 7.51	+ 7.49	+ 7.46	− 3	+ 5
1	−2	+ 2.59	+ 2.49	+ 2.59	+ 10	0
2	−2	+ 0.19	+ 0.16	. .	.	.
3	−2	+ 0.01	+ 0.01	. .	.	.
−1	−3	+ 0.02	+ 0.02	. .	.	.
0	−3	+ 0.08	+ 0.14	. .	.	.
1	−3	+ 0.05	+ 0.03	. .	.	.
$2\omega - 2\omega'$						
−1	0	− 0.01	− 0.01	. .	.	.
0	0	− 0.23	− 0.16	. .	.	.
1	0	− 2.54	− 2.22	− 2.35	+ 13	+ 19
2	0	− 0.19	− 0.15	− 0.15	0	+ 4
3	0	− 0.01	− 0.01	. .	.	.
−2	−1	+ 0.02	. .	. .	.	.
−1	−1	+ 0.18	+ 0.07	. .	.	.
0	−1	+ 2.52	+ 1.87	+ 2.27	+ 40	+ 25
1	−1	− 28.56	− 29.50	− 28.32	−1.18	+ 24
2	−1	− 24.45	− 24.60	− 24.50	− 10	− 5
3	−1	− 2.93	− 2.96	− 2.96	0	− 3
4	−1	− 0.29	− 0.27	. .	.	.
5	−1	− 0.02	− 0.02	. .	.	.
−3	−2	+ 0.07	+ 0.06	. .	.	.
−2	−2	+ 0.95	+ 0.91	+ 1.00	+ 9	− 5
−1	−2	+ 13.19	+ 13.15	+ 13.32	+ 17	− 13
0	−2	+ 211.71	+ 211.46	+ 211.84	+ 38	− 13
1	−2	+ 4586.56	+ 4586.24	+ 4586.44	+ 20	+ 12
2	−2	+ 2369.75	+ 2369.74	+ 2369.74	0	+ 1
3	−2	+ 191.95	+ 192.00	+ 192.00	0	− 5
4	−2	+ 14.38	+ 14.[illegible]0	+ 14.40	0	− 2
5	−2	+ 1.06	+ 1.06	+ 1.06	0	0
6	−2	+ 0.08	+ 0.08	. .	.	.

TABLE III.—*Reduced Coefficients of Longitude, &c.*—Continued.

g	g'	*Hansen.*	*Delaunay* (1).	*Delaunay* (2).	$D_2 - D_1$	$H - D_2$
$2\omega - 2\omega'$		″	″	″		
−2	−3	+ 0.03	+ 0.03	. .	.	.
−1	−3	+ 0.48	+ 0.49	+ 0.49	0	− 1
0	−3	+ 8.66	+ 8.66	+ 8.66	0	0
1	−3	+ 206.46	+ 206.54	+ 206.34	− 20	+ 12
2	−3	+ 165.52	+ 165.55	+ 165.55	0	− 3
3	−3	+ 14.60	+ 14.59	+ 14.66	+ 7	− 6
4	−3	+ 1.18	+ 1.11	+ 1.15	+ 4	+ 3
5	−3	+ 0.10	+ 0.08	. .	.	.
−1	−4	+ 0.02	+ 0.01	. .	.	.
0	−4	+ 0.28	+ 0.28	. .	.	.
1	−4	+ 7.44	+ 7.50	+ 7.50	0	− 6
2	−4	+ 8.13	+ 8.06	+ 8.06	0	+ 7
3	−4	+ 0.76	+ 0.68	+ 0.72	+ 4	+ 4
4	−4	+ 0.06	+ 0.05	. .	.	.
0	−5	+ 0.01	. .	. .	.	.
1	−5	+ 0.26	+ 0.19	. .	.	.
2	−5	+ 0.34	+ 0.25	. .	.	.
3	−5	+ 0.03	+ 0.01	. .	.	.
2ω						
0	1	+ 0.01	+ 0.02	. .	.	.
1	1	− 0.09	− 0.09	. .	.	.
2	1	+ 0.42	+ 0.42	+ 0.42	0	0
3	1	+ 0.27	+ 0.26	. .	.	.
4	1	+ 0.04	+ 0.04	. .	.	.
−1	0	+ 0.07	+ 0.05	+ 0.05	0	+ 2
0	0	+ 1.09	+ 1.39	+ 1.38	− 1	− 29
1	0	− 39.58	− 39.54	− 39.54	0	+ 4
2	0	− 411.60	− 411.63	− 411.63	0	− 3
3	0	− 45.09	− 45.12	− 45.12	0	− 3
4	0	− 4.00	− 4.01	− 4.01	0	− 1
5	0	− 0.33	− 0.33	− 0.33	0	0
6	0	− 0.03	− 0.03	. .	.	.
0	−1	+ 0.07	− 0.01	. .	.	.
1	−1	+ 0.08	+ 0.12	. .	.	.
2	−1	− 0.08	− 0.09	. .	.	.
3	−1	− 0.30	− 0.28	. .	.	.
4	−1	− 0.05	− 0.04	. .	.	.
5	−1	− 0.01	. .	. .	.	.
$2\omega'$						
−1	4	− 0.01	+ 0.01	. .	.	.
0	4	− 0.07	− 0.07	. .	.	.
1	4	+ 0.01	. .	. .	.	.
−2	3	+ 0.03	+ 0.03	. .	.	.
−1	3	+ 0.40	+ 0.37	+ 0.37	0	+ 3
0	3	− 2.15	− 2.17	− 2.17	0	− 2
1	3	+ 0.06	+ 0.05	. .	.	.
2	3	+ 0.03	+ 0.02	. .	.	.

TABLE III.—*Reduced Coefficients of Longitude*, &c.—Continued.

g	*g′*	*Hansen.*	*Delaunay* (1).	*Delaunay* (2).	$D_2 - D_1$	$H - D_2$
$2\omega'$		″	″	″		
−3	2	+ 0.03	+ 0.03	. .	.	.
−2	2	+ 0.43	+ 0.45	+ 0.45	0	− 2
−1	2	+ 6.36	+ 6.37	+ 6.37	0	− 1
0	2	− 55.25	− 55.20	− 55.17	− 3	+ 8
1	2	− 0.18	− 0.18	− 0.14	− 4	+ 4
2	2	+ 0.56	+ 0.54	+ 0.54	0	+ 2
3	2	+ 0.10	+ 0.08	. .	.	.
4	2	+ 0.01	+ 0.01	. .	.	.
−2	1	− 0.01	− 0.01	. .	.	.
−1	1	− 0.08	− 0.10	. .	.	.
0	1	+ 1.55	+ 1.43	+ 1.43	0	+ 12
1	1	+ 0.01	+ 0.01	. .	.	.
2	1	+ 0.01	. .	. .	.	.
0	0	+ 0.01	+ 0.02	. .	.	.
$2\omega + 2\omega'$						
1	2	− 0.03	. .	. .	.	.
2	2	+ 0.08	+ 0.08	. .	.	.
3	2	+ 0.01	+ 0.02	. .	.	.
$\omega - \omega'$						
0	1	+ 0.01	. .	. .	.	.
1	1	− 0.03	− 0.04	. .	.	.
2	1	0.00	− 0.01	. .	.	.
−2	0	+ 0.02	+ 0.02	. .	.	.
−1	0	+ 0.38	+ 0.26	. .	.	.
0	0	+ 1.33	+ 0.87	+ 0.87	0	+ 46
1	0	+ 18.09	+ 18.08	+ 18.08	0	+ 1
2	0	+ 1.27	+ 1.22	+ 1.21	− 1	+ 6
3	0	+ 0.09	+ 0.09	. .	.	.
4	0	+ 0.01	+ 0.01	. .	.	.
−3	−1	− 0.01	. .	. .	.	.
−2	−1	− 0.12	− 0.09	. .	.	.
−1	−1	− 1.78	− 1.50	− 1.59	+ 9	+ 19
0	−1	− 18.70	− 18.35	− 18.76	+ 41	− 6
1	−1	− 125.43	− 125.49	− 125.98	+ 49	− 55
2	−1	− 8.48	− 8.45	− 8.54	+ 9	− 6
3	−1	− 0.59	− 0.57	− .60	+ 3	− 1
4	−1	− 0.04	− 0.04	. .	.	.
−1	−2	− 0.01	− 0.01	. .	.	.
0	−2	− 0.17	− 0.14	− 0.14	0	+ 3
1	−2	− 0.60	− 0.55	− 0.56	+ 1	+ 4
2	−2	− 0.13	− 0.08	. .	.	.
3	−2	− 0.02	− 0.01	. .	.	.
1	−3	+ 0.04	+ 0.05	. .	.	.
$3\omega - 3\omega'$						
1	−2	− 0.04	− 0.01	. .	.	.
2	−2	+ 0.28	+ 0.27	. .	.	.
3	−2	+ 0.15	+ 0.14	. .	.	.
4	−2	+ 0.02	+ 0.02	. .	.	.

TABLE III.—*Reduced Coefficients of Longitude, &c.*—Continued.

g	g′	Hansen.	Delaunay (1).	Delaunay (2).	$D_2 - D_1$	$H - D_2$
$3\omega - 3\omega'$		″	″	″		
0	−3	− 0.06	− 0.04	. .	.	.
1	−3	− 1.22	− 1.17	− 1.23	+ 6	− 1
2	−3	− 3.23	− 2.98	− 3.12	+ 14	+ 11
3	−3	+ 0.41	+ 0.57	+ 0.54	− 3	− 13
4	−3	. .	+ 0.04	+ 0.01	− 3	− 1
5	−3	. .	− 0.01	. .	.	.
1	−4	− 0.08	− 0.07	. .	.	.
2	−4	− 0.23	− 0.18	. .	.	.
3	−4	+ 0.06	+ 0.11	. .	.	.
4	−4	. .	+ 0.01	. .	.	.
2	−5	− 0.01	. .	. .	.	.
3	−5	+ 0.01	+ 0.01	. .	.	.
$\omega + \omega'$						
1	2	. .	+ 0.01	. .	.	.
−1	1	+ 0.01	. .	. .	.	.
0	1	+ 0.08	+ 0.04	. .	.	.
1	1	+ 0.55	+ 0.59	+ 0.59	0	− 4
2	1	+ 0.01	+ 0.03	. .	.	.
0	0	+ 0.05	. .	. .	.	.
1	0	+ 0.06	. .	. .	.	.
$3\omega - \omega'$						
3	0	− 0.04	− 0.04	. .	.	.
4	0	− 0.01	− 0.01	. .	.	.
2	−1	+ 0.02	+ 0.02	. .	.	.
3	−1	+ 0.25	+ 0.24	. .	.	.
4	−1	+ 0.04	+ 0.04	. .	.	.
$\omega - 3\omega'$						
1	−2	. .	+ 0.01	. .	.	.
0	−3	. .	− 0.03	. .	.	.
1	−3	− 0.32	− 0.26	− 0.25	− 1	+ 7
2	−3	− 0.01	− 0.01	. .	.	.
1	−4	. .	− 0.02	. .	.	.
$4\omega - 4\omega'$						
2	−2	− 0.03	− 0.01	. .	.	.
3	−2	− 0.02	− 0.01	. .	.	.
1	−3	+ 0.04	− 0.02	. .	.	.
2	−3	− 0.36	− 0.67	− 0.67	0	− 31
3	−3	− 0.64	− 0.83	− 0.83	0	− 19
4	−3	− 0.29	− 0.29	− 0.30	+ 1	− 1
5	−3	− 0.05	− 0.04	. .	.	.
−1	−4	− 0.01	. .	. .	.	.
0	−4	− 0.03	. .	. .	.	.
1	−4	+ 1.18	+ 0.96	+ 1.08	+ 12	+ 10
2	−4	+ 30.78	+ 30.52	+ 30.72	+ 20	+ 6
3	−4	+ 38.43	+ 38.31	+ 38.48	+ 17	− 5
4	−4	+ 13.90	+ 13.89	+ 13.98	+ 9	− 8

TABLE III.—*Reduced Coefficients of Longitude, &c.*—Continued.

g	g'	*Hansen.*	*Delaunay* (1).	*Delaunay* (2).	$D_2 - D_1$	$H - D_2$
$4\,\omega-4\,\omega'$		″	″	″		
5	−4	+ 1.98	+ 1.86	+ 1.88	+ 2	+ 10
6	−4	+ 0.22	+ 0.18	+ 0.20	+ 2	+ 2
7	−4	+ 0.02	+ 0.01	. .	.	.
1	−5	+ 0.07	+ 0.06	. .	.	.
2	−5	+ 2.75	+ 2.69	+ 2.75	+ 6	0
3	−5	+ 4.41	+ 4.28	+ 4.34	+ 6	+ 7
4	−5	+ 1.89	+ 1.67	+ 1.71	+ 4	+ 18
5	−5	+ 0.29	+ 0.20	. .	.	.
6	−5	+ 0.03	+ 0.01	. .	.	.
2	−6	+ 0.16	+ 0.11	. .	.	.
3	−6	+ 0.31	+ 0.22	. .	.	.
4	−6	+ 0.15	+ 0.10	. .	.	.
5	−6	+ 0.02	+ 0.01	. .	.	.
3	−7	+ 0.02	. .	. .	.	.
4	−7	+ 0.01	. .	. .	.	.
$4\,\omega-2\,\omega'$						
2	−1	. .	+ 0.01	. .	.	.
3	−1	+ 0.07	+ 0.11	. .	.	.
4	−1	+ 0.06	+ 0.07	. .	.	.
5	−1	+ 0.02	+ 0.01	. .	.	.
1	−2	+ 0.01	. .	. .	.	.
2	−2	− 0.54	− 0.54	− 0.53	− 1	+ 1
3	−2	− 9.37	− 9.34	− 9.39	+ 5	− 2
4	−2	− 5.74	− 5.73	− 5.73	0	+ 1
5	−2	− 0.99	− 0.98	− 1.00	+ 2	− 1
6	−2	− 0.12	− 0.12	. .	.	.
7	−2	− 0.01	− 0.01	. .	.	.
2	−3	− 0.02	− 0.02	. .	.	.
3	−3	− 0.43	− 0.43	− 0.43	0	0
4	−3	− 0.38	− 0.37	. .	.	.
5	−3	− 0.08	− 0.06	. .	.	.
6	−3	− 0.01	. .	. .	.	.
3	−4	− 0.02	− 0.01	. .	.	.
4	−4	− 0.02	− 0.01	. .	.	.
$2\,\omega-4\,\omega'$						
1	−3	− 0.03	− 0.02	. .	.	.
2	−3	− 0.02	. .	. .	.	.
0	−4	. .	+ 0.01	. .	.	.
1	−4	+ 0.22	+ 0.34	+ 0.34	0	− 12
2	−4	. .	− 0.01	− 0.01	0	− 1
3	−4	− 0.03	− 0.06	. .	.	.
4	−4	− 0.01	− 0.01	. .	.	.
1	−5	+ 0.03	+ 0.03	. .	.	.
2	−5	+ 0.04	+ 0.01	+ 0.01	0	+ 3
$5\,\omega-5\,\omega'$						
3	−5	− 0.06	− 0.02	. .	.	.
4	−5	− 0.01	+ 0.02	. .	.	.
5	−5	+ 0.01	+ 0.02	. .	.	.
3	−6	− 0.01	. .	. .	.	.

TABLE III.—*Reduced Coefficients of Longitude, &c.*—Continued.

g	g'	*Hansen.*	*Delaunay* (1).	*Delaunay* (2).	$D_2 - D_1$	$H - D_2$
$6\omega - 6\omega'$		"	"	"		
4	−5	− 0.01	− 0.01	. .	.	.
5	−5	− 0.01	. .	. .	.	.
2	−6	+ 0.01	. .	. .	.	.
3	−6	+ 0.29	+ 0.20	. .	.	.
4	−6	+ 0.57	+ 0.40	+ 0.51	+ 11	+ 6
5	−6	+ 0.40	+ 0.26	. .	.	.
6	−6	+ 0.13	+ 0.07	. .	.	.
7	−6	+ 0.02	+ 0.01	. .	.	.
3	−7	+ 0.04	+ 0.01	. .	.	.
4	−7	+ 0.09	+ 0.03	. .	.	.
5	−7	+ 0.07	+ 0.02	. .	.	.
6	−7	+ 0.02	. .	. .	.	.
$6\omega - 4\omega'$						
3	−4	− 0.01	− 0.01	. .	.	.
4	−4	− 0.17	− 0.14	. .	.	.
5	−4	− 0.20	− 0.16	. .	.	.
6	−4	− 0.09	− 0.06	. .	.	.
7	−4	− 0.02	− 0.01	. .	.	.
4	−5	− 0.01	− 0.01	. .	.	.
5	−5	− 0.02	− 0.01	. .	.	.
6	−5	− 0.01	. .	. .	.	.
4ω						
2	0	− 0.01	. .	. .	.	.
3	0	+ 0.08	+ 0.08	. .	.	.
4	0	+ 0.42	+ 0.42	+ 0.42	0	0
5	0	+ 0.09	+ 0.09	. .	.	.
6	0	+ 0.01	+ 0.01	. .	.	.
$6\omega - 2\omega'$						
5	−2	+ 0.02	+ 0.02	. .	.	.
6	−2	+ 0.01	+ 0.01	. .	.	.
$5\omega - 3\omega'$						
4	−3	+ 0.01	+ 0.01	. .	.	.

TABLE IV.—*The Moon's Latitude.*

g	g'	$\sin I \sin (f+\omega)$	s	$\sin \beta$	$\beta - \sin \beta$	β *Hansen.*	β *Delaunay* (1).	β *Delaunay* (2).
ω		″	″	″	″	″	″	″
0	3	− .002	. .	− .002	. .	− .002	. .	. .
1	3	− .003	. .	− .003	. .	− .003	− .003	. .
2	3	− .001	. .	− .001	. .	− .001	. .	. .
−1	2	− .001	. .	− .001	. .	− .001	− .004	. .
0	2	− .092	− .016	− .108	. .	− .108	− .075	− 0.08
1	2	− .317	+ .262	− .055	. .	− .055	− .072	− 0.07
2	2	− .064	+ .009	− .055	. .	− .055	− .055	− 0.06
3	2	− .006	. .	− .006	. .	− .006	− .006	− 0.01
−2	1	− .004	− .020	− .024	. .	− .024	− .024	− 0.02
−1	1	− .071	− .233	− .304	− .001	− .305	− .300	− 0.30
0	1	− 5.089	− .573	− 5.662	− .005	− 5.667	− 5.370	− 5.50
1	1	− 30.067	+ 23.578	− 6.489	− .008	− 6.497	− 6.471	− 6.33
2	1	− 6.610	+ 1.279	− 5.331	− .005	− 5.336	− 5.254	− 5.25
3	1	− .720	+ .080	− .640	. .	− .640	− .617	− 0.62
4	1	− .068	+ .005	− .063	. .	− .063	− .056	− 0.06
5	1	− .004	. .	− .004	. .	− .004	− .004	. .
−4	0	. .	− .006	− .006	. .	− .006	− .006	− 0.01
−3	0	− .012	− .080	− .092	. .	− .092	− .095	− 0.09
−2	0	− .254	− 1.328	− 1.582	− .003	− 1.585	− 1.590	− 1.59
−1	0	− 6.933	− 24.787	− 31.720	− .049	− 31.769	− 31.788	− 31.79
0	0	− 1020.614	+ 21.919	− 998.695	− .991	− 999.686	− 999.747	− 999.75
1	0	+ 18444.607	0	+ 18444.607	+ 18.641	+ 18463.248	+ 18461.26	+ 18461.26
2	0	+ 1010.337	− 1.216	+ 1009.121	+ 1.052	+ 1010.173	+ 1010.233	+ 1010.19
3	0	+ 61.915	− .055	+ 61.860	+ .041	+ 61.901	+ 61.990	+ 61.99
4	0	+ 3.983	− .003	+ 3.980	− .001	+ 3.979	+ 4.013	+ 4.01
5	0	+ .263	. .	+ .263	. .	+ .263	+ .272	+ 0.27
6	0	+ .019	. .	+ .019	. .	+ .019	+ .019	+ 0.02
7	0	+ .001	. .	+ .001	. .	+ .001	. .	. .
−2	−1	+ .004	+ .021	+ .025	. .	+ .025	+ .024	+ 0.02
−1	−1	+ .065	+ .246	+ .311	+ .001	+ .312	+ .316	+ 0.32
0	−1	+ 3.266	+ 1.853	+ 5.119	+ .006	+ 5.125	+ 5.014	+ 5.07
1	−1	+ 29.641	− 24.763	+ 4.878	+ .002	+ 4.880	+ 4.955	+ 4.80
2	−1	+ 8.151	− 1.396	+ 6.755	+ .008	+ 6.763	+ 6.519	+ 6.62
3	−1	+ .880	− .083	+ .797	+ .001	+ .798	+ .744	+ 0.74
4	−1	+ .081	− .005	+ .076	. .	+ .076	+ .064	+ 0.06
5	−1	+ .005	. .	+ .005	. .	+ .005	+ .004	. .
0	−2	+ .040	+ .017	+ .057	. .	+ .057	+ .061	+ 0.06
1	−2	+ .355	− .336	+ .019	. .	+ .019	+ .037	+ 0.04
2	−2	+ .142	− .026	+ .116	. .	+ .116	+ .099	+ 0.10
3	−2	+ .018	. .	+ .018	. .	+ .018	+ .011	+ 0.01
4	−2	+ .001	. .	+ .001	. .	+ .001	. .	. .
1	−3	+ .004	. .	+ .004	. .	+ .004	+ .002	. .
2	−3	+ .003	. .	+ .003	. .	+ .003	. .	. .
$\omega - 2\omega'$								
−2	0	− .002	. .	− .002	. .	− .002	. .	. .
−1	0	− .015	. .	− .015	. .	− .015	− .007	− 0.01
0	0	− .111	+ .007	− .104	. .	− .104	− .083	− 0.08
1	0	− .003	− .128	− .131	. .	− .131	− .122	− 0.12
2	0	. .	− .005	− .005	. .	− .005	− .006	− 0.01

TABLE IV.—*The Moon's Latitude*—Continued.

g	g'	$\sin I \sin (f+\omega)$	s	$\sin \beta$	$\beta - \sin \beta$	β *Hansen.*	β *Delaunay* (1).	β *Delaunay* (2).
$\omega - 2\omega'$		″	″	″	″	″	″	″
−3	−1	+ .004	. .	+ .004	. .	+ .004	− .001	. .
−2	−1	+ .026	. .	+ .026	. .	+ .026	− .011	− 0.01
−1	−1	+ .151	− .062	+ .089	. .	+ .089	− .073	− 0.07
0	−1	− .961	+ .164	− .797	. .	− .797	− 1.195	− 1.00
1	−1	− .940	− 11.183	− 12.123	− .017	− 12.140	− 12.179	− 12.18
2	−1	− .041	− .789	− .830	− .002	− .832	− .820	− 0.82
3	−1	− .003	− .058	− .061	. .	− .061	− .060	− 0.06
−5	−2	+ .001	. .	+ .001	. .	+ .001	. .	. .
−4	−2	+ .013	. .	+ .013	. .	+ .013	+ .007	+ 0.01
−3	−2	+ .153	− .019	+ .134	. .	+ .134	+ .116	+ 0.12
−2	−2	+ 1.822	− .303	+ 1.519	. .	+ 1.519	+ 1.459	+ 1.49
−1	−2	+ 21.041	− 5.490	+ 15.551	+ .009	+ 15.560	+ 15.362	+ 15.51
0	−2	+ 210.540	− 44.062	+ 166.478	+ .125	+ 166.603	+ 166.395	+ 166.57
1	−2	+ 99.960	+ 522.584	+ 622.544	+ 1.158	+ 623.702	+ 623.622	+ 623.62
2	−2	+ 2.716	+ 30.588	+ 33.304	+ .065	+ 33.369	+ 33.382	+ 33.38
3	−2	+ .116	+ 2.027	+ 2.143	+ .003	+ 2.146	+ 2.160	+ 2.16
4	−2	+ .005	+ .140	+ .145	. .	+ .145	+ .149	+ 0.15
5	−2	. .	+ .009	+ .009	. .	+ .009	+ .011	+ 0.01
−3	−3	+ .005	. .	+ .005	. .	+ .005	+ .003	. .
−2	−3	+ .063	− .007	+ .056	. .	+ .056	+ .052	+ 0.05
−1	−3	+ .826	− .171	+ .655	. .	+ .655	+ .719	+ 0.79
0	−3	+ 9.249	− 1.779	+ 7.470	+ .005	+ 7.475	+ 7.462	+ 7.50
1	−3	+ 6.910	+ 22.773	+ 29.683	+ .053	+ 29.736	+ 29.683	+ 29.68
2	−3	+ .198	+ 1.574	+ 1.772	+ .004	+ 1.776	+ 1.754	+ 1.75
3	−3	+ .009	+ .112	+ .121	. .	+ .121	+ .124	+ 0.12
−2	−4	+ .002	. .	+ .002	. .	+ .002	+ .001	. .
−1	−4	+ .026	. .	+ .026	. .	+ .026	+ .019	+ 0.02
0	−4	+ .330	− .054	+ .276	. .	+ .276	+ .256	+ 0.26
1	−4	+ .337	+ .756	+ 1.093	+ .002	+ 1.095	+ 1.083	+ 1.08
2	−4	+ .011	+ .052	+ .063	. .	+ .063	+ .065	+ 0.06
−1	−5	+ .001	. .	+ .001	. .	+ .001	. .	. .
0	−5	+ .011	. .	+ .011	. .	+ .011	+ .004	. .
1	−5	+ .014	. .	+ .014	. .	+ .014	+ .028	+ 0.03
$\omega + 2\omega'$								
1	4	− .005	. .	− .005	+ .001	− .004	− .003	. .
0	3	+ .029	− .002	+ .027	− .002	+ .025	+ .016	+ 0.02
1	3	− .142	+ .026	− .116	+ .028	− .088	− .091	− 0.09
2	3	− .021	− .001	− .022	+ .010	− .012	− .012	− 0.01
3	3	− .002	. .	− .002	+ .002	0	− .001	. .
−1	2	+ .005	+ .003	+ .008	− .002	+ .006	+ .011	+ 0.01
0	2	+ .506	− .186	+ .320	− .034	+ .286	+ .291	+ 0.29
1	2	− 3.565	+ .768	− 2.797	+ .603	− 2.194	− 2.190	− 2.19
2	2	− .601	+ .043	− .558	+ .232	− .326	− .313	− 0.31
3	2	− .063	. .	− .063	+ .040	− .023	− .023	− 0.02
4	2	− .006	. .	− .006	+ .005	− .001	− .002	. .
0	1	− .009	. .	− .009	+ .001	− .008	− .005	− 0.01
1	1	+ .085	− .008	+ .077	− .012	+ .065	+ .054	+ 0.05
2	1	+ .006	. .	+ .006	− .002	+ .004	+ .005	. .
3	1	+ .001	. .	+ .001	. .	+ .001	. .	. .

Table IV.—*The Moon's Latitude*—Continued.

g	g'	$\sin I \sin(f+\omega)$	s	$\sin\beta$	$\beta - \sin\beta$	β *Hansen.*	β *Delaunay* (1).	β *Delaunay* (2).
$\omega - 4\omega'$		″	″	″	″	″	″	″
−2	−4	− .002	. .	− .002	. .	− .002	. .	. .
−1	−4	− .010	. .	− .010	+ .002	− .008	− .002	. .
0	−4	− .033	+ .004	− .029	+ .005	− .024	− .002	. .
1	−4	− .004	+ .054	+ .050	+ .018	+ .068	+ .064	+ 0.06
2	−4	− .001	. .	− .001	+ .001	0	− .005	. .
−1	−5	− .001	. .	− .001	. .	− .001	. .	. .
0	−5	− .003	. .	− .003	+ .001	− .002	. .	. .
1	−5	. .	− .003	− .003	+ .002	− .001	+ .006	+ 0.01
3ω								
1	1	+ .001	. .	+ .001	. .	+ .001	. .	. .
2	1	− .011	. .	− .011	+ .004	− .007	− .006	− 0.01
3	1	+ .005	. .	+ .005	+ .008	+ .013	+ .013	+ 0.01
4	1	. .	. .	. .	+ .006	+ .006	+ .007	+ 0.01
5	1	. .	. .	. .	+ .001	+ .001	+ .001	. .
0	0	+ .005	+ .003	+ .008	− .001	+ .007	+ .001	. .
1	0	+ .268	− .137	+ .131	− .023	+ .108	+ .133	+ 0.13
2	0	− 3.818	+ .002	− 3.816	+ 1.010	− 2.806	− 2.697	− 2.70
3	0	− .252	. .	− .252	− 6.051	− 6.303	− 6.297	− 6.30
4	0	− .021	. .	− .021	− 1.000	− 1.021	− 1.018	− 1.02
5	0	− .002	. .	− .002	− .117	− .119	− .119	− 0.12
6	0	. .	. .	. .	− .012	− .012	− .012	− 0.01
7	0	. .	. .	. .	− .001	− .001	. .	. .
1	−1	+ .002	. .	+ .002	. .	+ .002	. .	. .
2	−1	+ .006	. .	+ .006	− .004	+ .002	+ .008	+ 0.01
3	−1	+ .003	. .	+ .003	− .003	0	. .	. .
4	−1	. .	. .	. .	− .007	− .007	− .006	− 0.01
5	−1	. .	. .	. .	− .001	− .001	− .001	. .
$3\omega - 2\omega'$								
1	0	− .005	. .	− .005	. .	− .005	− .001	. .
2	0	− .116	. .	− .116	. .	− .116	− .090	− 0.09
3	0	− .015	. .	− .015	. .	− .015	− .009	− 0.01
4	0	− .001	. .	− .001	. .	− .001	. .	. .
0	−1	− .002	+ .003	+ .001	. .	+ .001	− 001	. .
1	−1	+ .112	− .052	+ .060	. .	+ .060	+ .021	+ 0.02
2	−1	− 1.574	+ .256	− 1.318	− .003	− 1.321	− 1.802	− 1.50
3	−1	− 1.430	+ .150	− 1.280	+ .003	− 1.277	− 1.382	− 1.38
4	−1	− .259	+ .017	− .242	+ .002	− .240	− .239	− 0.24
5	−1	− .034	. .	− .034	. .	− .034	− .023	− 0.02
6	−1	− .003	. .	− .003	. .	− .003	. .	. .
−1	−2	+ .005	+ .029	+ .034	. .	+ .034	+ .025	+ 0.03
0	−2	− .004	+ .273	+ .269	. .	+ .269	+ .246	+ 0.25
1	−2	− 1.858	+ .236	− 1.622	− .001	− 1.623	− 1.739	− 1.68
2	−2	+ 199.476	− .296	+ 199.180	+ .303	+ 199.483	+ 199.277	+ 199.42
3	−2	+ 117.753	− .096	+ 117.657	− .399	+ 117.258	+ 117.188	+ 117.19
4	−2	+ 15.207	− .015	+ 15.192	− .077	+ 15.115	+ 15.105	+ 15.11
5	−2	+ 1.531	− .002	+ 1.529	− .010	+ 1.519	+ 1.502	+ 1.50
6	−2	+ .141	. .	+ .141	− .001	+ .140	+ .132	+ 0.13
7	−2	+ .012	. .	+ .012	. .	+ .012	+ .008	+ 0.01

TABLE IV.—*The Moon's Latitude*—Continued.

g	g'	$\sin I \sin(f+\omega)$	s	$\sin\beta$	$\beta - \sin\beta$	β *Hansen.*	β *Delaunay* (1).	β *Delaunay* (2).
$3\omega - 2\omega'$		"	"	"	"	"	"	"
0	−3	. .	+ .010	+ .010	. .	+ .010	+ .010	+ 0.01
1	−3	− .071	+ .010	− .061	. .	− .061	− .078	− 0.08
2	−3	+ 9.184	− .286	+ 8.898	+ .014	+ 8.912	+ 8.968	+ 9.00
3	−3	+ 8.180	− .168	+ 8.012	− .015	+ 7.997	+ 7.946	+ 7.95
4	−3	+ 1.166	− .022	+ 1.144	− .004	+ 1.140	+ 1.082	+ 1.08
5	−3	+ .128	. .	+ .128	− .001	+ .127	+ .100	+ 0.10
6	−3	+ .013	. .	+ .013	. .	+ .013	+ .006	+ 0.01
1	−4	− .003	. .	− .003	. .	− .003	− .003	. .
2	−4	+ .334	− .014	+ .320	. .	+ .320	+ .311	+ 0.31
3	−4	+ .401	− .011	+ .390	. .	+ .390	+ .362	+ 0.36
4	−4	+ .061	. .	+ .061	. .	+ .061	+ .043	+ 0.04
5	−4	+ .007	. .	+ .007	. .	+ .007	+ .002	. .
2	−5	+ .012	. .	+ .012	. .	+ .012	+ .005	+ 0.01
3	−5	+ .017	. .	+ .017	. .	+ .017	+ .006	+ 0.01
4	−5	+ .002	. .	+ .002	. .	+ .002	. .	. .
$3\omega - 4\omega'$								
2	−2	− .001	. .	− .001	. .	− .001	− .002	. .
0	−3	+ .001	. .	+ .001	. .	+ .001	. .	. .
1	−3	− .003	+ .002	− .001	. .	− .001	− .014	− 0.01
2	−3	− .010	− .143	− .153	. .	− .153	− .199	− 0.20
3	−3	− .005	− .098	− .103	. .	− .103	− .114	− 0.11
4	−3	− .001	− .012	− .013	. .	− .013	− .014	− 0.01
−2	−4	− .003	. .	− .003	. .	− .003	. .	. .
−1	−4	− .012	. .	− .012	. .	− .012	− .002	. .
0	−4	− .043	+ .045	+ .002	. .	+ .002	− .005	. .
1	−4	+ .220	+ .404	+ .624	+ .001	+ .625	+ .582	+ 0.61
2	−4	+ .603	+ 5.957	+ 6.560	+ .016	+ 6.576	+ 6.532	+ 6.56
3	−4	+ .236	+ 3.451	+ 3.687	− .008	+ 3.679	+ 3.651	+ 3.65
4	−4	+ .020	+ .448	+ .468	− .002	+ .466	+ .464	+ 0.46
5	−4	+ .001	+ .047	+ .048	. .	+ .048	+ .044	+ 0.04
−1	−5	− .002	. .	− .002	. .	− .002	. .	. .
0	−5	− .005	. .	− .005	. .	− .005	. .	. .
1	−5	+ .018	+ .011	+ .029	. .	+ .029	+ .042	+ 0.04
2	−5	+ .066	+ .450	+ .516	+ .001	+ .517	+ .576	+ 0.60
3	−5	+ .031	+ .379	+ .410	. .	+ .410	+ .395	+ 0.40
4	−5	+ .003	+ .050	+ .053	. .	+ .053	+ .046	+ 0.05
2	−6	+ .004	+ .021	+ .025	. .	+ .025	+ .028	+ 0.03
3	−6	+ .002	+ .019	+ .021	. .	+ .021	+ .022	+ 0.02
$5\omega - 6\omega'$								
1	−6	− .001	. .	− .001	. .	− .001	. .	. .
2	−6	− .002	+ .005	+ .003	. .	+ .003	+ .003	. .
3	−6	+ .005	+ .068	+ .073	. .	+ .073	+ .061	+ 0.06
4	−6	+ .005	+ .086	+ .091	. .	+ .091	+ .071	+ 0.07
5	−6	+ .001	+ .035	+ .036	. .	+ .036	+ .024	+ 0.02
6	−6	. .	+ .002	+ .002	. .	+ .002	+ .002	. .
4	−7	+ .001	. .	+ .001	. .	+ .001	+ .005	. .

Table IV.—*The Moon's Latitude*—Continued.

g	g'	$\sin I \sin (f+\omega)$	s	$\sin \beta$	$\beta - \sin \beta$	β *Hansen.*	β *Delaunay* (1)	β *Delaunay* (2).
$-\omega'$		″	″	″	″	″	″	″
−3	0	+ .002	. .	+ .002	. .	+ .002	+ .001	. .
−2	0	+ .022	. .	+ .022	. .	+ .022	+ .010	+ 0.01
−1	0	+ .096	− .080	+ .016	. .	+ .016	+ .024	+ 0.02
0	0	+ .777	+ .015	+ .792	+ .001	+ .793	+ .794	+ 0.79
1	0	+ .009	+ .004	+ .013	. .	+ .013	+ .035	+ 0.03
−4	−1	− .001	. .	− .001	. .	− .001	. .	. .
−3	−1	− .015	. .	− .015	. .	− .015	− .005	− 0.01
−2	−1	− .151	+ .041	− .110	. .	− .110	− .080	− 0.08
−1	−1	− 1.174	+ .751	− .423	. .	− .423	− .381	− 0.38
0	−1	− 5.504	+ .821	− 4.683	− .005	− 4.688	− 4.756	− 4.83
1	−1	− .083	− .500	− .583	− .001	− .584	− .603	− 0.60
2	−1	− .004	− .030	− .034	. .	− .034	− .039	− 0.04
−2	−2	+ .001	. .	+ .001	. .	+ .001	+ .002	. .
−1	−2	. .	+ .019	+ .019	. .	+ .019	+ .010	+ 0.01
0	−2	− .015	− .005	− .020	. .	− .020	− .005	− 0.01
1	−2	− .002	− .008	− .010	. .	− .010	− .018	− 0.02
0	−3	+ .002	. .	+ .002	. .	+ .002	+ .002	. .
$2\omega - \omega'$								
2	1	− .002	. .	− .002	. .	− .002	− .002	. .
0	0	+ .013	+ .004	+ .017	. .	+ .017	+ .014	+ 0.01
1	0	+ .018	− .069	− .051	. .	− .051	− .043	− 0.04
2	0	+ .796	− .009	+ .787	+ .001	+ .788	+ .791	+ 0.79
3	0	+ .101	. .	+ .101	. .	+ .101	+ .101	+ 0.10
4	0	+ .010	. .	+ .010	. .	+ .010	+ .008	+ 0.01
0	−1	− .023	− .043	− .066	. .	− .066	− .062	− 0.06
1	−1	− .452	+ .577	+ .125	. .	+ .125	+ .141	+ 0.14
2	−1	− 5.431	+ .186	− 5.245	− .006	− 5.251	− 5.325	− 5.40
3	−1	− .659	+ .009	− .650	. .	− .650	− .647	− 0.67
4	−1	− .063	. .	− .063	. .	− .063	− .058	− 0.06
5	−1	− .006	. .	− .006	. .	− .006	− .004	. .
1	−2	− .014	+ .009	− .005	. .	− .005	. .	. .
2	−2	− .038	+ .026	− .012	. .	− .012	− .013	− 0.01
3	−2	− .011	. .	− .011	. .	− .011	− .004	. .
4	−2	− .002	. .	− .002	. .	− .002	. .	. .
2	−3	+ .002	. .	+ .002	. .	+ .002	+ .002	. .
$2\omega - 3\omega'$								
0	−2	− .002	. .	− .002	. .	− .002	. .	. .
1	−2	+ .003	. .	+ .003	. .	+ .003	+ .012	+ 0.01
2	−2	+ .001	+ .020	+ .021	. .	+ .021	+ .033	+ 0.03
−1	−3	− .004	. .	− .004	. .	− .004	− .002	. .
0	−3	− .046	− .002	− .048	. .	− .048	− .034	− 0.03
1	−3	− .071	− .224	− .295	. .	− .295	− .290	− 0.29
2	−3	+ .041	− .391	− .350	− .001	− .351	− .339	− 0.34
3	−3	+ .001	− .045	− .044	. .	− .044	− .031	− 0.03
0	−4	− .003	. .	− .003	. .	− .003	− .001	. .
1	−4	− .006	. .	− .006	. .	− .006	− .017	− 0.02
2	−4	+ .006	. .	+ .006	. .	+ .006	− .021	− 0.02

TABLE IV.—*The Moon's Latitude*—Continued.

g	g'	sin I sin $(f+\omega)$	s	sin β	$\beta-\sin\beta$	β *Hansen.*	β *Delaunay* (1).	β *Delaunay* (2).
		″	″	″	″	″	″	″
$5\omega-4\omega'$								
3	−2	− .002	. .	− .002	. .	− .002	. .	. .
4	−2	− .001	. .	− .001	. .	− .001	. .	. .
2	−3	+ .005	. .	+ .005	. .	+ .005	. .	. .
3	−3	− .030	. .	− .030	. .	− .030	− .058	− 0.06
4	−3	− .055	. .	− .055	. .	− .055	− .058	− 0.06
5	−3	− .027	. .	− .027	. .	− .027	− .020	− 0.02
6	−3	− .005	. .	− .005	. .	− .005	− .001	. .
1	−4	− .003	. .	− .003	. .	− .003	. .	. .
2	−4	+ .019	. .	+ .019	. .	+ .019	+ .006	+ 0.01
3	−4	+ 2.415	. .	+ 2.415	+ .004	+ 2.419	+ 2.257	+ 2.32
4	−4	+ 3.017	. .	+ 3.017	− .013	+ 3.004	+ 2.816	+ 2.89
5	−4	+ 1.204	. .	+ 1.204	− .011	+ 1.193	+ 1.079	+ 1.09
6	−4	+ .216	. .	+ .216	− .002	+ .214	+ .162	+ 0.16
7	−4	+ .029	. .	+ .029	. .	+ .029	+ .012	+ 0.01
8	−4	+ .003	. .	+ .003	. .	+ .003	. .	. .
3	−5	+ .218	. .	+ .218	. .	+ .218	+ .168	+ 0.17
4	−5	+ .347	. .	+ .347	− .001	+ .346	+ .256	+ 0.26
5	−5	+ .162	. .	+ .162	− .001	+ .161	+ .102	+ 0.10
6	−5	+ .031	. .	+ .031	. .	+ .031	+ .010	+ 0.01
7	−5	+ .003	. .	+ .003	. .	+ .003	. .	. .
3	−6	+ .012	. .	+ .012	. .	+ .012	+ .005	+ 0.01
4	−6	+ .024	. .	+ .024	. .	+ .024	+ .008	+ 0.01
5	−6	+ .013	. .	+ .013	. .	+ .013	+ .003	. .
6	−6	+ .002	. .	+ .002	. .	+ .002	. .	. .
4	−7	+ .001	. .	+ .001	. .	+ .001	. .	. .
$4\omega-3\omega'$								
2	−2	− .002	. .	− .002	. .	− .002	. .	. .
3	−2	+ .021	. .	+ .021	. .	+ .021	+ .018	+ 0.02
4	−2	+ .014	. .	+ .014	. .	+ .014	+ .011	+ 0.01
5	−2	+ .002	. .	+ .002	. .	+ .002	+ .002	. .
2	−3	− .054	. .	− .054	. .	− .054	− .043	− 0.04
3	−3	− .208	. .	− .208	. .	− .208	− .165	− 0.17
4	−3	− .030	. .	− .030	+ .001	− .029	+ .005	+ 0.01
5	−3	− .007	. .	− .007	. .	− .007	+ .005	+ 0.01
6	−3	− .001	. .	− .001	. .	− .001	+ .001	. .
2	−4	− .003	. .	− .003	. .	− .003	− .002	. .
3	−4	− .014	. .	− .014	. .	− .014	− .005	− 0.01
$2\omega+\omega'$								
1	1	+ .001	. .	+ .001	. .	+ .001	+ .001	. .
2	1	+ .035	. .	+ .035	− .005	+ .030	+ .032	+ 0.03
3	1	+ .004	. .	+ .004	− .001	+ .003	+ .004	. .
1	0	+ .002	. .	+ .002	. .	+ .002	. .	. .
2	0	+ .001	. .	+ .001	+ .001	+ .002	. .	. .
$4\omega-\omega'$								
4	0	. .	. .	. .	− .001	− .001	. .	. .
3	−1	+ .003	. .	+ .003	− .001	+ .002	+ .002	. .
4	−1	. .	. .	. .	+ .005	+ .005	+ .005	+ 0.01
5	−1	. .	. .	. .	+ .001	+ .001	. .	. .

TABLE IV.—*The Moon's Latitude*—Continued.

g	g'	sin I sin $(f+\omega)$	s	sin β	$\beta-$ sin β	β *Hansen.*	β *Delaunay* (1).	β *Delaunay* (2).
$-3\omega'$		″	″	″	″	″	″	″
−1	−3	− .002	. .	− .002	. .	− .002	− .002	. .
0	−3	− .015	. .	− .015	. .	− .015	− .010	− 0.01
$5\omega-2\omega'$								
4	−1	. .	. .	. .	+ .001	+ .001	+ .003	. .
5	−1	. .	. .	. .	+ .001	+ .001	+ .002	. .
2	−2	+ .004	. .	+ .004	. .	+ .004	+ .002	. .
3	−2	− .089	. .	− .089	+ .024	− .065	− .068	− 0.07
4	−2	− .060	. .	− .060	− .186	− .246	− .246	− 0.25
5	−2	− .008	. .	− .008	− .137	− .145	− .142	− 0.14
6	−2	. .	. .	. .	− .030	− .030	− .028	− 0.03
7	−2	. .	. .	. .	− .004	− .004	− .003	. .
3	−3	− .004	. .	− .004	+ .001	− .003	− .003	. .
4	−3	− .004	. .	− .004	− .008	− .012	− .011	− 0.01
5	−3	. .	. .	. .	− .009	− .009	− .008	− 0.01
6	−3	. .	. .	. .	− .002	− .002	− .001	. .
$4\omega-5\omega'$								
1	−5	+ .001	. .	+ .001	. .	+ .001	. .	. .
2	−5	− .001	. .	− .001	. .	− .001	− .002	. .
3	−5	+ .001	. .	+ .001	. .	+ .001	− .004	. .
$6\omega-5\omega'$								
4	−5	− .005	. .	− .005	. .	− .005	. .	. .
5	−5	− .002	. .	− .002	. .	− .002	+ .001	. .
$7\omega-6\omega'$								
4	−6	+ .031	. .	+ .031	. .	+ .031	+ .011	+ 0.01
5	−6	+ .060	. .	+ .060	. .	+ .060	+ .020	+ 0.02
6	−6	+ .043	. .	+ .043	. .	+ .043	+ .012	+ 0.01
7	−6	+ .014	. .	+ .014	. .	+ .014	+ .002	. .
8	−6	+ .002	. .	+ .002	. .	+ .002	. .	. .
4	−7	+ .004	. .	+ .004	. .	+ .004	. .	. .
5	−7	+ .010	. .	+ .010	. .	+ .010	. .	. .
6	−7	+ .008	. .	+ .008	. .	+ .008	. .	. .
7	−7	+ .001	. .	+ .001	. .	+ .001	. .	. .
$7\omega-4\omega'$								
4	−4	− .001	. .	− .001	. .	− .001	. .	. .
5	−4	− .001	. .	− .001	− .004	− .005	− .002	. .
6	−4	. .	. .	. .	− .006	− .006	− .002	. .
7	−4	. .	. .	. .	− .003	− .003	. .	. .
$3\omega+2\omega'$								
2	2	. .	. .	. .	− .001	− .001	+ .001	. .
3	2	. .	. .	. .	+ .002	+ .002	+ .002	. .
5ω								
4	0	. .	. .	. .	+ .001	+ .001	+ .002	. .
5	0	. .	. .	. .	+ .006	+ .006	+ .006	+ 0.01
6	0	. .	. .	. .	+ .002	+ .002	+ .002	. .

TABLE V.—*The Moon's Parallax.*

g	g'	$\frac{D(1+e\cos f)}{a(1-e^2)}$	Pert.	*Hansen's* sine Parallax.	*Delaunay's* sine Parallax. (1)	*Delaunay's* sine Parallax. (2)	D_2-D_1	$H-D_2$	*Adams'* sine Parallax.
		″	″	″	″	″			″
0	0	3399.682	+ 22.405	3422.09	3422.7	3422.7	0	− 61	3422.32
1	0	+ 186.547	− .064	+ 186.483	+ 186.587	+ 186.55	− 4	− 7	+ 186.51
2	0	+ 10.220	− .059	+ 10.161	+ 10.198	+ 10.20	0	− 4	+ 10.17
3	0	+ .627	− .007	+ .620	+ .631	+ 0.63	0	− 1	+ 0.63
4	0	+ .040	. .	+ .040	+ .041	+ 0.04	0	0	+ 0.04
5	0	+ .003	. .	+ .003	+ .003	. .	.	.	. .
−4	−1	− .001	. .	− .001	. .	. .	.	.	. .
−3	−1	− .007	− .003	− .010	− .006	− 0.01	0	0	. .
−2	−1	− .067	− .055	− .122	− .092	− 0.09	0	+ 3	− 0.10
−1	−1	− .304	− .657	− .961	− .912	− 0.93	+ 2	+ 3	− 0.95
0	−1	− .018	− .375	− .393	− .427	− 0.43	0	− 4	− 0.40
1	−1	+ .299	+ .845	+ 1.144	+ 1.052	+ 1.11	+ 6	+ 3	+ 1.16
2	−1	+ .082	+ .067	+ .149	+ .103	+ 0.10	0	+ 5	+ 0.12
3	−1	+ .009	+ .003	+ .012	+ .006	+ 0.01	0	0	. .
4	−1	+ .001	. .	+ .001	. .	. .	.	.	. .
−1	−2	− .003	− .007	− .010	− .010	− 0.01	0	0	. .
0	−2	. .	− .008	− .008	− .012	− 0.01	0	0	. .
1	−2	+ .003	+ .009	+ .012	+ .013	+ 0.01	0	0	. .
2	−2	+ .001	. .	+ .001	. .	. .	.	.	. .
$2\omega-2\omega'$									
0	0	+ .001	. .	+ .001	. .	. .	.	.	. .
1	0	. .	− .021	− .021	− .013	− 0.01	0	+ 1	. .
2	0	− .001	− .001	− .002	. .	. .	.	.	. .
−1	−1	− .001	. .	− .001	+ .001	. .	.	.	. .
0	−1	+ .010	− .012	− .002	+ .002	. .	.	.	. .
1	−1	+ .010	− .237	− .227	− .379	− 0.38	0	− 15	− 0.23
2	−1	− .015	− .286	− .301	− .328	− 0.33	0	− 3	− 0.31
3	−1	− .015	− .034	− .049	− .040	− 0.04	0	+ 1	. .
4	−1	− .002	− .002	− .004	− .002	. .	.	.	. .
−3	−2	− .002	. .	− .002	. .	. .	.	.	. .
−2	−2	− .018	+ .004	− .014	− .008	− 0.01	0	0	. .
−1	−2	− .213	+ .092	− .121	− .101	− 0.10	0	+ 2	− 0.12
0	−2	− 2.128	+ 1.826	− .302	− .277	− 0.28	0	+ 2	− 0.31
1	−2	− .992	+ 35.301	+ 34.309	+ 34.166	+ 34.29	+ 12	+ 2	+ 34.30
2	−2	+ 1.990	+ 26.235	+ 28.225	+ 28.179	+ 28.20	+ 2	+ 3	+ 28.23
3	−2	+ 1.190	+ 1.894	+ 3.084	+ 3.064	+ 3.07	+ 1	+ 1	+ 3.09
4	−2	+ .154	+ .129	+ .283	+ .271	+ 0.27	0	+ 1	+ 0.28
5	−2	+ .015	+ .008	+ .023	+ .018	+ 0.02	0	0	. .
6	−2	+ .001	. .	+ .001	. .	. .	.	.	. .
−2	−3	− .001	. .	− .001	. .	. .	.	.	. .
−1	−3	− .008	+ .004	− .004	− .003	. .	.	.	. .
0	−3	− .094	+ .075	− .019	− .013	− 0.01	0	+ 1	. .
1	−3	− .069	+ 1.516	+ 1.447	+ 1.452	+ 1.47	+ 2	− 2	+ 1.45
2	−3	+ .091	+ 1.829	+ 1.920	+ 1.876	+ 1.91	+ 3	+ 1	+ 1.92
3	−3	+ .082	+ .147	+ .229	+ .197	+ 0.22	+ 2	+ 1	+ 0.22
4	−3	+ .012	+ .010	+ .022	+ .012	+ 0.01	0	+ 1	. .
5	−3	+ .001	. .	+ .001	. .	. .	.	.	. .

TABLE V.—*The Moon's Parallax*—Continued.

g	g'	$\frac{D(1 - e \cos f)}{a(1 - e^2)}$	Pert.	*Hansen's* sine Parallax.	*Delaunay's* sine Parallax. (1)	*Delaunay's* sine Parallax. (2)	$D_2 - D_1$	$H - D_2$	*Adams'* sine Parallax.
$2\omega - 2\omega'$		″	″	″	″	″			″
0	−4	− .003	+ .002	− .001	. .	. .	.	.	. .
1	−4	− .004	+ .053	+ .049	+ .045	+ 0.04	0	+ 1	+ 0.05
2	−4	+ .003	+ .089	+ .092	+ .076	+ 0.10	+ 2	− 1	+ 0.09
3	−4	+ .004	+ .008	+ .012	+ .005	+ 0.01	0	0	. .
4	−4	+ .001	. .	+ .001	. .	. .	.	.	. .
1	−5	. .	+ .001	+ .001	. .	. .	.	.	. .
2	−5	. .	+ .004	+ .004	. .	. .	.	.	. .
$4\omega - 4\omega'$									
2	−3	. .	− .004	− .004	− .007	− 0.01	0	0	. .
3	−3	. .	− .009	− .009	− .008	− 0.01	0	0	. .
4	−3	. .	− .004	− .004	− .002	. .	.	.	. .
1	−4	− .002	+ .010	+ .008	+ .004	. .	.	.	. .
2	−4	− .005	+ .377	+ .372	+ .310	+ 0.31	0	+ 6	+ 0.37
3	−4	+ .022	+ .577	+ .599	+ .499	+ 0.50	0	+ 10	+ 0.60
4	−4	+ .030	+ .231	+ .261	+ .196	+ 0.20	0	+ 6	+ 0.26
5	−4	+ .012	+ .031	+ .043	+ .019	+ 0.02	0	+ 2	. .
6	−4	+ .002	+ .002	+ .004	. .	. .	.	.	. .
2	−5	− .001	+ .033	+ .032	+ .016	+ 0.02	0	+ 1	. .
3	−5	+ .002	+ .067	+ .069	+ .030	+ 0.03	0	+ 4	+ 0.06
4	−5	+ .004	+ .031	+ .035	+ .011	+ 0.01	0	+ 2	. .
5	−5	+ .002	+ .005	+ .007	. .	. .	.	.	. .
2	−6	. .	+ .002	+ .002	.	. .	.	.	. .
3	−6	. .	+ .004	+ .004	. .	. .	.	.	. .
4	−6	. .	+ .002	+ .002	. .	. .	.	.	. .
$6\omega - 6\omega'$									
3	−6	. .	+ .004	+ .004	. .	. .	.	.	. .
4	−6	. .	+ .010	+ .010	. .	. .	.	.	. .
5	−6	. .	+ .007	+ .007	. .	. .	.	.	. .
6	−6	. .	+ .002	+ .002	. .	. .	.	.	. .
4	−7	. .	+ .001	+ .001	. .	. .	.	.	. .
5	−7	. .	+ .001	+ .001	. .	. .	.	.	. .
2ω									
1	1	. .	− .003	− .003	− .002	. .	.	.	. .
2	1	. .	+ .001	+ .001	+ .002	. .	.	.	. .
−1	0	+ .002	− .002	0	. .	. .	.	.	. .
0	0	+ .038	− .038	0	. .	. .	.	.	. .
1	0	. .	− .709	− .709	− .708	− 0.71	0	0	− 0.71
2	0	− .039	+ .027	− .012	− .009	− 0.01	0	0	. .
3	0	− .002	+ .002	0	. .	. .	.	.	. .
1	−1	. .	+ .002	+ .002	+ .002	. .	.	.	. .
2	−1	. .	+ .001	+ .001	+ .002	. .	.	.	. .
$2\omega'$									
−1	3	+ .001	− .004	− .003	− .002	. .	.	.	. .
0	3	. .	− .007	− .007	− .007	− 0.01	0	0	. .
1	3	− .001	− .001	− .002	− .003	. .	.	.	. .

Table V.—*The Moon's Parallax*—Continued.

g	g'	$\dfrac{D(1 + e\cos f)}{a(1 - e^2)}$	Pert.	*Hansen's* sine Parallax.	*Delaunay's* sine Parallax. (1)	*Delaunay's* sine Parallax. (2)	$D_2 - D_1$	$H - D_2$	*Adams'* sine Parallax.
		"	"	"	"	"			"
$2\omega'$									
−2	2	. .	− .004	− .004	− .006	− 0.01	0	0	. .
−1	2	+ .038	− .086	− .048	− .050	− 0.05	0	0	. .
0	2	+ .007	− .112	− .105	− .109	− 0.11	0	0	− 0.11
1	2	− .036	− .047	− .083	− .082	− 0.08	0	0	− 0.09
2	2	− .006	− .003	− .009	− .008	− 0.01	0	0	. .
3	2	− .001	. .	− .001	. .	. .	.	.	. .
−1	1	− .001	+ .001	0	+ .001	. .	.	.	. .
0	1	. .	+ .001	+ .001	+ .001	. .	.	.	. .
1	1	+ .001	. .	+ .001	+ .001	. .	.	.	. .
$4\omega - 2\omega'$									
2	−2	. .	− .014	− .014	− .015	− 0.01	0	0	. .
3	−2	− .001	− .010	− .011	− .011	− 0.01	0	0	. .
4	−2	− .001	. .	− .001	. .	. .	.	.	. .
$2\omega - 4\omega'$									
1	−4	. .	+ .001	+ .001	+ .003	. .	.	.	. .
2	−4	. .	+ .002	+ .002	+ .004	. .	.	.	. .
$\omega - \omega'$									
−1	0	− .001	− .002	− .003	− .002	. .	.	.	. .
0	0	− .008	+ .007	− .001	. .	. .	.	.	. .
1	0	. .	+ .146	+ .146	+ .151	+ 0.15	0	0	+ 0.14
2	0	+ .008	+ .008	+ .016	+ .013	+ 0.01	0	0	. .
3	0	+ .001	. .	+ .001	+ .002	. .	.	.	. .
−2	−1	+ .001	. .	+ .001	. .	. .	.	.	. .
−1	−1	+ .012	+ .003	+ .015	+ .007	+ 0.01	0	+ 1	. .
0	−1	+ .055	− .044	+ .011	+ .008	+ 0.01	0	0	+ 0.01
1	−1	− .004	− .949	− .953	− .938	− 0.97	+ 3	− 2	− 0.95
2	−1	− .055	− .051	− .106	− .097	− 0.10	0	+ 1	− 0.11
3	−1	− .007	− .003	− .010	− .006	− 0.01	0	0	. .
4	−1	− .001	. .	− .001	. .	. .	.	.	. .
1	−2	. .	− .004	− .004	− .002	. .	.	.	. .
2	−2	. .	− .001	− .001	+ .001	. .	.	.	. .
$3\omega - 3\omega'$									
2	−2	. .	+ .003	+ .003	+ .003	. .	.	.	. .
3	−2	. .	+ .002	+ .002	+ .002	. .	.	.	. .
1	−3	. .	− .009	− .009	− .005	. .	.	.	. .
2	−3	− .001	− .036	− .037	− .020	− 0.02	0	+ 2	. .
3	−3	− .002	+ .005	+ .003	+ .016	+ 0.02	0	− 2	. .
2	−4	. .	− .002	− .002	. .	. .	.	.	. .
3	−4	. .	+ .001	+ .001	+ .002	. .	.	.	. .
$\omega + \omega'$									
1	1	. .	+ .007	+ .007	+ .007	+ 0.01	0	0	. .
$\omega - 3\omega'$									
1	−3	. .	− .002	− .002	− .002	. .	.	.	. .

EXPERIMENTAL DETERMINATION

OF THE

VELOCITY OF LIGHT

MADE AT THE

U. S. NAVAL ACADEMY, ANNAPOLIS.

BY

ALBERT A. MICHELSON,

MASTER U. S. NAVY.

NOTE.

The probability that the most accurate method of determining the solar parallax now available is that resting on the measurement of the velocity of light, has led to the acceptance of the following paper as one of the series having in view the increase of our knowledge of the celestial motions. The researches described in it, having been made at the United States Naval Academy, though at private expense, were reported to the Honorable Secretary of the Navy, and referred by him to this Office. At the suggestion of the writer, the paper was reconstructed with a fuller general discussion of the processes, and with the omission of some of the details of individual experiments.

To prevent a possible confusion of this determination of the velocity of light with another now in progress under official auspices, it may be stated that the credit and responsibility for the present paper rests with Master Michelson.

SIMON NEWCOMB,
Professor, U. S. Navy,
Superintendent Nautical Almanac.

NAUTICAL ALMANAC OFFICE,
BUREAU OF NAVIGATION,
NAVY DEPARTMENT,
Washington, February 20, 1880.

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EXPERIMENTAL DETERMINATION OF THE VELOCITY OF LIGHT.

By Albert A. Michelson, *Master*, *U. S. N.*

INTRODUCTION.

In Cornu's elaborate memoir upon the determination of the velocity of light, several objections are made to the plan followed by Foucault, which will be considered in the latter part of this work. It may, however, be stated that the most important among these was that the deflection was too small to be measured with the required degree of accuracy. In order to employ this method, therefore, it was absolutely necessary that the deflection should be increased.

In November, 1877, a modification of Foucault's arrangement suggested itself, by which this result could be accomplished. Between this time and March of the following year a number of preliminary experiments were performed in order to familiarize myself with the optical arrangements The first experiment tried with the revolving mirror produced a deflection considerably greater than that obtained by Foucault. Thus far the only apparatus used was such as could be adapted from the apparatus in the laboratory of the Naval Academy.

At the expense of $10 a revolving mirror was made, which could execute 128 turns per second. The apparatus was installed in May, 1878, at the laboratory. The distance used was 500 feet, and the deflection was about twenty times that obtained by Foucault.*

These experiments, made with very crude apparatus and under great difficulties, gave the following table of results for the velocity of light in miles per second:

	186730
	188820
	186330
	185330
	187900
	184500
	186770
	185000
	185800
	187940
Mean	186500 ± 300 miles per second.
or	300140 kilometers per second

* See Proc. Am. Assoc. Adv. Science, Saint Louis meeting.

In the following July the sum of $2,000 was placed at my disposal by a private gentleman for carrying out these experiments on a large scale. Before ordering any of the instruments, however, it was necessary to find whether or not it was practicable to use a large distance. With a distance (between the revolving and the fixed mirror) of 500 feet, in the preliminary experiments, the field of light in the eye-piece was somewhat limited, and there was considerable indistinctness in the image, due to atmospheric disturbances.

Accordingly, the same lens (39 feet focus) was employed, being placed, together with the other pieces of apparatus, along the north sea-wall of the Academy grounds, the distance being about 2,000 feet. The image of the slit, at noon, was so confused as not to be recognizable, but toward sunset it became clear and steady, and measurements were made of its position, which agreed within one one-hundredth of a millimeter. It was thus demonstrated that with this distance and a deflection of 100 millimeters this measurement could be made within the ten-thousandth part.

In order to obtain this deflection, it was sufficient to make the mirror revolve 250 times per second and to use a "radius" of about 30 feet In order to use this large radius (distance from slit to revolving mirror), it was necessary that the mirror should be large and optically true; also, that the lens should be large and of great focal length Accordingly the mirror was made 1¼ inches in diameter, and a new lens, 8 inches in diameter, with a focal length of 150 feet was procured.

In January, 1879, an observation was taken, using the old lens, the mirror making 128 turns per second. The deflection was about 43 millimeters. The micrometer eye-piece used was substantially the same as Foucault's, except that part of the inclined plate of glass was silvered, thus securing a much greater quantity of light. The deflection having reached 43 millimeters, the inclined plate of glass could be dispensed with, the light going past the observer's head through the slit, and returning 43 millimeters to the left of the slit, where it could be easily observed.

Thus the micrometer eye-piece is much simplified, and many possible sources of error are removed.

The field was quite limited, the diameter being, in fact, but little greater than the width of the slit. This would have proved a most serious objection to the new arrangement. With the new lens, however, this difficulty disappeared, the field being about twenty times the width of the slit. It was expected that, with the new lens, the image would be less distinct; but the difference, if any, was small, and was fully compensated by the greater size of the field.

The first observation with the new lens was made January 30, 1879. The deflection was 70 millimeters. The image was sufficiently bright to be observed without the slightest effort. The first observation with the new micrometer eye-piece was made April 2, the deflection being 115 millimeters.

The first of the final series of observations was made on June 5. All the observations previous to this, thirty sets in all, were rejected. After this time, no set of observations nor any single observation was omitted.

THEORY OF NEW METHOD.

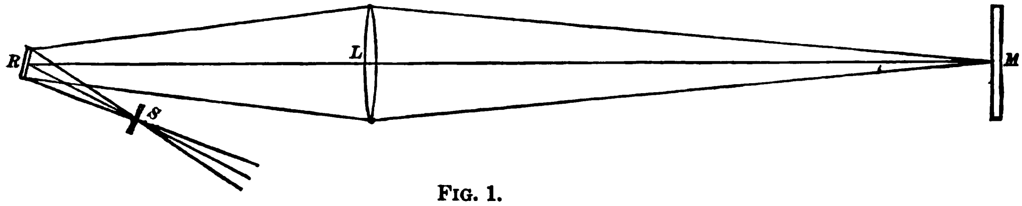

FIG. 1.

Let S, Fig. 1, be a slit, through which light passes, falling on R, a mirror free to rotate about an axis at right angles to the plane of the paper; L, a lens of great focal length, upon which the light falls which is reflected from R. Let M be a plane mirror whose surface is perpendicular to the line R, M, passing through the centers of R, L, and M, respectively. If L be so placed that an image of S is formed on the surface of M, then, this image acting as the object, its image will be formed at S, and will coincide, point for point, with S.

If, now, R be turned about the axis, so long as the light falls upon the lens, an image of the slit will still be formed on the surface of the mirror, though on a different part, and as long as the returning light falls on the lens an image of this image will be formed at S, notwithstanding the change of position of the first image at M. This result, namely, the production of a stationary image of an image in motion, is absolutely necessary in this method of experiment. It was first accomplished by Foucault, and in a manner differing apparently but little from the foregoing.

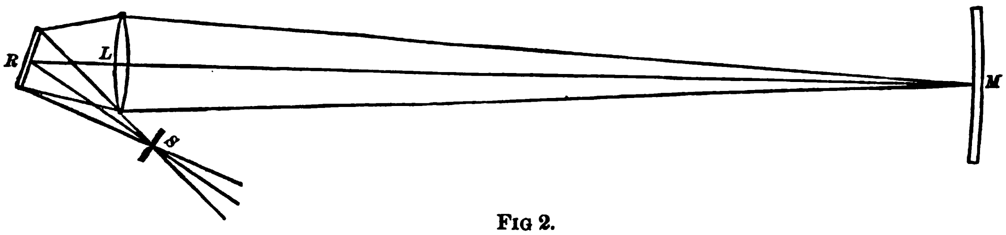

FIG 2.

In his experiments L, Fig. 2, served simply to form the image of S at M, and M, the returning mirror, was spherical, the center coinciding with the axis of R. The lens L was placed as near as possible to R. The light forming the return image lasts, in this case, while the first image is sweeping over the face of the mirror, M. Hence, the greater the distance R M, the larger must be the mirror in order that the same amount of light may be preserved, and its dimensions would soon become inordinate. The difficulty was partly met by Foucault, by using five concave reflectors instead of one, but even then the greatest distance he found it practicable to use was only 20 meters.

Returning to Fig. 1, suppose that R is in the principal focus of the lens L; then, if the plane mirror M have the same diameter as the lens, the first, or moving image, will remain upon M as long as the axis of the pencil of light remains on the lens, and *this will be the case no matter what the distance may be.*

When the rotation of the mirror R becomes sufficiently rapid, then the flashes of light which produce the second or stationary image become blended, so that the image appears to be continuous. But now it no longer coincides with the slit, but is *deflected* in the direction of rotation, and through twice the angular distance described by the

mirror, during the time required for light to travel twice the distance between the mirrors. This displacement is measured by the tangent of the arc it subtends. To make this as large as possible, the distance between the mirrors, the radius, and the speed of rotation should be made as great as possible.

The second condition conflicts with the first, for the radius is the difference between the focal length for parallel rays, and that for rays at the distance of the fixed mirror. The greater the distance, therefore, the smaller will be the radius.

There are two ways of solving the difficulty: first, by using a lens of great focal length; and secondly, by placing the revolving mirror within the principal focus of the lens. Both means were employed. The focal length of the lens was 150 feet, and the mirror was placed about 15 feet within the principal focus. A limit is soon reached, however, for the quantity of light received diminishes very rapidly as the revolving mirror approaches the lens.

ARRANGEMENT AND DESCRIPTION OF APPARATUS.

SITE AND PLAN.

The site selected for the experiments was a clear, almost level, stretch along the north sea-wall of the Naval Academy. A frame building was erected at the western end of the line, a plan of which is represented in Fig. 3.

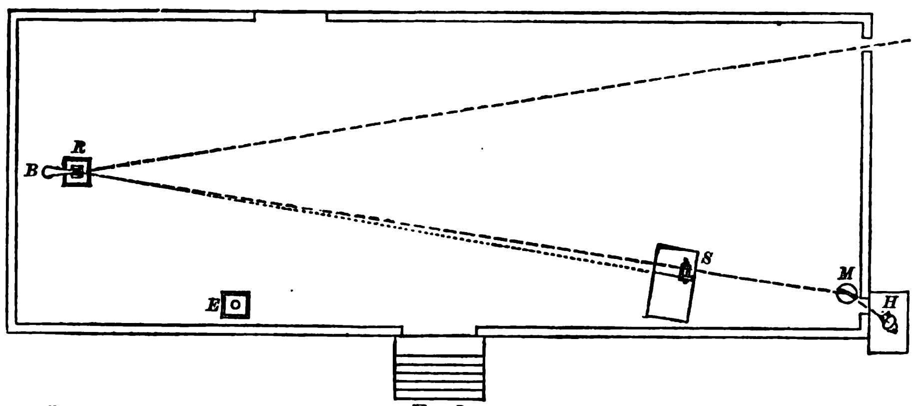

FIG 3.

The building was 45 feet long and 14 feet wide, and raised so that the line along which the light traveled was about 11 feet above the ground. A heliostat at H reflected the sun's rays through the slit at S to the revolving mirror R, thence through a hole in the shutter, through the lens, and to the distant mirror.

THE HELIOSTAT.

The heliostat was one kindly furnished by Dr. Woodward, of the Army Medical Museum, and was a modification of Foucault's form, designed by Keith. It was found to be accurate and easy to adjust. The light was reflected from the heliostat to a plane mirror, M, Fig. 3, so that the former need not be disturbed after being once adjusted.

THE REVOLVING MIRROR.

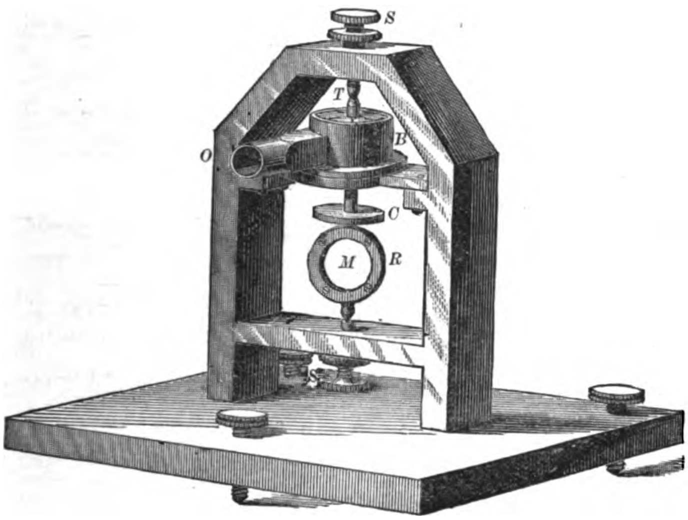

FIG. 4.

The revolving mirror was made by Fauth & Co., of Washington. It consists of a cast-iron frame resting on three leveling screws, one of which was connected by cords to the table at S, Fig. 3, so that the mirror could be inclined forward or backward while making the observations.

Two binding screws, S, S, Fig. 4, terminating in hardened steel conical sockets, hold the revolving part. This consists of a steel axle, X, Y, Figs. 4 and 5, the pivots being conical and hardened. The axle expands into a ring at R, which holds the mirror M. The latter was a disc of plane glass, made by Alvan Clark & Sons, about $1\frac{1}{4}$ inch in diameter and 0.2 inch thick. It was silvered on one side only, the reflection taking place from the outer or front surface. A species of turbine wheel, T, is held on the axle by friction. This wheel has six openings for the escape of air; a section of one of them is represented in Fig. 6.

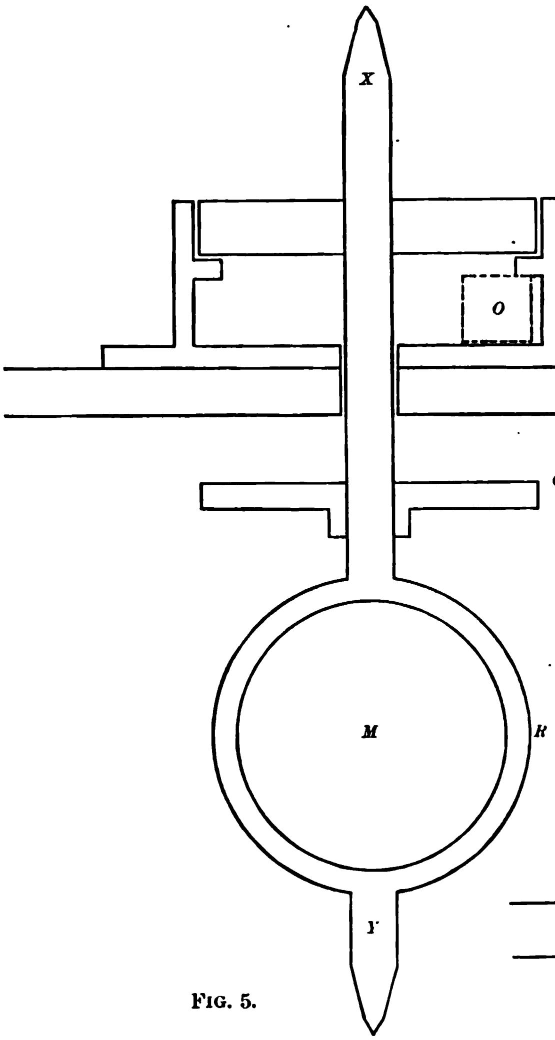

FIG. 5.

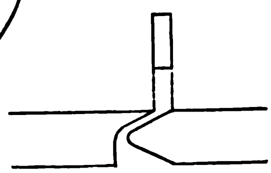

FIG. 6.

ADJUSTMENT OF THE REVOLVING MIRROR.

The air entering on one side at O, Fig. 5, acquires a rotary motion in the box B, B, carrying the wheel with it, and this motion is assisted by the reaction of the air in escaping. The disc C serves the purpose of bringing the center of gravity in the axis

of rotation. This was done, following Foucault's plan, by allowing the pivots to rest on two inclined planes of glass, allowing the arrangement to come to rest, and filing away the lowest part of the disc; trying again, and so on, till it would rest in indifferent equilibrium. The part corresponding to C, in Foucault's apparatus, was furnished with three vertical screws, by moving which the axis of figure was brought into coincidence with the axis of rotation. This adjustment was very troublesome. Fortunately, in this apparatus it was found to be unnecessary.

When the adjustment is perfect the apparatus revolves without giving any sound, and when this is accomplished, the motion is regular and the speed great. A slight deviation causes a sound due to the rattling of the pivots in the sockets, the speed is very much diminished, and the pivots begin to wear. In Foucault's apparatus oil was furnished to the pivots, through small holes running through the screws, by pressure of a column of mercury. In this apparatus it was found sufficient to touch the pivots occasionally with a drop of oil.

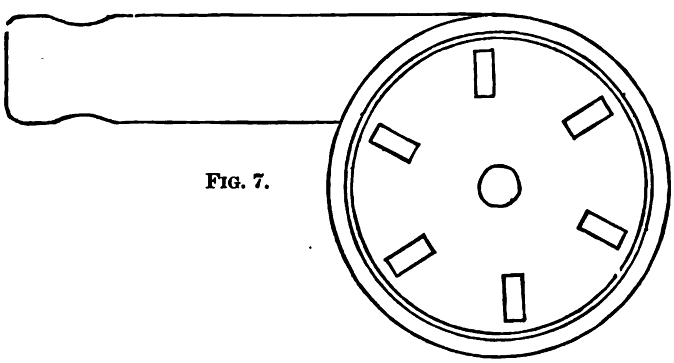

Fig. 7.

Fig. 7 is a view of the turbine, box, and supply-tube, from above. The quantity of air entering could be regulated by a valve to which was attached a cord leading to the observer's table.

The instrument was mounted on a brick pier.

THE MICROMETER.

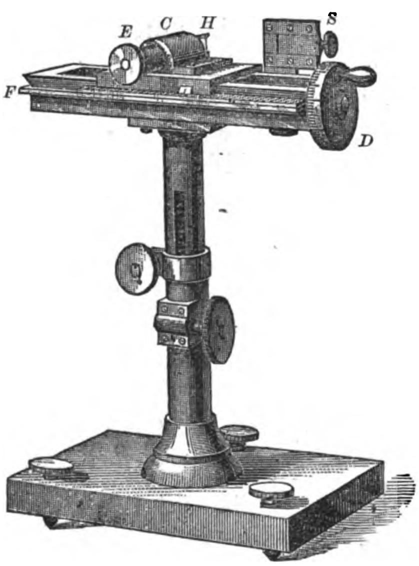

Fig. 8.

The apparatus for measuring the deflection was made by Grunow, of New York.

This instrument is shown in perspective in Fig. 8, and in plan by Fig. 9. The adjustable slit S is clamped to the frame F. A long millimeter-screw, not shown in Fig. 8, terminating in the divided head D, moves the carriage C, which supports the eye-piece E. The frame is furnished with a brass scale at F for counting revolutions, the head counting hundredths. The eye-piece consists of a single achromatic lens, whose focal length is about two inches At its focus, in H, and in nearly the same plane as the face of the slit, is a single vertical silk fiber. The apparatus is furnished with a standard with rack and pinion, and the base furnished with leveling screws.

MANNER OF USING THE MICROMETER.

In measuring the deflection, the eye-piece is moved till the cross-hair bisects the slit, and the reading of the scale and divided head gives the position. This measurement need not be repeated unless the position or width of the slit is changed. Then

the eye-piece is moved till the cross-hair bisects the deflected image of the slit; the reading of scale and head are again taken, and the difference in readings gives the deflection. The screw was found to have no lost motion, so that readings could be taken with the screw turned in either direction.

MEASUREMENT OF SPEED OF ROTATION.

To measure the speed of rotation, a tuning-fork, bearing on one prong a steel mirror, was used. This was kept in vibration by a current of electricity from five "gravity" cells. The fork was so placed that the light from the revolving mirror was reflected to a piece of plane glass, in front of the lens of the eye-piece of the micrometer, inclined at an angle of 45°, and thence to the eye. When fork and revolving mirror are both at rest, an image of the revolving mirror is seen. When the fork vibrates, this image is drawn out into a band of light.

When the mirror commences to revolve, this band breaks up into a number of moving images of the mirror; and when, finally, the mirror makes as many turns as the fork makes vibrations, these images are reduced to one, which is stationary. This is also the case when the number of turns is a submultiple. When it is a multiple or simple ratio, the only difference is that there are more images. Hence, to make the mirror execute a certain number of turns, it is simply necessary to pull the cord attached to the valve to the right or left till the images of the revolving mirror come to rest.

The electric fork made about 128 vibrations per second. No dependence was placed upon this rate, however, but at each set of observations it is compared with a standard Ut_3 fork, the temperature being noted at the same time. In making the comparison the sound-beats produced by the forks were counted for 60 seconds. It is interesting to note that the electric fork, as long as it remained untouched and at the same temperature, did not change its rate more than one or two hundredths vibrations per second.

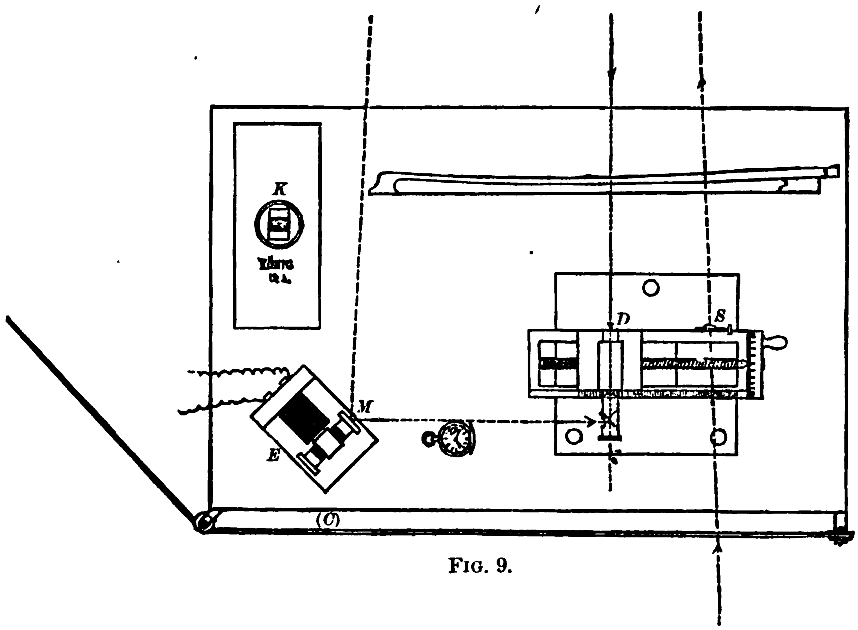

FIG. 9.

THE OBSERVER'S TABLE.

Fig. 9 represents the table at which the observer sits. The light from the heliostat passes through the slit at S, goes to the revolving mirror, &c., and, on its return, forms an image of the slit at D, which is observed through the eye-piece. E represents the electric fork (the prongs being vertical) bearing the steel mirror M. K is the standard fork on its resonator. C is the cord attached to the valve supplying air to the turbine.

THE LENS.

The lens was made by Alvan Clark & Sons. It was 8 inches in diameter; focal length, 150 feet; not achromatic. It was mounted in a wooden frame, which was placed on a support moving on a slide, about 16 feet long, placed about 80 feet from the building. As the diameter of the lens was so small in comparison with its focal length, its want of achromatism was inappreciable. For the same reason, the effect of "parallax" (due to want of coincidence in the plane of the image with that of the silk fiber in the eye-piece) was too small to be noticed.

THE FIXED MIRROR.

The fixed mirror was one of those used in taking photographs of the transit of Venus. It was about 7 inches in diameter, mounted in a brass frame capable of adjustment in a vertical and a horizontal plane by screw motion. Being wedge-shaped, it had to be silvered on the front surface. To facilitate adjustment, a small telescope furnished with cross-hairs was attached to the mirror by a universal joint. The heavy frame was mounted on a brick pier, and the whole surrounded by a wooden case to protect it from the sun.

ADJUSTMENT OF THE FIXED MIRROR.

The adjustment was effected as follows: A theodolite was placed at about 100 feet in front of the mirror, and the latter was moved about by the screws till the observer at the theodolite saw the image of his telescope reflected in the center of the mirror. Then the telescope attached to the mirror was pointed (without moving the mirror itself) at a mark on a piece of card-board attached to the theodolite. Thus the line of collimation of the telescope was placed at right angles to the surface of the mirror. The theodolite was then moved to 1,000 feet, and, if found necessary, the adjustment was repeated. Then the mirror was moved by the screws till its telescope pointed at the hole in the shutter of the building. The adjustment was completed by moving the mirror, by signals, till the observer, looking through the hole in the shutter, through a good spy-glass, saw the image of the spy-glass reflected centrally in the mirror.

The whole operation was completed in a little over an hour.

Notwithstanding the wooden case about the pier, the mirror would change its position between morning and evening; so that the last adjustment had to be repeated before every series of experiments.

APPARATUS FOR SUPPLYING AND REGULATING THE BLAST OF AIR.

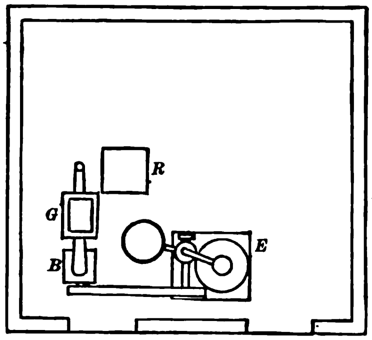

FIG. 10.

Fig. 10 represents a plan of the lower floor of the building. E is a three-horse power Lovegrove engine and boiler, resting on a stone foundation; B, a small Roots' blower; G, an automatic regulator. From this the air goes to a delivery-pipe, up through the floor, and to the turbine. The engine made about 4 turns per second and the blower about 15. At this speed the pressure of the air was about half a pound per square inch.

The regulator, Fig. 11, consists of a strong bellows supporting a weight of 370 pounds, partly counterpoised by 80 pounds in order to prevent the bellows from sagging. When the pressure of air from the blower exceeds the weight, the bellows commences to rise, and, in so doing, closes the valve V.

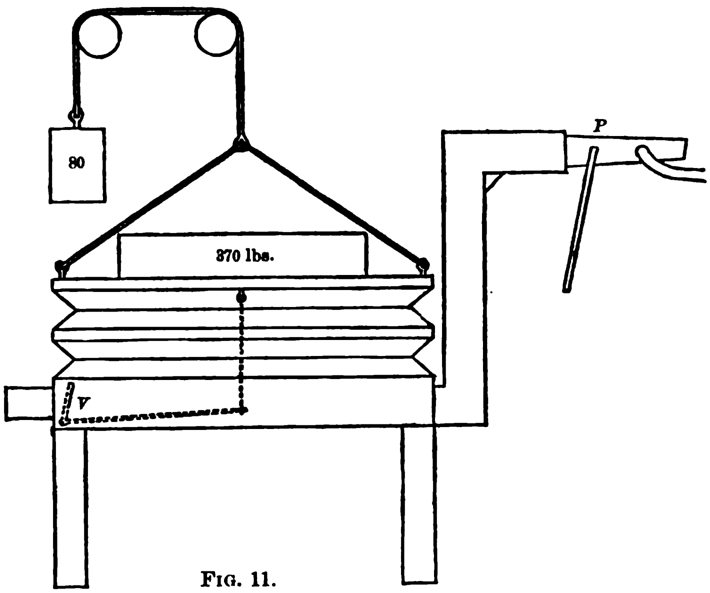

FIG. 11.

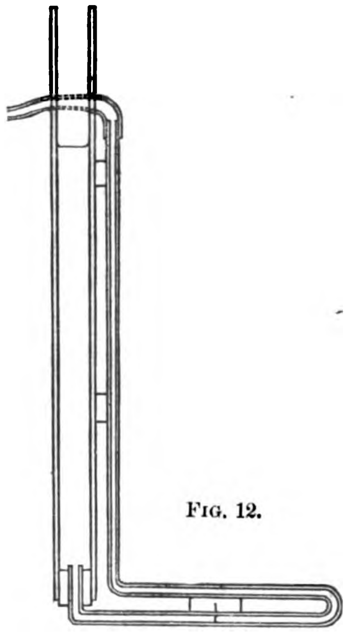
FIG. 12.

This arrangement was found in practice to be insufficient, and the following addition was made: A valve was placed at P, and the pipe was tapped a little farther on, and a rubber tube led to a water-gauge, Fig 12. The column of water in the smaller tube is depressed, and, when it reaches the horizontal part of the tube, the slightest variation of pressure sends the column from one end to the other. This is checked by an assistant at the valve; so that the column of water is kept at about the same place, and the pressure thus rendered very nearly constant. The result was satisfactory, though not in the degree anticipated. It was possible to keep the mirror at a constant speed for three or four seconds at a time, and this was sufficient for an observation. Still it would have been more convenient to keep it so for a longer time.

I am inclined to think that the variations were due to changes in the friction of the pivots rather than to changes of pressure of the blast of air.

It may be mentioned that the test of uniformity was very delicate, as a change of speed of one or two hundredths of a turn per second could easily be detected.

METHOD FOLLOWED IN EXPERIMENT.

It was found that the only time during the day when the atmosphere was sufficiently quiet to get a distinct image was during the hour after sunrise, or during the hour before sunset. At other times the image was "boiling" so as not to be recognizable. In one experiment the electric light was used at night, but the image was no more distinct than at sunset, and the light was not steady.

The method followed in experiment was as follows: The fire was started half an hour before, and by the time everything was ready the gauge would show 40 or 50 pounds of steam. The mirror was adjusted by signals, as before described. The heliostat was placed and adjusted. The revolving mirror was inclined to the right or left, so that the *direct* reflection of light from the slit, which otherwise would flash into the eye-piece at every revolution, fell either above or below the eye-piece.*

The revolving mirror was then adjusted by being moved about, and inclined forward and backward, till the light was seen reflected back from the distant mirror. This light was easily seen through the coat of silver on the mirror.

The distance between the front face of the revolving mirror and the cross-hair of the eye-piece was then measured by stretching from the one to the other a steel tape, making the drop of the catenary about an inch, as then the error caused by the stretch of the tape and that due to the curve just counterbalance each other.

The position of the slit, if not determined before, was then found as before described. The electric fork was started, the temperature noted, and the sound-beats between it and the standard fork counted for 60 seconds. This was repeated two or three times before every set of observations.

The eye-piece of the micrometer was then set approximately† and the revolving mirror started. If the image did not appear, the mirror was inclined forward or backward till it came in sight.

The cord connected with the valve was pulled right or left till the images of the revolving mirror, represented by the two bright round spots to the left of the cross-hair, came to rest. Then the screw was turned till the cross-hair bisected the deflected image of the slit. This was repeated till ten observations were taken, when the mirror was stopped, temperature noted, and beats counted. This was called a set of observations. Usually five such sets were taken morning and evening.

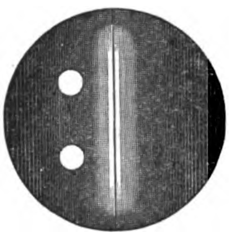

FIG. 13.

Fig. 13 represents the appearance of the image of the slit as seen in the eye-piece magnified about five times.

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* Otherwise this light would overpower that which forms the image to be observed. As far as I am aware, Foucault does not speak of this difficulty. If he allowed this light to interfere with the brightness of the image, he neglected a most obvious advantage. If he did incline the axis of the mirror to the right or left, he makes no allowance for the error thus introduced.

† The deflection being measured by its tangent, it was necessary that the scale should be at right angles to the radius (the radius drawn from the mirror to one or the other end of that part of the scale which represents this tangent). This was done by setting the eye-piece approximately to the expected deflection, and turning the whole micrometer about a vertical axis till the cross-hair bisected the circular field, of light reflected from the revolving mirror. The axis of the eye-piece being at right angles to the scale, the latter would be at right angles to radius drawn to the cross-hair.

DETERMINATION OF THE CONSTANTS.

COMPARISON OF THE STEEL TAPE WITH THE STANDARD YARD.

The steel tape used was one of Chesterman's, 100 feet long. It was compared with Wurdeman's copy of the standard yard, as follows:

Temperature was 55° Fahr.

The standard yard was brought under the microscopes of the comparator; the cross-hair of the unmarked microscope was made to bisect the division marked 0, and the cross-hair of the microscope, marked I, was made to bisect the division marked 36. The reading of microscope I was taken, and the other microscope was not touched during the experiment. The standard was then removed and the steel tape brought under the microscopes and moved along till the division marked 0.1 (feet) was bisected by the cross-hair of the unmarked microscope. The screw of microscope I was then turned till its cross-hair bisected the division marked 3.1 (feet), and the reading of the screw taken. The difference between the original reading and that of each measurement was noted, care being taken to regard the direction in which the screw was turned, and this gave the difference in length between the standard and each succesive portion of the steel tape in terms of turns of the micrometer-screw.

To find the value of one turn, the cross-hair was moved over a millimeter scale, and the following were the values obtained:

Turns of screw of microscope I in 1^{mm}—

7.68	7.73	7.60	7.67
7.68	7.62	7.65	7.57
7.72	7.70	7.64	7.69
7.65	7.59	7.63	7.64
7.55	7.65	7.61	7.63

Mean = 7.65

Hence one turn = 0.1307^{mm}.

or = 0.0051 inch.

The length of the steel tape from 0.1 to 99.1 was found to be greater than 33 yards, by 7.4 turns = $.96^{mm}$ -	+.003 feet.
Correction for temperature - - - - - -	+.003 feet.
Length - - - - - - - - - - - - -	100.000 feet.
Corrected length - - - - - -	100.006 feet.

DETERMINATION OF THE VALUE OF MICROMETER.

Two pairs of lines were scratched on one slide of the slit, about 38^{mm} apart, *i. e.*, from the center of first pair to center of second pair. This distance was measured at intervals of 1^{mm} through the whole length of the screw, by bisecting the interval between each two pairs by the vertical silk fiber at the end of the eye-piece. With these values a curve was constructed which gave the following values for this distance, which we shall call $D_{,}$:

	Turns of screw.
At 0 of scale $D_{,}$	= 38.155
10 of scale $D_{,}$	38.155
20 of scale $D_{,}$	38.150
30 of scale $D_{,}$	38 150
40 of scale $D_{,}$	38.145
50 of scale $D_{,}$	38.140
60 of scale $D_{,}$	38.140
70 of scale $D_{,}$	38.130
80 of scale $D_{,}$	38.130
90 of scale $D_{,}$	38.125
100 of scale $D_{,}$	38.120
110 of scale $D_{,}$	38 110
120 of scale $D_{,}$	38.105
130 of scale $D_{,}$	38.100
140 of scale $D_{,}$	38.100

Changing the form of this table, we find that—

For the *first* 10 turns the *average* value of $D_{,}$ is	38.155
20 turns	38.153
30 turns	38.152
40 turns	38.151
50 turns	38.149
60 turns	38.148
70 turns	38.146
80 turns	38.144
90 turns	38.142
100 turns	38.140
110 turns	38.138
120 turns	38.135
130 turns	38.132
140 turns	38.130

On comparing the scale with the standard meter, the temperature being 16°.5 C., 140 divisions were found to $= 139.462^{mm}$. This multiplied by $(1 + .0000188 \times 16.5) = 139.505^{mm}$.

One hundred and forty divisions were found to be equal to 140.022 turns of the screw, whence 140 turns of the screw = 139 483^{mm}, or 1 turn of the screw = 0.996305^{mm}.

This is the *average* value of one turn in 140.

But the average value of $D_{,}$ for 140 turns is, from the preceding table, 38.130.

Therefore, the true value of $D_{,}$ is $38.130 \times .996305^{mm}$, and the average value of one turn for 10, 20, 30, etc., turns, is found by dividing $38.130 \times .996305$ by the values of $D_{,,}$ given in the table.

This gives the value of a turn—

		mm.
For the first	10 turns	0.99570
	20 turns	0 99570
	30 turns	0 99573
	40 turns	0.99577
	50 turns	0.99580
	60 turns	0.99583
	70 turns	0.99589
	80 turns	0 99596
	90 turns	0.99601
	100 turns	0.99606
	110 turns	0.99612
	120 turns	0 99618
	130 turns	0.99625
	140 turns	0.99630

NOTE.—The micrometer has been sent to Professor Mayer, of Hoboken, to test the screw again, and to find its value. The steel tape has been sent to Professor Rogers, of Cambridge, to find its length again. (See page 145.)

MEASUREMENT OF THE DISTANCE BETWEEN THE MIRRORS.

Square lead weights were placed along the line, and measurements taken from the forward side of one to forward side of the next. The tape rested on the ground (which was very nearly level), and was stretched by a constant force of 10 pounds.

The correction for length of the tape (100.006) was +0.12 of a foot.

To correct for the stretch of the tape, the latter was stretched with a force of 15 pounds, and the stretch at intervals of 20 feet measured by a millimeter scale

	mm.
At 100 feet the stretch was	8.0
80 feet the stretch was	5.0
60 feet the stretch was	5 0
40 feet the stretch was	3.5
20 feet the stretch was	1.5
300	23.00

Weighted mean $= 7.7^{mm}$.
For 10 pounds, stretch $= 5.1^{mm}$.
$= 0.0167$ feet.
Correction for whole distance $= +0.33$ feet.

The following are the values obtained from five separate measurements of the distance between the caps of the piers supporting the revolving mirror and the distant reflector; allowance made in each case for effect of temperature:

1985 13 feet.
1985.17 feet.
1984.93 feet.
1985.09 feet.
1985.09 feet.

Mean = 1985.082 feet.
+.70. Cap of pier to revolving mirror.
+.33. Correction for stretch of tape.
+.12. Correction for length of tape.

1986.23. True distance between mirrors.

RATE OF STANDARD Ut_3 FORK.

The rate of the standard Ut_3 fork was found at the Naval Academy, but as so much depended on its accuracy, another series of determinations of its rate was made, together with Professor Mayer, at the Hoboken Institute of Technology.

Set of determinations made at Naval Academy.

The fork was armed with a tip of copper foil, which was lost during the experiments and replaced by one of platinum having the same weight, 4.6 mgr. The fork, on its resonator, was placed horizontally, the platinum tip just touching the lamp-blacked cylinder of a Schultze chronoscope. The time was given either by a sidereal break-circuit chronometer or by the break-circuit pendulum of a mean-time clock. In the former case the break-circuit worked a relay which interrupted the current from three Grove cells. The spark from the secondary coil of an inductorium was delivered from a wire near the tip of the fork. Frequently two sparks near together were given, in which case the first alone was used. The rate of the chronometer, the record of which was kept at the Observatory, was very regular, and was found by observations of transits of stars during the week to be + 1.3 seconds per day, which is the same as the recorded rate.

SPECIMEN OF A DETERMINATION OF RATE OF Ut_3 FORK.

Temp. = 27° C. Column 1 gives the number of the spark or the number of the second. Column 2 gives the number of sinuosities or vibrations at the corresponding second. Column 3 gives the difference between 1 and 11, 2 and 12, 3 and 13, etc.

July 4, 1879.

1	0.1	2552.0
2	255.3	2551.7
3	510.5	2551.9
4	765.6	2551.9
5	1020.7	2552.1
6	1275.7	2552.0
7	1530.7	2551.8
8	1786.5	2551.4
9	2041.6	2551.7
10	2297.0	2551.5
11	2552.1	255.180 = mean ÷ 10.
12	2807.0	+ .699 = reduction for mean time.
13	3062.4	+ .003 = correction for rate.
14	3317.5	+ .187 = correction for temperature.
15	3572.8	256.069 = number of vibrations per second at 65° Fahr.
16	3827.7	
17	4082.5	
18	4335.9	
19	4593.3	
20	4848.5	

The correction for temperature was found by Professor Mayer by counting the sound-beats between the standard and another Ut_3 fork, at different temperatures. His result is +.012 vibrations per second for a diminution of 1° Fahr. Using the same method, I arrived at the result +.0125. Adopted +.012.

Résumé of determinations made at Naval Academy.

In the following table the first column gives the date, the second gives the total number of seconds, the third gives the result uncorrected for temperature, the fourth gives the temperature (centigrade), the fifth gives the final result, and the sixth the difference between the greatest and least values obtained in the several determinations for intervals of ten seconds:

July 4	20	255.882	27.0	256.069	0.07
5	19	255.915	26.4	256.089	0.05
5	18	255.911	26.0	256.077	0.02
6	21	255.874	24.7	256.012	0.13
6	9	255.948	24.8	256.087	0.24
7	22	255.938	24.6	256.074	0.05
7	21	255.911	25.3	256.061	0.04
8	20	255.921	26.6	256.100	0.02
8	20	255.905	26.6	256.084	0.06
8	20	255.887	26.6	256.066	0.03
			Mean =	256.072	

In one of the preceding experiments, I compared the two Vt_3 forks while the standard was tracing its record on the cylinder, and also when it was in position as for use in the observations. The difference, if any, was less than .01 vibration per second.

Second determination.

(Joint work with Professor A. M. Mayer, Stevens Institute, Hoboken.)

The fork was wedged into a wooden support, and the platinum tip allowed to rest on lampblacked paper, wound about a metal cylinder, which was rotated by hand Time was given by a break-circuit clock, the rate of which was ascertained, by comparisons with Western Union time-ball, to be 9.87 seconds. The spark from secondary coil of the inductorium passed from the platinum tip, piercing the paper. The size of the spark was regulated by resistances in primary circuit.

The following is a specimen determination:

Column 1 gives the number of the spark or the number of seconds. Column 2 gives the corresponding number of sinuosities or vibrations. Column 3 gives the difference between the 1st and 7th ÷ 6, 2nd and 8th ÷ 6, etc.

1	0.3	255.83	
2	256.1	255.90	
3	511.7	255.90	
4	767.9	255.93	
5	1023.5	255.92	
6	1289.2	256.01	
7	1535.3	255.95	
8	1791.5	255.920	= mean.
9	2047.1	— .028	= correction for rate.
10	2303 5	255.892	
11	2559.0	+ .180	= correction for temperature.
12	2825.3	256.072	= number of vibrations per second at 65° Fahr.
13	3071.0		

In the following *résumé*, column 1 gives the number of the experiments. Column 2 gives the total number of seconds. Column 3 gives the result not corrected for temperature. Column 4 gives the temperature Fahrenheit. Column 5 gives the final result. Column 6 gives the difference between the greatest and least values:

1	13	255.892	80	256.072	0.18
2	11	255.934	81	256.126	0.17
3	13	255.899	81	256.091	0.12
4	13	255.988	75	256.108	0.13
5	11	255.948	75	256.068	0.05
6	12	255.970	75	256.090	0.05
7	12	255.992	75	256.112	0.20
8	11	255.992	76	256.124	0.03
9	11	255.888	81	256.080	0.13
10	13	255.878	81	256.070	0.13
			Mean =	256.094	

EFFECT OF SUPPORT AND OF SCRAPING.

The standard Vt_3 fork held in its wooden support was compared with another fork on a resonator loaded with wax and making with standard about five beats per second. The standard was free from the cylinder. The beats were counted by coincidences with the $\frac{1}{5}$ second beats of a watch.

Specimen.

Coincidences were marked—

At 32 seconds.
37 seconds.
43.5 seconds.
49 seconds.
54.5 seconds.
61.5 seconds.
$61.5 - 32 = 29.5$.
$29.5 \div 5 = 5.9 =$ time of one interval.

Résumé.

1 - - - - -	5.9
2 - - - - -	6.2
3 - - - - -	6.2
4 - - - - -	6.2

Mean $= 6.13 =$ time of one interval between coincidences.

In this time the watch makes $6.13 \times 5 = 30.65$ beats, and the forks make $30.65 + 1 = 31.65$ beats.

Hence the number of beats per second is $31.65 \div 6.13 = 5.163$.

Specimen.

Circumstances the same as in last case, except that standard Vt_3 fork was allowed to trace its record on the lampblacked paper, as in finding its rate of vibration.

Coincidences were marked at—

59 seconds.
04 seconds.
10.5 seconds.
17 seconds.

$77 - 59 = 18$.
$18 \div 3 = 6.0 =$ time of one interval.

Résumé.

No. 1 - - - -	6.0 seconds.	
2 - - - -	6.0 seconds.	$6.31 \times 5 = 31.55$
3 - - - -	6.7 seconds.	$+ \ 1.00$
4 - - - -	6.3 seconds.	32 55
5 - - - -	6.5 seconds.	$32.55 \div 6.31 = 5.159$
6 - - - -	6.7 seconds.	With fork free 5.163
7 - - - -	6 0 seconds.	
		Effect of scrape $= -$ .004
Mean $= 6.31$ seconds.		

Specimen.

Circumstances as in first case, except that both forks were on their resonators. Coincidences were observed at—

21 seconds.
28 seconds.
36 seconds.
44 seconds.
51 seconds.
60 seconds.
$60 - 21 = 39$
$39 \div 5 = 7.8$ = time of one interval.

Résumé.

No. 1 - -	7.8	seconds.
2 - -	7.1	seconds.
3 - -	7.6	seconds.
4 - -	7.4	seconds.
5 - -	7.2	seconds.

Mean = 7.42 seconds.

$$7.42 \times 5 = 37.10$$
$$+ 1.00$$
$$38.10$$
$$38.10 \div 7.42 = 5.133$$
(Above) 5.159

Effect of support and scrape = —.026

Mean of second determination was - - - - - - -	256.094
Applying correction (scrape, etc.) - - - - - - -	—.026
Corrected mean - - - - - - - -	256.068
Result of first determination - - - - - - -	256.072
Final value - - - - - - - - - -	256.070

Note.—The result of first determination excludes all work except the series commencing July 4. If previous work is included, and also the result first obtained by Professor Mayer, the result would be 256.089.

256 180
256.036
256 072
256.068
Mean = 256.089

The previous work was omitted on account of various inaccuracies and want of practice, which made the separate results differ widely from each other.

THE FORMULÆ.

The formulæ employed are—

$$(1) \quad \tan \varphi = \frac{d_{,}}{r}$$

$$(2) \quad V = \frac{2592000'' \times D \times n}{\varphi''}$$

φ = angle of deflection.
$d_{,}$ = corrected displacement (linear).
r = radius of measurement.
D = twice the distance between the mirrors.
n = number of revolutions per second.

α = inclination of plane of rotation.
d = deflection as read from micrometer.
B = number of beats per second between electric Vt_2 fork and standard Vt_3
Cor = correction for temperature of standard Vt_3.
V = velocity of light.
T = value of one turn of screw (Table, page 126.)

Substituting for d, its value or $d \times T \times \sec \alpha$ (log sec α = .00008), and for D its value 3972.46, and reducing to kilometers, the formulæ become—

$$(3) \quad \tan \varphi = c, \frac{dT}{r}; \quad \log c, = .51607$$

$$(4) \quad V = c \frac{n}{\varphi}; \quad \log c = .49670$$

D and r are expressed in feet and $d,$ in millimeters.
Vt_3 fork makes 256.070 vibrations per second at 65° Fahr.
D = 3972.46 feet.
tan α = tangent of angle of inclination of plane of rotation = 0.02 in all but the last twelve observations, in which it was 0.015.
log $c,$ = .51607 (.51603 in last twelve observations.).
log c = .49670.
The electric fork makes $\frac{1}{2}$ (256.070 + B + cor.) vibrations per second, and n is a multiple, submultiple, or simple ratio of this.

OBSERVATIONS.

SPECIMEN OBSERVATION.

June 17. sunset. Image good; best in column (4).
The columns are sets of readings of the micrometer for the deflected image of slit.

	112.81	112.80	112.83	112.74	112.79
	81	81	81	76	78
	79	78	78	74	74
	80	75	74	76	74
	79	77	74	76	77
	82	79	72	78	81
	82	73	76	78	77
	76	78	81	79	75
	83	79	74	83	82
	78	73	76	78	82
Mean =	112.801	112.773	112.769	112.772	112.779
Zero =	0.260	0.260	0.260	0.260	0.260
d =	112.541	112.513	112.509	112.512	112.519
Temp =	77°	77°	77°	77°	77°
B =	+ 1.500				
Cor =	− .144				
	+ 1.365				
	256.070				
n =	257.426	257.43	257.43	257.43	257 43
r =	28.157	28.157	28.157	28.157	28.157

The above specimen was selected because in it the readings were all taken by another and noted down without divulging them till the whole five sets were completed.

The following is the calculation for V:

	1st set.	2d, 3d, and 4th sets.	5th set.
log	$c_{\prime} = 51607$	51607	51607
"	$T = 99832$	99832	99832
"	$d = 05131$	05119	05123
	56570	56558	56562
"	$r = 44958$	44958	44958
" tan	$\varphi = 11612$	11600	11604
	$\varphi = 2694''.7$	$2694''.1$	$2694''.3$
"	$c = 49670$	49670	49670
"	$n = 41066$	41066	41066
	90736	90736	90736
"	$\varphi = 43052$	43042	43046
"	$V = 47684$	47694	47690
	$V = 299800$	299880	299850

In the following table, the numbers in the column headed "Distinctness of Image" are thus translated: 3, good; 2, fair; 1, poor. These numbers do not, however, show the relative weights of the observations

The numbers contained in the columns headed "Position of Deflected Image," "Position of Slit," and displacement of image in divisions were obtained as described in the paragraph headed "Micrometer," page 120.

The column headed "B" contains the number of "beats" per second between the electric Vt_2 fork and the standard Vt_3 as explained in the paragraph headed "Measurement of the Speed of Rotation." The column headed "Cor." contains the correction of the rate of the standard fork for the difference in temperature of experiment and 65° Fahr., for which temperature the rate was found. The numbers in the column headed "Number of revolutions per second" were found by applying the corrections in the two preceding columns to the rate of the standard, as explained in the same paragraph.

The "radius of measurement" is the distance between the front face of the revolving mirror and the cross-hair of the micrometer.

The numbers in the column headed "Value of one turn of the screw" were taken from the table, page 127.

Date.	Distinctness of image.	Temperature, Fahr.	Position of deflected image.	Position of slit.	Displacement of image in divisions.	Difference between greatest and least values.	B.	Cor.	Number of revolutions per second.	Radius of measurement, in feet.	Value of one turn of the screw.	Velocity of light in air, in kilometers.	Remarks.
June 5 . .	3	76	114.85	0.300	114.55	0.17	+ 1.423	— 0.132	257.36	28.672	0.99614	299850	Electric light.
7 . .	2	72	114.64	0.074	114.56	0.10	1.533	— 0.084	257.52	28.655	0.99614	299740	P. M. Frame inclined at various angles.
7 . .	2	72	114.58	0.074	114.50	0.08	1.533	— 0.084	257.52	28.647	0.99614	299900	P. M. Frame inclined at various angles.
7 . .	2	72	85.91	0.074	85.84	0.12	1.533	— 0.084	193.14	28.647	0.99598	300070	P. M. Frame inclined at various angles.
7 . .	2	72	85.97	0.074	85.89	0.07	1.533	— 0.084	193.14	28.650	0.99598	299930	P. M. Frame inclined at various angles.
7 . .	2	72	114.61	0.074	114.53	0.07	1.533	— 0.084	257.42	28.650	0.99614	299850	P. M. Frame inclined at various angles.
9 . .	3	83	114.54	0.074	114.47	0.07	1.533	— 0.216	257.39	28.658	0.99614	299950	P. M. Frame inclined at various angles.
9 . .	3	83	114.54	0.074	114.46	0.10	1.533	— 0.216	257.39	28.658	0.99614	299980	P. M. Frame inclined at various angles.
9 . .	3	83	114.57	0.074	114.47	0.08	1.533	— 0.216	257.39	28.662	0.99614	299980	P. M. Frame inclined at various angles.
9 . .	3	83	114.57	0.074	114.50	0.06	1.533	— 0.216	257.39	28.660	0.99614	299880	P. M. Frame inclined at various angles.
9 . .	2	83	114.61	0.074	114.53	0.13	1.533	— 0.216	257.39	28.678	0.99614	300000	P. M. Frame inclined at various angles.
10 . .	2	90	114.60	0.074	114.52	0.11	1.517	— 0.300	257.29	28.685	0.99614	299980	P. M.
10 . .	2	90	114.62	0.074	114.54	0.08	1.517	— 0.300	257.29	28.685	0.99614	299930	P. M.
12 . .	2	71	114.81	0.074	114.74	0.09	1.450	— 0.072	257.45	28.690	0.99614	299650	A. M.
12 . .	2	71	114.78	0.074	114.70	0.05	1.450	— 0.072	257.45	28.690	0.99614	299760	A. M.
12 . .	1	71	114.76	0.074	114.68	0.09	1.450	— 0.072	257.45	28.690	0.99614	299810	A. M.
13 . .	3	72	112.64	0.074	112.56	0.09	1.500	— 0.084	257.49	28.172	0.99614	300000	A. M.
13 . .	3	72	112.63	0.074	112.56	0.10	1.500	— 0.084	257.49	28.172	0.99614	300000	A. M.
13 . .	2	72	112.65	0.074	112.57	0.08	1.500	— 0.084	257.49	28.172	0.99614	299960	A. M.
13 . .	3	79	112.82	0.260	112.56	0.06	1.517	— 0.168	257.42	28.178	0.99614	299960	P. M.
13 . .	3	79	112.82	0.260	112.56	0.13	1.517	— 0.168	257.42	28.178	0.99614	299960	P. M.
13 . .	3	79	112.83	0.260	112.57	0.07	1.517	— 0.168	257.42	28.178	0.99614	299940	P. M.
13 . .	3	79	112.82	0.260	112.56	0.06	1.517	— 0.168	257.42	28.178	0.99614	299960	P. M.
13 . .	3	79	112.83	0.260	112.57	0.11	1.517	— 0.168	257.42	28.178	0.99614	299940	P. M.
13 . .	3	79	113.41	0.260	113.15	11	1.517	— 0.168	258.70	28.152	0.99614	299880	P. M. Set micrometer and counted oscillations.

Date.	Distinctness of image.	Temperature, Fahr.	Position of deflected image.	Position of slit.	Displacement of image in divisions.	Difference between greatest and least values.	B.	Cor.	Number of revolutions per second.	Radius of measurement, in feet.	Value of one turn of the screw.	Velocity of light in air, in kilometers.	Remarks.
June 13 . .	3	79	112.14	0.260	111.88	6	+ 1.517	— 0.168	255.69	28.152	0.99614	299800	Oscillations of image of revolving mirror.
14 . .	1	64	112.83	0.260	112.57	0.12	1.500	+ 0.012	257.58	28.152	0.99614	299850	A. M.
14 . .	1	64	112.83	0.260	112.57	0.05	1.517	+ 0.012	257.60	28.152	0.99614	299880	A. M.
14 . .	1	65	112.81	0.260	112.55	0.11	1.517	0.000	257.59	28.152	0.99614	299900	A. M.
14 . .	1	66	112.83	0.260	112.57	0.09	1.517	— 0.012	257.57	28.152	0.99614	299840	A. M.
14 . .	1	67	112.83	0.260	112.57	0.12	1.517	— 0.024	257.56	28.152	0.99614	299830	A. M.
14 . .	1	84	112.78	0.260	112.52	0.06	1.517	— 0.228	257.36	28.159	0.99614	299790	P. M. Readings taken by Lieut. Nazro.
14 . .	1	85	112.76	0.260	112.50	0.08	1.500	— 0.240	257.33	28.159	0.99614	299810	P. M. Readings taken by Lieut. Nazro.
14 . .	1	84	112.72	0.260	112.46	0.08	1.483	— 0.228	257.32	28.159	0.99614	299880	P. M. Readings taken by Lieut. Nazro.
14 . .	1	84	112.73	0.260	112.47	0.09	1.483	— 0.228	257.32	28.159	0.99614	299880	P. M.
14 . .	1	84	112.75	0.260	112.49	0.09	1.483	— 0.228	257.32	28.159	0.99614	299830	P. M.
17 . .	2	62	112.85	0.260	112.59	0.09	1.517	+ 0.036	257.62	28.149	0.99614	299800	A. M.
17 . .	2	63	112.84	0.260	112.58	0.06	1.500	+ 0.024	257.59	28.149	0.99614	299790	A. M.
17 . .	1	64	112.85	0.260	112.59	0.07	1.500	+ 0.012	257.58	28.149	0.99614	299760	A. M.
17 . .	3	77	112.80	0.260	112.54	0.07	1.500	— 0.144	257.43	28.157	0.99614	299800	P. M. Readings taken by Mr. Clason.
17 . .	3	77	112.77	0.260	112.51	0.08	1.500	— 0.144	257.43	28.157	0.99614	299880	P. M. Readings taken by Mr. Clason.
17 . .	3	77	112.77	0.260	112.51	0.11	1.500	— 0.144	257.43	28.157	0.99614	299880	P. M. Readings taken by Mr. Clason.
17 . .	3	77	112.77	0.260	112.51	0.09	1.500	— 0.144	257.43	28.157	0.99614	299880	P. M. Readings taken by Mr. Clason.
17 . .	3	77	112.78	0.260	112.52	0.08	1.500	— 0.144	257 43	28.157	0.99614	299860	P. M. Readings taken by Mr. Clason.
18 . .	1	58	112.90	0.265	112.64	0.07	1.500	+ 0.084	257.65	28.150	0.99614	299720	A. M.
18 . .	1	58	112.90	0.265	112.64	0.10	1.500	+ 0.084	257.65	28.150	0.99614	299720	A. M.
18 . .	1	59	112.92	0.265	112.66	0.07	1.483	+ 0.072	257.62	28.150	0.99614	299620	A. M.
18 . .	2	75	112.79	0.265	112.52	0.09	1.483	— 0.120	257.43	28.158	0.99614	299860	P. M.
18 . .	2	75	112.75	0.265	112.48	0.10	1.483	— 0.120	257.43	28.158	0.99614	299970	P. M.
18 . .	2	75	112.76	0.265	112.49	0.08	1.483	— 0.120	257.43	28.158	0.99614	299950	P. M.

Date.	Distinctness of image.	Temperature, Fahr.	Position of deflected image.	Position of slit.	Displacement of image in divisions.	Difference between greatest and least values.	B.	Cor.	Number of revolutions per second.	Radius of measurement, in feet.	Value of one turn of the screw.	Velocity of light in air, in kilometers.	Remarks.
June 20 . .	3	60	112.94	0.265	112.67	0.07	+1.517	+ 0.063	257.65	28.172	9.99614	299880	A. M.
20 . .	3	61	112.92	0.265	112.65	0.09	1.517	+ 0.048	257.63	28.172	0.99614	299910	A. M.
20 . .	2	62	112.94	0.265	112.67	0.07	1.517	+ 0.036	257.62	28.172	0.99614	299850	A. M.
20 . .	2	63	112.93	0.265	112.66	0.03	1.517	+ 0.024	257.61	28.172	0.99614	299870	A. M.
20 . .	2	78	133.48	0.265	133.21	0.13	1.450	— 0.156	257.36	33.345	0.99627	299840	P. M.
20 . .	2	79	133.49	0.265	133.23	0.09	1.500	— 0.168	257.40	33.345	0.99627	299840	P. M.
20 . .	2	80	133.49	0.265	133.22	0.07	1.500	— 0.180	257.39	33.345	0.99627	299850	P. M.
20 . .	2	79	133.50	0.265	133.24	0.13	1.483	— 0.168	257.39	33.345	0.99627	299840	P. M.
20 . .	2	79	133.49	0.265	133.22	0.06	1.483	— 0.168	257.38	33.345	0.99627	299840	P. M.
20 . .	2	79	133.49	0.265	133.22	0.10	1.483.	— 0.168	257.38	33.345	0.99627	299840	P. M.
21 . .	2	61	133.56	0.265	133.29	0.12	1.533	+ 0.048	257.65	33.332	0.99627	299890	A. M.
21 . .	2	62	133.58	0.265	133.31	0.08	1.533	+ 0.036	257.64	33.332	0.99627	299810	A. M.
21 . .	2	63	133.57	0.265	133.31	0.09	1.533	+ 0.024	257.63	33.332	0.99627	299810	A. M.
21 . .	2	64	133.57	0.265	133.30	0.11	1.533	+ 0.012	257.61	33.332	0.99627	299820	A. M.
21 . .	2	65	133.56	0.265	133.30	0.13	1.533	0.000	257.60	33.332	0.99627	299800	A. M.
21 . .	3	80	133.48	0.265	133.21	0.06	1.533	— 0.180	257.42	33.330	0.99627	299770	P. M.
21 . .	3	81	133.46	0.265	133.19	0.10	1.500	— 0.192	257.38	33.330	0.99627	299760	P. M.
21 . .	3	82	133.46	0.265	133.20	0.05	1.500	— 0.204	257.37	33.330	9.99627	299740	P. M.
21 . .	3	82	133.46	0.265	133.20	0.08	1.517	— 0.204	257.38	33.330	0.99627	299750	P. M.
21 . .	3	81	133.46	0.265	133.19	0.08	1.500	— 0.192	257.38	33.330	0.99627	299760	P. M.
23 . .	3	89	133.43	0.265	133.16	0.08	1.542	— 0.288	257.32	33.345	0.99627	299910	P. M.
23 . .	3	89	133.42	0.265	133.15	0.06	1.550	— 0.288	257.33	33.345	0.99627	299920	P. M.
23 . .	3	90	133.43	0.265	133.17	0.09	1.550	— 0.300	257.32	33.345	0.99627	299890	P. M.
23 . .	3	90	133.43	0.265	133.16	0.07	1.533	— 0.300	257.30	33.345	0.99627	299860	P. M.
23 . .	3	90	133.42	0.265	133.16	0.07	1.517	— 0.300	257.29	33.345	0.99627	299880	P. M.

Date.	Distinctness of image.	Temperature, Fahr.	Position of deflected image.	Position of slit.	Displacement of image in divisions.	Difference between greatest and least values.	B.	Cor.	Number of revolutions per second.	Radius of measurement, in feet.	Value of one turn of the screw.	Velocity of light in air, in kilometers.	Remarks.
June 24 . .	3	72	133.47	0.265	133.20	0.15	+ 1.517	− 0.084	257.50	33.319	0.99627	299720	A. M.
24 . .	3	73	133.44	0.265	133.17	0.04	1.517	− 0.096	257.49	33.319	0.99627	299840	A. M.
24 . .	3	74	133.42	0.265	133.16	0.11	1.517	− 0.108	257.48	33.319	0.99627	299850	A. M.
24 . .	3	75	133.42	0.265	133.16	0.06	1.517	− 0.120	257.47	33.319	0.99627	299850	A. M.
24 . .	3	76	133.44	0.265	133.18	0.10	1.517	− 0.132	257.45	33.319	0.99627	299780	A. M.
26 . .	2	86	133.42	0.265	133.15	0.05	1.508	− 0.252	257.33	33.339	0.99627	299890	P. M.
26 . .	2	86	133.44	0.265	133.17	0.08	1.508	− 0.252	257.33	33.339	0.99627	299840	P. M.
27 . .	3	73	133.49	0.265	133.22	0.11	1.483	− 0.096	257.46	33.328	0.99627	299780	A. M.
27 . .	3	74	133.47	0.265	133.20	0.06	1.483	− 0.108	257.44	33.328	0.99627	299810	A. M.
27 . .	3	75	133.47	0.265	133.21	0.09	1.483	− 0.120	257.43	33.328	0.99627	299760	A. M.
27 . .	3	75	133.45	0.265	133.19	0.09	1.467	− 0.120	257.42	33.328	0.99627	299810	A M.
27 . .	3	76	133.47	0.265	133.20	0.08	1.483	− 0.132	257.42	33.328	0.99627	299790	A. M.
27 . .	3	76	133.45	0.265	133.19	0.10	1.483	− 0.132	257.42	33.328	0.99627	299810	A. M.
30 . .	2	85	35.32	135.00	99.68	0.05	1.500	− 0.240	193.00	33.274	0.99645	299820	P. M. Mirror inverted.
30 . .	2	86	35.34	135.00	99.67	0.06	1.508	− 0.252	193.00	33.274	0.09645	299850	P. M. Mirror inverted.
30 . .	2	86	35.34	135.00	99.66	0.10	1.508	− 0.252	193.00	33.274	0.99645	299870	P. M. Mirror inverted.
30 . .	2	86	35.34	135.00	99.66	0.09	1.517	− 0.252	193.00	33.274	0.99645	299870	P. M. Mirror inverted.
July 1 . .	2	83	02.17	135.145	132.98	0.07	1.500	− 0.216	257.35	33.282	0.99627	299810	P. M. Mirror inverted.
1 . .	2	84	02.15	135.145	133.00	0.09	1.500	− 0.228	257.34	33.282	0.09627	299740	P. M. Mirror inverted.
1 . .	2	86	02.14	135.145	133.01	0.06	1.467	− 0.252	257.28	33.311	0.99627	299810	P. M. Mirror inverted.
1 . .	2	86	02.14	135.145	133.00	0.08	1.467	− 0.252	257.28	33.311	0.99627	299940	P. M. Mirror inverted.
2 . .	3	86	99.85	0.400	99.45	0.05	1.450	− 0.252	192.95	33.205	0.99606	299950	P. M. Mirror erect.
2 . .	3	86	66.74	0.400	66.34	0.03	1.450	− 0.252	128.63	33.205	0.99586	299800	P. M. Mirror erect.
2 . .	3	86	50.16	0.400	47.96	0.07	1.467	− 0.252	96.48	33.205	0.99580	299810	P. M. Mirror erect.
2 . .	3	85	33.57	0.400	33.17	0.06	1.450	− 0.240	64.32	33.205	0.99574	299870	P. M. Mirror erect.

In the last two sets of June 13, the micrometer was fixed at 113.41 and 112.14 respectively. The image was bisected by the cross-hair, and kept as nearly as possible in this place, meantime counting the number of seconds required for the image of the revolving mirror to complete 60 oscillations. In other words, instead of measuring the deflection, the speed of rotation was measured. In column 7 for these two sets, the numbers 11 and 6 are the differences between the greatest and the smallest number of seconds observed.

In finding the mean value of V from the table, the sets are all given the same weight. The difference between the result thus obtained and that from any system of weights is small, and may be neglected.

The following table gives the result of different groupings of sets of observations. Necessarily some of the groups include others:

Electric light (1 set)	299850
Set micrometer counting oscillations (2)	299840
Readings taken by Lieutenant Nazro (3)	299830
Readings taken by Mr. Clason (5)	299860
Mirror inverted (8)	299840
Speed of rotation, 192 (7)	299990
Speed of rotation, 128 (1)	299800
Speed of rotation, 96 (1)	299810
Speed of rotation, 64 (1)	299870
Radius, 28.5 feet (54)	299870
Radius, 33.3 feet (46)	299830
Highest temperature, 90° Fahr. (5)	299910
Mean of lowest temperatures, 60° Fahr. (7)	299800
Image, good (46)	299860
Image, fair (39)	299860
Image, poor (15)	299810
Frame, inclined (5)	299960
Greatest value	300070
Least value	299650
Mean value	299852
Average difference from mean	60
Value found for π	3.26
Probable error	± 5

DISCUSSION OF ERRORS.

The value of V depends on three quantities D, n, and φ. These will now be considered in detail.

THE DISTANCE.

The distance between the two mirrors may be in error, either by an erroneous determination of the length of the steel tape used, or by a mistake in the measurement of the distance by the tape.

The first may be caused by an error in the copy of the standard yard, or in the comparison between the standard and the tape. An error in this copy, of .00036 inch, which, for such a copy, would be considered large, would produce an error of only .00001 in the final result. Supposing that the bisections of the divisions are correct to .0005 inch, which is a liberal estimate, the error caused by supposing the error in each yard to be in the same direction would be only .000014; or the total error of the tape, if both errors were in the same direction, would be 000024 of the whole length.

The calculated probable error of the five measurements of the distance was $\pm$ .000015; hence the total error due to D would be at most .00004. The tape has been sent to Professor Rogers, of Cambridge, for comparison, to confirm the result.

THE SPEED OF ROTATION.

This quantity depends on three conditions It is affected, first, by an error in the rate of the standard; second, by an error in the count of the sound beats between the forks; and third, by a false estimate of the moment when the image of the revolving mirror is at rest, at which moment the deflection is measured.

The calculated probable error of the rate is .000016. If this rate should be questioned, the fork can be again rated and a simple correction applied. The fork is carefully kept at the Stevens Institute, Hoboken, and comparisons were made with two other forks, in case it was lost or injured.

In counting the sound beats, experiments were tried to find if the vibrations of the standard were affected by the other fork, but no such effect could be detected. In each case the number of beats was counted correctly to .02, or less than .0001 part, and in the great number of comparisons made this source of error could be neglected.

The error due to an incorrect estimate of the exact time when the images of the revolving mirror came to rest was eliminated by making the measurement sometimes when the speed was slowly increasing, and sometimes when slowly decreasing. Further, this error would form part of the probable error deduced from the results of observations.

We may then conclude that the error, in the measurement of n, was less than .00002.

THE DEFLECTION.

The angle of deflection φ was measured by its tangent, tan. $\varphi = \frac{d}{r}$; d was measured by the steel screw and brass scale, and r by the steel tape.

The value of one turn of the screw was found by comparison with the standard meter for all parts of the screw. This measurement, including the possible error of the copy of the standard meter, I estimate to be correct to .00005 part. The instrument is at the Stevens Institute, where it is to be compared with a millimeter scale made by Professor Rogers, of Cambridge.

The deflection was read to within three or four hundredths of a turn at each observation, and this error appears in the probable error of the result.

The deflection is also affected by the inclination of the plane of rotation to the horizon. This inclination was small, and its secant varies slowly, so that any slight error in this angle would not appreciably affect the result.

The measurement of r is affected in the same way as D, so that we may call the greatest error of this measurement .00004. It would probably be less than this, as the mistakes in the individual measurements would also appear in the probable error of the result.

The measurement of φ was not corrected for temperature. As the corrections would be small they may be applied to the final result. For an increase of 1° F. the correction to be applied to the screw for unit length would be $-.0000066$. The correction for the brass scale would be $+.0000105$, or the whole correction for the micrometer would be $+.000004$. The correction for the steel tape used to measure r would be $+.0000066$. Hence the correction for tan. φ would be $-.000003\ t$. The average temperature of the experiments is 75°.6 F. $75.6 - 62.5 = 13.1$. $-.000003 \times 13.1 = -.00004$

Hence φ should be divided by 1.00004, or the final result should be multiplied by 1.00004 This would correspond to a correction of + 12 kilometers.

The greatest error, excluding the one just mentioned, would probably be less than .00009 in the measurement of φ.

Summing up the various errors, we find, then, that the total constant error, in the most unfavorable case, where the errors are all in the same direction, would be .00015. Adding to this the probable error of the result, .00002, we have for the limiting value of the error of the final result $\pm .00017$. This corresponds to an error of ± 51 kilometers.

The correction for the velocity of light in vacuo is found by multiplying the speed in air by the index of refraction of air, at the temperature of the experiments. The error due to neglecting the barometric height is exceedingly small. This correction, in kilometers, is + 80.

FINAL RESULT.

The mean value of V from the tables is - - -	299852
Correction for temperature - - - - - - - -	+ 12
Velocity of light in air - - - - - - - - -	299864
Correction for vacuo - - - - - - - - - -	80
Velocity of light in vacuo - - - - - - -	299944 ± 51

The final value of the velocity of light from these experiments is then—

299940 kilometers per second,
or 186380 miles per second.

OBJECTIONS CONSIDERED.

MEASUREMENT OF THE DEFLECTION.

The chief objection, namely, that in the method of the revolving mirror the deflection is small, has already been sufficiently answered. The same objection, in another form, is that the image is more or less indistinct. This is answered by a glance at the tables. These show that in each individual observation the average error was only three ten-thousandths of the whole deflection.

UNCERTAINTY OF LAWS OF REFLECTION AND REFRACTION IN MEDIA IN RAPID ROTATION.

What is probably hinted at under the above heading is that there may be a possibility that the rapid rotation of the mirror throws the reflected pencil in the direction of rotation. Granting that this is the case, an inspection of Fig. 14 shows that the deflection will not be affected.

In this figure let $m\ m$ be the position of the mirror when the light first falls on it from the slit at a, and $m_,\ m_,$ the position when the light returns.

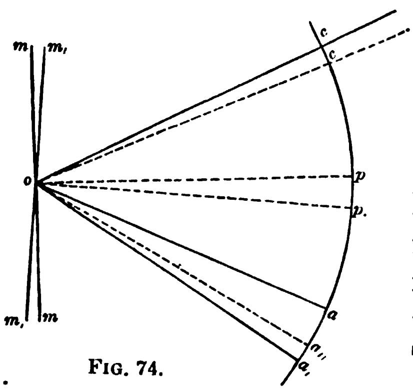

FIG. 74.

From the axis o draw $op\ op_,$ perpendicular to $m\,m$ and to $m_,m_{,,}$ respectively. Then, supposing there is no such effect, the course of the axis of the pencil of light would be $a\,o\,c$ mirror $c\,o\,a_,$. That is, the angle of deflection would be $a\,o\,a_,$, double the angle $p\,o\,p_,$. If now the mirror be supposed to carry the pencil with it, let $o\,c_,$ be the direction of the pencil on leaving the mirror $m\,m$; *i. e.*, the motion of the mirror has changed the direction of the reflected ray through the angle $c\,o\,c_,$. The course would then be $a\,o\,c_,$ mirror $c_,\,o$. From o the reflection would take place in the direction $a_{,,}$, making the angles $c_,\,o\,p_,$ and $p_,\,o\,a_{,,}$ equal. But the angle $c\,o\,c_,$ must be added to $p\,o\,a_{,,}$, in consequence of the motion of the mirror, or the angle of deviation will be $a\,o\,a_{,,} + c\,o\,c_,$; or $a\,o\,a_{,,} + c\,o\,c_, = d$. (1)

By construction—

$$c\,o\,p_, = p_,\,o\,a_, \quad (2)$$
$$c_,\,o\,p_, = p_,\,o\,a_{,,} \quad (3)$$

Subtracting (3) from (2) we have—

$$c\,o\,p_, - c_,\,o\,p_, = p_,\,o\,a_, - p'\,o\,a_{,,} \text{ or}$$
$$c\,o\,c_, = a_,\,o\,a_{,,}$$

Substituting $a_,\,o\,a_{,,}$ for $c\,o\,c_,$ in (1) we have—

$$a\,o\,a_{,,} + a_,\,o\,a_{,,} = a\,o\,a_, = d.$$

Or the deflection has remained unaltered.

RETARDATION CAUSED BY REFLECTION.

Cornu, in answering the objection that there may be an unknown retardation by reflection from the distant mirror, says that if such existed the error it would introduce in his own work would be only $\frac{1}{7000}$ that of Foucault, on account of the great distance used, and on account of there being in his own experiments but one reflection instead of twelve.

In my own experiments the same reasoning shows that if this possible error made a difference of 1 per cent. in Foucault's work (and his result is correct within that amount), then the error would be but .00003 part.

DISTORTION OF THE REVOLVING MIRROR.

It has been suggested that the distortion of the revolving mirror, either by twisting or by the effect of centrifugal force, might cause an error in the deflection.

The only plane in which the deflection might be affected is the plane of rotation. Distortions in a vertical plane would have simply the effect of raising, lowering, or extending the slit.

Again, if the *mean* surface is plane there will be no effect on the deflection, but simply a blurring of the image.

Even if there be a distortion of any kind, there would be no effect on the deflection if the rays returned to the same portion whence they were reflected.

The only case which remains to be considered, then, is that given in Fig. 15, where the light from the slit *a*, falls upon a distorted mirror, and the return light upon a different portion of the same.

The one pencil takes the course $a\ b\ c\ d\ e\ f\ a_{,}$, while the other follows the path $a\ f\ g\ h\ i\ b\ a_{,,}$.

In other words, besides the image coinciding with *a*, there would be two images, one on either side of *a*, and in case there were more than two portions having different inclinations there would be formed as many images to correspond. If the surfaces are not plane, the only effect is to produce a distortion of the image.

As no multiplication of images was observed, and no distortion of the one image, it follows that the distortion of the mirror was too small to be noticed, and that even if it were larger it could not affect the deflection.

The figure represents the distorted mirror at rest, but the reasoning is the same when it is in motion, save that all the images will be deflected in the direction of rotation.

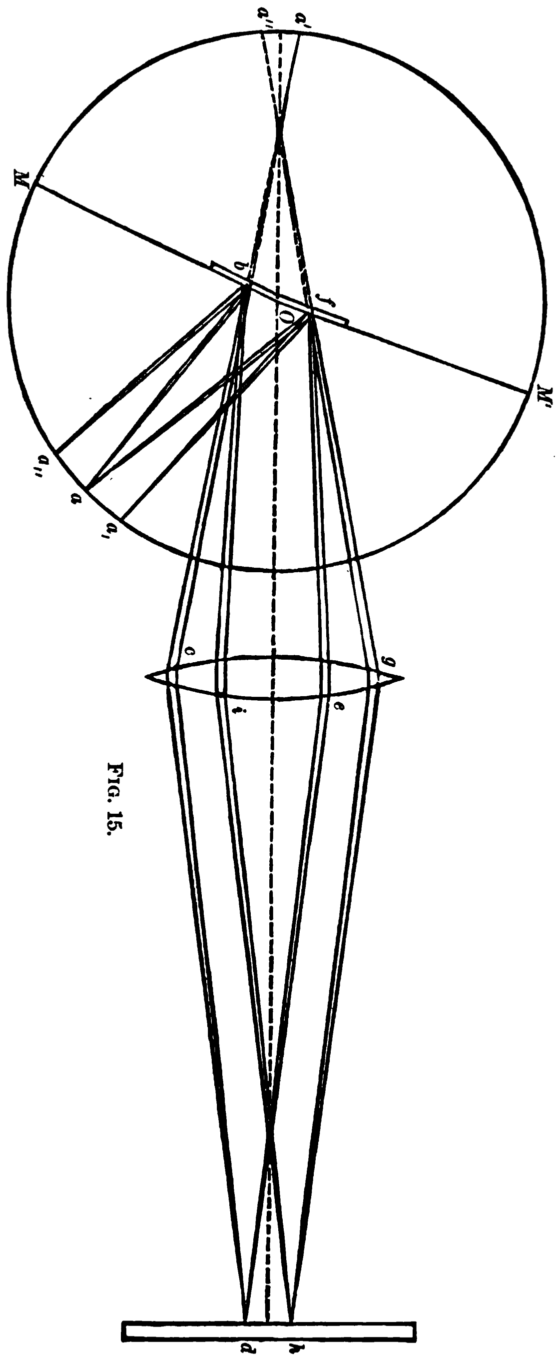

FIG. 15.

IMPERFECTION OF THE LENS.

It has also been suggested that, as the pencil goes through one-half of the lens and returns through the opposite half, if these two halves were not exactly similar,

the return image would not coincide with the slit when the mirror was at rest. This would undoubtedly be true if we consider but one-half of the original pencil. It is evident, however, that the other half would pursue the contrary course, forming another image which falls on the other side of the slit, and that both these images would come into view, and the line midway between them would coincide with the true position. No such effect was observed, and would be very unlikely to occur. If the lens was imperfect, the faults would be all over the surface, and this would produce simply an indistinctness of the image.

Moreover, in the latter part of the observations the mirror was inverted, thus producing a positive rotation, whereas the rotation in the preceding sets was negative. This would correct the error mentioned if it existed, and shows also that no constant errors were introduced by having the rotation constantly in the same direction, the results in both cases being almost exactly the same.

PERIODIC VARIATIONS IN FRICTION.

If the speed of rotation varied in the same manner in each revolution of the mirror, the chances would be that, at the particular time when the reflection took place, the speed would not be the same as the average speed found by the calculation. Such a periodic variation could only be caused by the influence of the frame or the pivots For instance, the frame would be closer to the ring which holds the mirror twice in every revolution than at other times, and it would be more difficult for the mirror to turn here than at a position 90° from this. Or else there might be a certain position, due to want of trueness of shape of the sockets, which would cause a variation of friction at certain parts of the revolution.

To ascertain if there were any such variations, the position of the frame was changed in azimuth in several experiments. The results were unchanged showing that any such variation was too small to affect the result.

CHANGE OF SPEED OF ROTATION.

In the last four sets of observations the speed was lowered from 256 turns to 192, 128, 96, and 64 turns per second. The results with these speeds were the same as with the greater speed within the limits of errors of experiment.

BIAS.

Finally, to test the question if there were any bias in taking these observations, eight sets of observations were taken, in which the readings were made by another, the results being written down without divulging them. Five of these sets are given in the "specimen," pages 133–134.

It remains to notice the remarkable coincidence of the result of these experiments with that obtained by Cornu by the method of the "toothed wheel."

Cornu's result was 300400 kilometers, or as interpreted by Helmert 299990 kilometers. That of these experiments is 299940 kilometers.

POSTSCRIPT.

The comparison of the micrometer with two scales made by Mr. Rogers, of the Harvard Observatory, has been completed The scales were both on the same piece of silver, marked "Scales No. 25, on silver. Half inch at 58° F., too short .000009 inch. Centimeter at 67° F., too short .00008 cm."

It was found that the ratio .3937079 could be obtained almost exactly, if, instead of the centimeter being too short, it were too *long* by .00008 cm. at 67°.

On this supposition the following tables were obtained. They represent the value of one turn of the micrometer in millimeters.

Table 1 is the result from centimeter scale.

Table 2 is the result from half-inch scale.

Table 3 is the result from page 31.

It is seen from the correspondence in these results, that the previous work is correct.

	(1)	(2)	(3)
From 0 to 13	.99563	.99562	.99570
25	.99562	.99564	.99571
38	.99560	.99572	.99576
51	99567	.99578	.99580
64	.99577	.99586	.99585
76	.99582	.99590	.99592
89	.99590	.99598	.99601
102	.99596	.99608	.99605
115	.99606	.99614	.99615
128	.99618	.99622	.99623
140	.99629	.99633	.99630

CATALOGUE

OF

1098 STANDARD CLOCK AND ZODIACAL STARS.

PREPARED UNDER THE DIRECTION OF

SIMON NEWCOMB,
PROFESSOR, U. S. N., SUPERINTENDENT AMERICAN EPHEMERIS.

PREFACE.

The preparation of the following catalogue was commenced at the Naval Observatory for the purpose of obtaining standard positions of reference stars for use in the lunar and planetary theories, especially in the reduction of the older occultations. It originally included only time stars, and stars occultations of which by the moon had been well observed.

In 1877 the unfinished work, along with other material pertaining to the lunar theory, was courteously turned over to the office of the American Ephemeris by Rear-Admiral RODGERS, United States Navy, the Superintendent of the Observatory. It was then found advisable to greatly enlarge the catalogue, so as to include all the standard stars of the American Ephemeris, and all the stars, down to the sixth magnitude, which could be occulted by the moon.

The work of reconstructing and completing the catalogue has been nearly all performed, under the personal direction of the writer, by Master CHAUNCEY THOMAS, United States Navy, to whose care and accuracy is due much of its value.

CONTENTS.

POSITIONS OF STANDARD CLOCK AND ZODIACAL STARS.

§ 1. INTRODUCTION.

In the reduction of all the Washington meridian observations since 1862, and in all the investigations of the motions of the planets by the author up to and including that of Uranus in 1873, the right ascensions depend fundamentally upon Dr. GOULD's standard catalogue. The latter was published by the Coast Survey, and introduced into the American Ephemeris from the years 1865 to 1880.

When the work of reducing the older occultations of stars with modern data was undertaken at the Naval Observatory, it became necessary to have accurate positions of stars for dates much more remote than the time of BRADLEY, because a large number of the occultations selected were observed before 1700. As Dr. GOULD's proper motions depended largely on BESSEL's BRADLEY, which was to be superseded by AUWERS's re-reduction of BRADLEY's observations, and as much other material had become available for the determination of accurate proper motions, it became necessary for the work in hand to redetermine the positions of the fundamental time-stars. So far as the right ascensions are concerned, this was done for the "Maskelyne stars" in 1872 The resulting "*Right Ascensions of the Equatorial Fundamental Stars*" appeared as an appendix to the Washington Observations for 1870.

One result of this investigation was the discovery of a periodic error in the right ascensions of a number of modern catalogues, which seems to have had its origin in some one of POND's adopted çatalogues, and to have disseminated itself among the results of many observatories through the employment of the earlier Greenwich star positions, which depend fundamentally upon those of POND. When, after the practice of Professor AIRY, new fundamental positions depending entirely on recent observations are formed from time to time, the error in question is gradually cut down, and, as a matter of fact, it has disappeared from the recent Greenwich results. But so long as the same fundamental catalogue is used, it will in consequence of erroneous proper motions, tend to increase with the time rather than diminish. By referring to the tables on page 46 of the paper cited, and the formulæ of correction which precede them, it will be seen that in the cases of the Greenwich, Oxford, Paris, and Washington results, the right ascensions about 9^h are very generally too great relative to those about 21^h, the difference ranging from $0.^s10$ in the case of Oxford (Radcliffe, 1845) to $0.^s03$ in the case of the Greenwich 7-year catalogue for 1864.

The necessity of reobserving a large number of the occulted stars, as well as the pressure of other duties, caused the work to be laid aside until 1876, when the means for recommencing it became available. It was the original intention to reduce the

declinations to AUWERS's standard, copious tables for doing which are given in the *Astronomische Nachrichten.* But it was found that in the mean time a very exhaustive discussion of the declinations of the principal fixed stars, and of the systematic corrections necessary to reduce the declinations of the different catalogues to a fundamental system, had been undertaken by Mr. LEWIS BOSS, then of the Northern Boundary Survey, but now Director of the Dudley Observatory, Albany. An examination of Mr. Boss's work led me to believe that in the thoroughness with which the bases of all existing original catalogues of value were examined and discussed, and in the correctness of the general principles on which the work was being executed, it left little to be desired. The only serious deficiency seemed to be the absence of AUWERS's reduction of BRADLEY's declinations from the data employed, an absence which I regretted, but which could not be satisfactorily supplied. Altogether, I judged it best to adopt Mr. Boss's declinations as the standard of reduction, and have to express my indebtedness to Maj. W. J. TWINING, Corps of Engineers, U. S. A., chief astronomer of the American branch of the survey, as well as to Mr. Boss, for the communication of all the tables and data necessary to reduce the declinations of different catalogues to Mr. Boss's system.

The zodiacal stars in the original catalogue, above described, included only those of which occultations had been actually observed up to 1870. On taking charge of the *Americn Ephemeris* the need of a complete revision of the stars which might be occulted by the moon was found to be pressing. It was therefore decided to extend the catalogue so as to include all stars to the sixth magnitude, inclusive, which could be occulted by the moon. Stars below this magnitude were included only when found in BRADLEY's catalogue or when occultations had actually been observed.

In preparing the original list, which was that employed in investigating the motion of the moon before 1750, the provisional declinations of Dr. AUWERS, reduced to Boss's system, were used for the epoch 1755. In the mean time Dr. AUWERS had worked out his definitive results for BRADLEY's declinations, and it was deemed best to incorporate them in the whole catalogue. The original places were therefore modified so as to give the results which would have been reached had AUWERS's declinations been used in the first place.

The catalogue here presented may therefore be considered as including two classes of stars:

(1) All the standard stars of the *American Ephemeris*, omitting for the most part those added for field work.

(2) All stars to the sixth magnitude, inclusive, which can be occulted by the moon, together with stars below the sixth magnitude which had been observed by BRADLEY.

§ 2. FORMATION OF RIGHT ASCENSIONS.

Owing to the constant improvements still in progress in the art of determining star positions, the time has not yet arrived when a fundamental catalogue can be regarded as entirely definitive. It is not, therefore, deemed necessary to present in detail the deduction of the position of each separate star, but it is considered sufficient to give a general statement of the method pursued.

The method of forming the definitive right ascensions of the original catalogue was to compare the catalogue places with computed provisional places, and, assuming the corrections thus obtained to be of the form $a + b\,T$, to find the values of a and b by least squares. These quantities were the corrections to be applied to the provisional right ascensions and proper motions.

As a general check upon the accuracy of all the work, two fundamental epochs were adopted, namely, 1755.0, the epoch of BESSEL's and of AUWERS's reductions of BRADLEY, and 1850.0, that most generally adopted as the zero epoch for the theoretical astronomy of the present time. Approximate positions for these two epochs (supposed to be correct to $0^s.3$ of time in R. A, and to $0'.1$ in declination) were obtained for these epochs, generally from BESSEL's *Fundamenta* and the *British Association Catalogue.* The precessions and secular variations of the annual motion for each epoch were then independently computed. In these computations STRUVE's constant of precession and HILL's formulæ, as found in the *Star Tables of the American Ephemeris* and in my paper of 1872, already cited, were made use of. It will be remarked that the secular variations thus computed are not those of the precession simply, but of the annual variation. The difference, however, is not great, except in cases of stars having considerable proper motion or high declination. The annual precessions were computed to $0^s.0001$, and the variations in 100 years to the same order of units

The provisional right ascensions of the stars were then carried forward from AUWERS's *Bradley*, neglecting proper motion entirely, and assuming the precession to vary uniformly between 1755 and 1850. The computed values of the secular variation were therefore substantially unused in obtaining the provisional places, except as a check against serious error. Practically the adopted value of this variation was $\frac{20}{19}$ of the difference between the precession for 1755 and that for 1850. The residual corrections given by the several catalogues thus represented proper motions from 1755. To guard against an accumulation of small errors, the computations of the provisional places were carried to $.^s001$.

In the case of stars of the *American Ephemeris,* a course different in some respects was pursued.

The annual variations and secular variations for 1860 being given in the *Star Tables of the American Ephemeris,* it was not considered necessary to compute them for 1850. The secular variations were, however, computed for 1755 to five places of decimals, the difference between this and the corresponding quantity for 1860 giving the term depending on the third power of the time. The right ascensions were then carried back to the epochs of the catalogues, supposing the annual variation and secular variation of the *Star Tables* to be exact for 1860, and including the term depending on the third power of the time. The provisional proper motions were therefore included.

The computed places thus obtained for each class of stars were then compared with those given in the following catalogues.

1. *Bradley,* 1755.—The right ascensions were those of Dr. AUWERS's, as communicated in manuscript. In the case of a few stars, however, BESSEL's places, as given in the *Fundamenta Astronomiæ,* had to be used.

2. *Piazzi*, 1800.—*Precipuarum Stellarum Inerrantium Positiones Mediæ. Panormi*, 1814. This catalogue was used in the case of stars not observed by BRADLEY.

3. *Struve*, 1830.—Catalogue in the *Positiones Mediæ*.

4. *Argelander*, 1830.—DLX *Stellarum Fixarum Positiones Mediæ, ineunte anno* 1830. *Helsingfors*, 1835.

5. *Pond*, 1830.—Catalogue of 1112 stars. London, 1833.

6. *Airy*, 1830.—*First Cambridge catalogue of* 726 *stars* in the *Memoirs of the Royal Astronomical Society*, vol. xi.

7. *Johnson*, 1830.—St. Helena catalogue of 606 stars. London, 1835. (Used only for two or three southern stars.)

8. *Gilliss*, 1840.—Catalogue in *Observations made at the* [old] *Naval Observatory*. Washington, 1846.

9. *Armagh*, 1840.—Robinson's catalogue.

10. *Airy*, 1840.
11. *Airy*, 1845. } The Greenwich twelve-year catalogue.

12. *Pulkowa*, 1845.—Catalogue in vol. I of the Pulkowa observations, derived from observations with the transit instrument.

13. *Airy*, 1850.—Greenwich six-year catalogue for 1850.

14. *Pulkowa*, 1855.—Catalogue from observations with the meridian circle, communicated in manuscript by Director STRUVE.

15. *Airy*, 1860.—Greenwich seven-year catalogue.

16. *Yarnall*, 1860.—Washington catalogue. Appendix to Washington observations for 1871.

17. *Airy*, 1864.—Second Greenwich seven-year catalogue.

18. *Engelmann*, 1866.—*Resultate aus Beobachtungen am Meridiankreise der Sternwarte zu Leipzig, von Dr. Rudolph Engelmann.*

19. *Greenwich*, 1870.—Mean result from the Greenwich observations from 1868 to 1876, inclusive.

20. *Washington*, 1870.—Mean results from all observations with the Washington Transit circle from 1866 to 1873.

To the positions of the separate catalogues were applied the systematic corrections given on pages 43 to 47 of the paper on the right ascensions of the equatorial fundamental stars.

The weights assigned to the several catalogues, as dependent on the number of observations, were founded upon a consideration of the probable systematic and accidental errors of each catalogue. While such considerations do not constitute a refined discussion, I consider that the final results will be much nearer to those which would be given by the most refined discussion than to those given by the usual mode of combining catalogue results. I also consider that the former difference will be much less than the probable error of the best results. The following is the table made use of, the argument at the top being the number of observations.

Tables of adopted weights in right ascension.

Number of observation.	1	2	3	4	5	7	10	15	20	25	30	40	50	60	80	100
Bessel's Bradley	$\frac{1}{15}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{8}$	$\frac{1}{7}$	$\frac{1}{7}$	$\frac{1}{4}$	$\frac{1}{3}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$
Auwers's Bradley	$\frac{1}{10}$	$\frac{1}{5}$	$\frac{1}{4}$	$\frac{1}{3}$	$\frac{1}{2}$	$\frac{1}{2}$	1	1	1	2	2	3	3	3	4	4
Piazzi	$\frac{1}{4}$	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	1	1	1	1	1	1	1	1	1	1
Struve, 1825	$\frac{1}{2}$	1	1	1	2	2	3	3	4	4	5	5	6	6	7	8
Argelander, 1830	"	"	"	"	"	"	"	"	"	"	"	"	"	"	"	"
Pond	$\frac{1}{2}$	1	1	1	1	1	2	2	2	2	3	3	4	4	5	6
Johnson	"	"	"	"	"	"	"	"	"	"	"	"	"	"	"	"
Airy, Cambridge, 1830	"	"	"	"	"	"	"	"	"	"	"	"	"	"	"	"
Gilliss, 1840	"	"	"	"	"	"	"	"	"	"	"	"	"	"	"	"
Airy, Greenwich, 1840	"	"	"	"	"	"	"	"	"	"	"	"	"	"	"	"
Armagh, 1840	$\frac{1}{8}$	$\frac{1}{4}$	$\frac{1}{3}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	1	1	2	2	3	3	4	4	5	6
Pulkowa, 1845	1	2	2	3	4	5	7	7	8	10	15	20	20	25	25	30
Radcliffe, 1845	$\frac{1}{2}$	1	1	1	1	1	2	2	2	2	3	3	4	4	5	6
Airy, Greenwich, 1845	"	"	"	"	"	"	"	"	"	"	"	"	"	"	"	"
Airy, Greenwich, 1850	"	"	"	"	"	"	"	"	"	"	"	"	"	"	"	"
Pulkowa, 1850	1	2	2	3	4	5	7	7	8	10	15	15	20	25	25	30
Airy, Greenwich, 1860	"	"	"	"	"	"	"	"	"	"	"	"	"	"	"	"
Yarnall, Washington, 1860	"	"	"	"	"	"	"	"	"	"	"	"	"	"	"	"
Airy, Greenwich, 1864	"	"	"	"	"	"	"	"	"	"	"	"	"	"	"	"
Engelmann, Leipzig, 1866	"	"	"	"	"	"	"	"	"	"	"	"	"	"	"	"
Airy, Greenwich, 1870	"	"	"	"	"	"	"	"	"	"	"	"	"	"	"	"
Washington, 1870	"	"	"	"	"	"	"	"	"	"	"	"	"	"	"	"

Assuming the residuals to be represented by an expression of the form $a + b$T, T being the fraction of a century after 1850.0, the equations of condition thus obtained were solved by least squares. The definitive correction to the provisional right ascension for 1850 was then a, and to that for 1755 was $a - 0.95\,b$. These corrections being applied to the provisional places, corrected places for the two fundamental epochs would then be obtained.

The process thus described was not rigorous with respect to the third place of decimals in the seconds owing to three causes.

(1) The limitation of the adopted annual precession to the fourth decimal.

(2) The neglect of the secular variation of the proper motion, which would introduce a small term varying with the time.

(3) The assumption that the secular variation of the centennial motion was constant.

The errors thus introduced were entirely unimportant so far as the immediate purpose was concerned, because they were smaller than the necessary uncertainty of the results; but it was considered desirable that the relation between the final positions in the catalogue, and the precessions and proper motions, should correspond accurately to a uniform theory. The results were therefore checked and adjusted by the following process.

The centennial variation for 1850 was obtained in the first place by correcting that value of the precession or centennial variation for 1850, which was used in computing the provisional places, by the quantity b, derived from the equations of condition. The

value thus employed for correction was not generally the same as the definitive precession for 1850, because the latter was afterward computed to one more place of decimals. But the corrected result was considered as the definitive variation for the epoch 1850.

The centennial variation for 1755 was derived from that for 1850 by subtracting from the latter the quantity

$$\tfrac{1}{2}(s_1 + s_2) \times (1 - \tfrac{1}{20})$$

s_1 and s_2 being the secular variations for the respective fundamental epochs.

Having thus obtained the centennial variations, which we may call v_1 and v_2, for the two fundamental epochs, the change of right ascension between those two epochs was independently computed by the formula

$$\triangle \text{R.A.} = \tfrac{1}{2}(v_1 + v_2)(1 - \tfrac{1}{20}) - 0.075(s_2 - s_1)$$

Had the data and method of interpolation of the provisional places of the stars been perfectly consistent with the definitive quantities, the right ascension for 1755, obtained by subtracting △ R.A. from the right ascension for 1850, would have agreed exactly with that obtained by correcting the provisional place. But, owing to the want of a rigorous reduction already pointed out, small discordancies were to be expected. In a large majority of cases the discordance was less than $0^s.01$ and rarely or never amounted to $0^s.02$, unless from some error of computation to be rectified. It was then judged best to render the right ascensions for 1755 and the centennial variations consistent with each other by an adjustment. In general one-third the discordance was applied to the place for 1755 and two-thirds to the centennial variation. But this proportion was subject to change in exceptional cases. The general result aimed at was that the numbers should be as nearly as possible the same as if a rigorous theory had been adopted at the outset.

The above descriptions apply only to the original catalogue. In the extension of it made by Master CHAUNCEY THOMAS, U. S. N., it was considered better to use the more elegant process of reducing each catalogue place to 1850 by precession alone and then to obtain the position and proper motion for this epoch by the usual method.

In the original formation of a catalogue, assuming the proper motions to be entirely unknown, this is the preferable process. But in future it will probably be found more convenient, at least in the case of fundamental stars, to reduce the provisional places to the epoch of each catalogue and work only with the residual differences between the two positions. This is in fact using the general astronomical method of correcting elements.

Ulterior details respecting the construction of the catalogue, will be given in connection with it.

§3. FORMATION OF THE DECLINATIONS.

As already stated, the normal catalogue to which all the declinations are reduced is that of Mr. LEWIS BOSS. This catalogue has since been published as Appendix H to the American Report of the *Northern Boundary Commission.**

* Reports upon the Survey of the Boundary between the Territory of the United States and the Possessions of Great Britain from the Lake of the Woods to the Summit of the Rocky Mountains, authorized by an act of Congress approved March 19, 1872. Archibald Campbell, esq., Commissioner; Capt. W. J. Twining, Corps of Engineers, brevet major U. S. A., Chief Astronomer. Washington: Government Printing Office. 1878.

The most important modification which had to be made in using Mr. Boss's tables arose from the substitution of AUWERS's reduction of BRADLEY's observations for that of BESSEL. Mr. Boss's systematic corrections were applicable only to BESSEL's positions. It was therefore necessary to find the correction to be applied to AUWERS's declinations in order to reduce them to the same fundamental system. Boss's systematic correction to each of BRADLEY's zodiacal stars was taken from the table, which has since been published, page 496 [90] of Mr. Boss's paper, and the result compared with Dr. AUWERS's definitive reduction.

It would have been much better had all the zodiacal stars of Mr. Boss's catalogue been definitely reduced to 1755 and compared with AUWERS's corrections. This course was not, however, at the time practicable.

The following table shows the mean result for each hour of right ascension in the sense of Boss's correction to AUWERS's definitive declination. The argument 0° gives the mean result for all the stars between $23^h\ 30^m$ and $0^h\ 30^m$ of right ascension; the argument 15° the mean result from $0^h\ 30^m$ to $1^h\ 30^m$, etc.:

Right ascension.	Boss-Auwers.	Number of stars.	Right ascension.	Boss-Auwers.	Number of stars.
°	″		°	″	
0	+0.98	30	180	+2.58	17
15	1.70	37	195	2.48	25
30	1.65	28	210	2.99	19
45	1.61	35	225	2.29	26
60	1.45	67	240	2.10	31
75	0.89	34	255	1.64	26
90	1.57	45	270	1.47	26
105	0.74	39	285	1.06	30
120	1.14	44	300	1.60	25
135	1.71	37	315	0.65	36
150	2.14	33	330	0.71	47
165	+2.66	31	345	+0.66	37

It will be remarked that since the stars to which this table refers are on the average within 3° or 4° of the ecliptic the corrections are functions both of the right ascension and declination. Owing, however, to this arrangement, it is impossible to separate quantities depending on the right ascension from those depending on the declination. The best practical course, therefore, seems to be to leave in abeyance the general form of correction and to tabulate it as a function of the right ascension alone. Developing the residuals in the usual way the result is—

$$\text{Boss—Auwers} = +1''.60 - 0''.68 \cos \alpha + 0''.32 \cos 2\alpha - 0.10 \sin \alpha + 0.38 \sin 2\alpha$$

In cases of this sort the terms in 2α are generally to be regarded as accidental. It was therefore deemed best to omit them and to apply only the expression

$$+1''.60 - 0''.68 \cos \alpha - 0''.10 \sin \alpha$$

In applying this correction to AUWERS's results from BRADLEY's observations I do not wish to be considered as indorsing its reality, but have used it only in order that all the declinations might be reduced to the same system. I believe that considerable

weight would have been added to Boss's results had he been able to use AUWERS's *Bradley* as one of the normal catalogues. It may be expected that the additional data accumulated during the next fifteen or twenty years will lead to a more certain result.

Catalogues used for Declinations.—These were, in the main, the same as in the case of the right ascensions, with the following additions:

(1) *Cambridge*, 1840.—Mean results from the Cambridge observatories from 1836 to 1844, as found in the several annual volumes of observations.

(2) *Paris*, 1860.—Mean results from the Paris observations of 306 "étoiles fondamentales" made with the Gambey mural circle, 1854–'63, as found in the several annual volumes of observations.

(3) *Paris*, 1865.—Similar results from the observations with the new meridian instrument, 1863–'67.

(4) *Melbourne*, 1870.—First Melbourne General Catalogue of 1227 stars for the epoch 1870. Melbourne, 1874.

The several tables of systematic corrections which have been applied, and the weights, as dependent on the number of observations, will be found in Mr. Boss's work, pages 560–567.

The deduction of the definitive declinations has been carried out in the same way as in the case of the right ascensions. The most important modifications were these:

(1) An approximate proper motion was used in interpolating the provisional places compared with the several catalogues.

(2) In the same interpolation account was taken of the change in the secular variation of the annual motion; in other words, the term multiplied by the cube of the time was retained.

(3) All the results were computed to $0''.01$, with the definitive values of the annual motions.

In consequence of these changes, the average discrepancy between the places for 1755, as obtained by applying the computed correction, $a - 0.95\,b$, to the provisional place, and those obtained by direct computation from the definitive centennial motions is less than $0''.02$.

§4. POSITIONS OF THE NINE PRINCIPAL STARS OF THE PLEIADES.

The mode of treating the stars of this group was in some points exceptional. A question which naturally presents itself in investigating their positions is that of their relative proper motion. We might proceed on either of two hypotheses; first, that the place of each star is to be determined independently on the supposition that its proper motion is independent of that of the others; second, that they all have a common and equal proper motion. If the differences of the proper motions decidedly exceed the probable errors of the separate determinations, we should choose the first hypothesis; otherwise the second. On either hypothesis our first step must be to determine each star independently, and this was done in the same way as with all the other stars. It was thus found that there was no conclusive evidence of change from the meridian observations alone, and that the common proper motions $+0^s.088$ in R. A. and $-5''.87$ in declination for the entire group, would satisfy all these observations within their possible limits of error.

As a still further test of the invariableness of their relative positions, and a means of further correcting these positions, the triangulations of BESSEL and of WOLF were called into requisition. The former work is found in BESSEL'S *Astronomische Untersuchungen*, vol. 1, pp. 209–238, the latter in the *Comptes Rendus* of the French Academy for 1875. It has since appeared in *Annales de l'Observatoire de Paris, Memoires*, XIV.

The date of BESSEL'S triangulation is 1840, that of WOLF'S 1874, so that the elapsed time exceeds one-third of a century. Both of these sets of positions were reduced to 1850 with the common proper motion already given, and the results compared with the meridian observations. There was no marked resemblance between the signs of the differences WOLF—BESSEL and the signs of the relative proper motions indicated by the meridian observations; so that an additional proof of the unreality of these proper motions was obtained. I therefore conclude that although a certain amount of relative proper motion must exist in this group, yet the apparent motions, as observed, are as much due to errors of observation as to the actually existing motions, and when the latter shall finally be discovered they will, on the average, be found as near to zero as to the values indicated by all the observations yet made. Consequently, the most probable values of these relative proper motions must be regarded as zero.

It is evident that from the data described we shall have two classes of results for the position of each individual star of the group. The one is the result of the meridian observations of that particular star; the other the result of the triangulations between that and all the other stars, combined with the meridian observations of those other stars.

Since the triangulation can give only relative positions, the mean of the entire group should remain as determined by the meridian observations. We must therefore apply to the results of the triangulations such constant corrections that this result shall be attained. These corrections are:

In R. A.,	Bessel,	$-\ 0^s.03$
	Wolf,	$-\ 0.04$
In Dec.,	Bessel,	$+\ 0''.69$
	Wolf,	$+\ 0.04$

In combining the several results, the relative weights assigned were as follows:

In R. A.		In Dec.	
Mer. obs.,	Wt. = 1	Mer. obs.,	Wt. = 1
Bessel, "	Wt. = 2	Bessel, "	Wt. = 3
Wolf, "	Wt. = 1	Wolf, "	Wt. = 2

The several steps of the process thus described are shown in the following table. The small figures after the individual proper motions show the relative weights which have been assigned to them. The mean common proper motion of the group obtained by their combination is—

$$\text{In R. A., } \mu = +\ 0^s.088$$
$$\text{In Dec., } \mu = -\ 5''.87$$

Right ascensions of the Pleiades for 1850.0.

Name.	From meridian observations.		Seconds of right ascension from differential measures.		Concluded Right ascension, 1850.0.
	Right ascension, 1850.	Proper motion and weight.	Bessel. $-0^s.030$	Wolf. $-0^s.040$	
	h. m. s.	s.	s.	s.	h. m. s.
16 *g*, Celæno . .	3 35 53.716	$+0.212_1$	53.723	53.73	3 35 53.723
17 *b*, Electra . .	3 35 58.600	$+0.146_1$	58.595	58.60	3 35 58.598
18, *m*	3 36 13.299	$+0.020_1$	13.292	13.38	3 36 13.315
19 *e*, Tayzeta . .	3 36 17.226	-0.002_1	17.271	17.28	3 36 17.262
20 *c*, Maia . . .	3 36 54.549	$+0.116_3$	54.582	54.54	3 36 54.563
23 *d*, Merope . .	3 37 25.867	$+0.006_1$	25.893	25.90	3 37 25.888
25 *η*, Alcyone . .	3 38 34.588	$+0.136_4$	34.575	34.55	3 38 34.572
27 *ρ*, Atlas . . .	3 40 15.077	$+0.074_2$	15.053	15.05	3 40 15.058
28 *h*, Pleione . .	3 40 16.250	-0.068_1	16.216	16.21	3 40 16.223

Declinations of the Pleiades for 1850.

Name.	From meridian observations.		Seconds of declination from differential measures.		Concluded Declination, 1850.
	Declination, 1850.	Proper motion and weight.	Bessel. $+0''.69$	Wolf. $+0''.04$	
	° ′ ″	″	″	″	° ′ ″
16 *g*, Celæno . .	23 48 47.78	-6.53_1	47.87	47.83	23 48 47.84
17 *b*, Electra . .	23 38 14.76	-5.13_1	14.45	14.65	23 38 14.57
18, *m*	24 21 51.43	-6.31_1	50.40	50.86	24 21 50.73
19 *e*, Tayzeta . .	23 59 31.89	-6.26_1	32.12	32.18	23 59 32.10
20 *c*, Maia . . .	23 53 40.93	-4.82_1	40.68	40.71	23 53 40.73
23 *d*, Merope . .	23 28 36.07	-5.73_1	36.42	36.40	23 28 36.35
25 *η*, Alarone . .	23 38 13.13	-5.58_6	13.28	13.17	23 38 13.22
27 *ρ*, Atlas . . .	23 35 25.26	-5.85_1	25.58	25.44	23 35 25.48
28 *h*, Pleione . .	23 40 25.81	-7.96_1	25.76	25.64	23 40 25.73

§ 5. DECLINATIONS OF SIRIUS AND PROCYON.

SIRIUS.

In his researches on the variable proper motion of Sirius (*Publication VII der Astronomischen Gesellschaft, Leipzig*, 1868), AUWERS has found a correction, r, to its declination, defined as follows: Let r_1 and r_2 be the respective corrections to be applied to the declinations of Sirius in the *Tabulæ Regiomontanæ*, in order that this declination may be correct relatively to those of β Orionis and α Hydræ, respectively. Then AUWERS puts

$$r = \tfrac{1}{2}(r_1 + r_2)$$

It follows that if the corrections to the declinations of β Orionis and α Hydræ in the *Tabulæ Regiomontanæ* are respectively Δ_1 and Δ_2, the correction to the declination of Sirius in the same tables will be

$$\tfrac{1}{2}(\Delta_1 + \Delta_2) + r$$

By comparing the positions of β Orionis and α Hydræ in Boss's catalogue with those of the *Tabulæ Regiomontanæ*, we find:

$$\begin{aligned}
\Delta_1 &= +\overset{''}{1.48} + \overset{''}{2.44}\,\mathrm{T} + \overset{''}{0.01} \times \tfrac{1}{2}\mathrm{T}^2 - \overset{''}{0.22} \times \tfrac{1}{6}\mathrm{T}^3 \\
\Delta_2 &= +1.29 + 0.89 \quad + 0.35 \qquad\quad + 0.76 \\
\tfrac{1}{2}(\Delta_1 + \Delta_2) &= +1.38 + 1.66 \quad + 0.18 \qquad\quad + 0.27
\end{aligned}$$

T being counted from 1850. AUWERS finds for the value of r:

$$r = +0''.84 + 1''.47\,\mathrm{T} + \mathrm{P}'$$

P′ representing the purely periodic term arising from the orbital revolution of the companion of Sirius. The total correction to the place of Sirius *in the Tabulæ Regiomontanæ* would then be

$$+2''.23 + 3''.13\,\mathrm{T} + 0''.18 \times \tfrac{1}{2}\mathrm{T}^2 + 0''.27 \times \tfrac{1}{6}\mathrm{T}^3 + \mathrm{P}'$$

But by comparing the secular variation of the centennial motion, $-37''44 + 0''.13\,\mathrm{T}$, with that of BESSEL, $-38''.0$, it seems that the actual correction must be of the form—

$$a + b\mathrm{T} + 0''.56 \times \tfrac{1}{2}\mathrm{T}^2 + 0''.13 \times \tfrac{1}{6}\mathrm{T}^3 + \mathrm{P}'$$

The difference in the coefficients of T^2 will produce a difference of only $0''.17$ in the declinations for 1755; we may therefore omit any adjustment on account of it, and put for the total correction to the declination of Sirius—

$$+2''.23 + 3''.13\,\mathrm{T} + 0.56 \times \tfrac{1}{2}\mathrm{T}^2 + 0''.13 \times \tfrac{1}{6}\mathrm{T}^3 + \mathrm{P}'$$

PROCYON.

The declination of this star is determined on the same principle with that of Sirius, from the investigation of AUWERS in his paper.

The comparison is, however, made, not with the *Tabul Regiomontanæ*, but with the *Tabulæ Reductionum* of WOLFERS. The stars of comparison are α Ceti, α Orionis, α Serpentis, γ, α, and β Aquilæ, and α Aquarii, but the three stars of Aquila receive only the weight of two. In the value of Δ we may omit writing the terms depending on T^2 and T^3, since they are not used in obtaining the final result. By comparing

the corrections of Wolfers to the *Tabulæ Regiomontanæ* with the declinations of the present paper we find the following values of Δ:

		″	″
α Ceti - - -	$\Delta^1 =$	$+0.04$	$+1.88\,T$
α Orionis - - -	$\Delta_2 =$	$+0.18$	$+0.76$
α Serpentis - -	$\Delta_3 =$	-0.50	-2.66
γ Aquilæ - - -	$\Delta_4 =$	$-.0.35$	-1.45
α Aquilæ - - -	$\Delta_5 =$	-0.40	-1.35
β Aquilæ - - -	$\Delta_6 =$	-0.12	-1.02
α Aquarii - - -	$\Delta_7 =$	-0.24	-1.06
Mean by weights -	$\Delta =$	-0.18	$-0.605\,T$
Auwers's r - - -		$+0.39$	$+0.931\,T + P'$
Total correction - -		$+0.204$	$+0.326\,T + P'$

This correction, omitting P′, being applied to the place of the *Tabulæ Reductionum*, gives the declination in the table.

§ 6. CIRCUMPOLAR STARS.

In the case of stars within 30° of the pole an accurate reduction between epochs a century apart cannot be effected without other data than those given for the ecliptic and time stars. It was judged that the convenience of astronomers using the catalogue would be subserved by presenting data for the stars in the same general form as for others, with the addition of such intermediate epochs that the reductions could be effected without the employment of higher powers of the time. Hence stars between 10° and 30° from the pole have data given for each half century, or to speak more exactly, for the epochs 1755, 1800, 1850, and 1900. In the case of stars yet nearer the pole the epochs 1755, 1825, and 1875 are added.

The declinations of the circumpolar stars are all taken from Boss's catalogue for the epoch 1875.

The right ascensions have not been independently investigated, but are taken from the second edition of Dr. Gould's catalogue, published by the United States Coast Survey, and based upon Dr. Gould's extended investigations found in Volume VI of the Astronomical Journal. Although these right ascensions may be at the present time susceptible of correction, it was judged best to adhere to them for the following reasons:

1st. They had been retained in the American Ephemeris for 1881, in which new declinations had been introduced, and it was judged best to make changes only at few epochs. They had also been so extensively used by the Coast Survey and other authorities as to form a standard of reference which it was desirable not to change except when a great and permanent improvement was possible.

2d. Their definitive amelioration is not practicable until Dr. Auwers's reductions of Bessel's observations are available.

3d. Each astronomer can readily apply for himself such corrections as may appear necessary.

It will probably be found that the easiest way of making these corrections will

be to reduce each star to the epoch of the catalogue of observation and work with the correction thus indicated for that particular epoch. For all except two or three of the closest polar stars the correction of each co-ordinate may be assumed to increase uniformly with the time.

To form a set of data in which the positions, centennial variations, and secular variations for each epoch should be perfectly consistent throughout, several troublesome modifications were found necessary. The coefficients of reduction given by GOULD and BOSS, respectively, could not be used unchanged, because those for each co-ordinate depended upon the value of the other co-ordinate, and must therefore be changed with it. It was therefore necessary to compute anew for each epoch the constants corresponding to it and to combine these results in such a way as to secure homogeneity and consistency.

In the case of the close polar stars both the positions and the proper motions were reduced from 1855 to the several epochs by the rigorous trigonometrical formulæ, the constants being those founded on Struve's precession. These reductions were, in the first place, made with Dr. GOULD's proper motion in declination, but it was easy to correct them, so that they should give Boss's proper motion for the epoch 1875.

The positions and proper motions for this epoch include all the data necessary for computing the precessions and secular variations for the different epochs. The centennial variations were then found by applying the proper motion to the precession. The original reductions were next checked by computing the change of position between each pair of consecutive epochs from the centennial variations, secular variations, etc., and comparing it with the actual difference given by the trigonometrical reduction.

In the case of stars more than 15° from the pole the trigonometrical reduction was not necessary. Generally Dr. GOULD's coefficients gave results which need little correction, and this little, when necessary, was derived from the computed elements of motion for the different epochs.

§ 7. EXPLANATION OF THE CATALOGUE.

The catalogue is arranged so that all the data pertaining to the right ascension shall be on the left-hand pages, and those pertaining to the declinations on the right-hand pages.

In the case of the stars observed by BRADLEY, the positions and other data are given for the two fundamental Besselian epochs 1755.0 and 1850.0. In some cases stars not observed by BRADLEY have been given for both of these epochs. In the case of fundamental time stars the positions are also given for 1900. The precession and secular variation for each epoch are independently computed, so that their general agreement will serve as a check upon their accuracy.

On the left-hand page the fourth column gives, opposite the epoch 1755, the number of observations made by BRADLEY in right ascension. Opposite 1850 is given the number of observations made at Greenwich, Pulkowa, and Washington since 1840, which have been used in preparing the catalogue.

Observations at other observatories have been omitted in the enumeration,

although employed in obtaining the final result. In some cases, as, for instance, those of the Pulkowa fundamental time stars, no precise number of observations could be assigned. The object of this column is rather to give a general idea of the weight of the result than a precise enumeration of the observations.

Column *Right Ascension* gives the right ascension of each star for the several epochs, as already explained. The epoch 1850 has been taken as a fundamental one, and, for the most part, the positions for other epochs have been derived from those for 1850 by the centennial variation, etc., deduced from observations.

The equinox to which all the stars are reduced is that of my paper of 1872 on the *Right Ascensions of the Equatorial Fundamental Stars.* (Washington Observations for 1870, Appendix II.) The results obtained by Dr. AUWERS for BRADLEY's equinox, and the recent Greenwich observations, render it probable that the adopted equinox is nearly correct for 1850, but that the centennial variations require a general correction of perhaps — $0^s.05$. Further researches are, however, necessary before a definitive result for the motion of the equinox can be derived.

The right ascensions of the 32 Maskelyne stars in the investigation of 1872 are transferred without alterations to the present catalogue

The centennial variation derived in the first place for the epoch 1850 has usually been regarded as a fundamental one, and that for other epochs has been derived from it by the secular variations given in the following column.

The precessions are computed strictly from STRUVE's constant, using the formulæ given in the star tables of the American Ephemeris and in part reprinted on p. 172 of the present paper. But, as a general rule, the precessions and secular variations were derived before the definitive positions of the stars were worked out, and did not, therefore, in all cases accurately correspond to the finally concluded positions. In general, however, where any important discrepancy would thus be produced, the precessions and secular variations have been recomputed with the definitive data.

The proper motions are generally obtained by subtracting the precessions from the centennial variations. The differences among the proper motions thus found arise partly from incongruity of the data, imperfections of calculation, etc., but mostly from the change in the direction of the meridian produced by precession.

With a view of detecting any serious error in the proper motions the secular variation of the proper motion has been independently computed by the formula given in the present paper in the case of those stars of the American Ephemeris which have a considerable proper motion. The result of this computation is given in the last column.

In the case of circumpolar stars the above method of obtaining the centennial variations for the different epochs would not always have been reliable. In this case, therefore, the secular variation of the proper motion was carefully computed for several epochs and the proper motions for past and future epochs obtained by applying the changes thus indicated to the proper motions for the fundamental epoch. The precessions being also computed for the different epochs, the centennial variations were obtained by applying the proper motions to them.

On the right-hand pages the third column gives, for the epoch 1850, the magnitudes of the stars taken in the order of preference from the following authorities.

1. GOULD, *Uranometria Argentina.*

2. HEIS, *Atlas Coelestis Novus*, Köln, 1872.

3. ARGELANDER, *Bonner Sternverzeichniss*, commonly called the *Durchmusterung; Astronomische Beobachtungen auf der Sternwarte zu Bonn*, vols. iii, iv

Opposite epoch 1755 are given the magnitudes of BESSEL'S *Fundamenta.*

In both cases the fractions of a magnitude are expressed decimally.

The general method of arranging the data for the declinations is substantially the same as for the right ascensions, and therefore needs no additional explanation.

§ 8. FORMULÆ FOR REDUCING THE CATALOGUE PLACES TO OTHER EPOCHS.

It is supposed that the data given in connection with the place of each star will suffice for its reduction to any epoch between 1750 and 1900, by TAYLOR'S Theorem. To effect this we take the catalogue epoch nearest that to which the star is to be reduced, and put—

T, the interval, in units of a century.

α_0, the star position for the catalogue epoch.

c_0, the centennial variation for the same epoch.

s, the secular variation for the same epoch.

s', the derivative of s at this epoch, the unit of time being a century.

s'', the second derivative of s, etc.

Then:

$$\alpha = \alpha_0 + \mathrm{T}\, c_0 + \tfrac{1}{2}\mathrm{T}^2 s_0 + \tfrac{1}{6}\mathrm{T}^3 s'_0 + \tfrac{1}{24}\mathrm{T}^4 s''_0 + \text{etc.} \qquad (1)$$

The values of c_0 and s_0 are always given in the catalogue. Those of s'_0, s''_0, etc., will not always be required, but when required, are readily deduced from the values of s for different catalogue epochs.

In the most general case the values of s'_0, s''_0, etc., may be formed from the successive differences of s by the usual formulæ, namely, these differences being arranged according to the following usual scheme:

$$\begin{array}{llll}
s_{-2} & & & \\
 & \Delta'_{-\frac{3}{2}} & & \\
s_{-1} & & \Delta''_{-1} & \\
 & \Delta'_{-\frac{1}{2}} & & \Delta'''_{-\frac{1}{2}} \\
s_0 & & \Delta''_0 & \\
 & \Delta'_{\frac{1}{2}} & & \Delta'''_{\frac{1}{2}} \\
s_1 & & \Delta''_1 & \\
 & \Delta'_{\frac{3}{2}} & & \\
s_2 & & &
\end{array}$$

where

$$\begin{array}{ll}
\Delta'_{-\frac{3}{2}} = s_{-1} - s_{-2} & \Delta''_{-1} = \Delta'_{-\frac{1}{2}} - \Delta'_{-\frac{3}{2}} \\
\Delta'_{-\frac{1}{2}} = s_0 - s_{-1} & \Delta''_0 = \Delta'_{\frac{1}{2}} - \Delta'_{-\frac{1}{2}} \\
\Delta'_{\frac{1}{2}} = s_1 - s_0 & \Delta''_1 = \Delta'_{\frac{3}{2}} - \Delta'_{\frac{1}{2}} \\
\text{etc.,} \quad \text{etc.,} & \text{etc.,} \quad \text{etc.}
\end{array}$$

we put

$$\Delta_0' = \tfrac{1}{2}(\Delta_{-\frac{1}{2}}' + \Delta_{\frac{1}{2}}')$$
$$\Delta_0''' = \tfrac{1}{2}(\Delta_{-\frac{1}{2}}''' + \Delta_{\frac{1}{2}}''')$$
etc., etc.

and then find

$$s_0' = \frac{ds}{dT} = n\,(\Delta_0' - \tfrac{1}{6}\Delta_0''' + \tfrac{1}{30}\Delta_0^{v} - \text{etc.})$$
$$s_0'' = \frac{d^2s}{dT^2} = n^2\,(\Delta_0'' - \tfrac{1}{12}\Delta_0^{iv} + \text{etc.})$$
$$s_0''' = \frac{d^3s}{dT^3} = n^3(\Delta_0''' - \tfrac{1}{4}\Delta_0^{v})$$

n being the factor by which the interval between epochs must be multiplied to make 100 years. These values of s are to be introduced into the equation (1).

When several reductions are to be computed to the same epoch it may be a little more convenient to introduce Δ', Δ'', etc., directly into the formulæ instead of s', s'', etc. If we make this substitution, stopping at s''' and Δ_0''', the result will be

$$\alpha = \alpha_0 + Tc_0 + \tfrac{1}{2}T^2 s_0 + \frac{n}{6}T^3\,\Delta_0' + \frac{n^2}{24}T^4\,\Delta_0'' + \left(\frac{n^3}{120}T^5 - \frac{n}{36}T^3\right)\Delta_0'''$$

It will be remarked that the coefficients of Δ_0', Δ_0'' and Δ_0''' will be very minute fractions, so that these quantities are not required with great precision. When, owing to the epoch being near the end of the series, their values are not given by differencing, they may be found with sufficient accuracy by extending the successive orders of differences by induction.

When the interval is 95 years, $n = \frac{20}{19}$
When the interval is 50 years, $n = 2$
When the interval is 45 years, $n = \frac{20}{9}$
When the interval is 25 years, $n = 4$
When the interval is 20 years, $n = 5$

REDUCTION BETWEEN TWO CATALOGUE EPOCHS.

As a check upon the numbers of the catalogue it is desirable to compute the change of position between two catalogue epochs in order to see whether it agrees with the difference between the assigned positions. The following is a simple way of effecting this. Put

c_0, s_0, etc., the centennial variation, etc., for the first epoch;
c_1, s_1, etc, the same for the second epoch;
t the fraction of a century between the epochs;
$\alpha_{\frac{1}{2}}$ the position for the middle of the elapsed interval. Then—

$$\alpha_{\frac{1}{2}} = \alpha_0 + \frac{t}{2}c_0 + \frac{t^2}{8}s_0 + \frac{t^3}{48}s_0' + \frac{t^4}{384}s'' + \frac{t^5}{3840}s_0'''$$
$$\alpha_{\frac{1}{2}} = \alpha_1 - \frac{t}{2}c_1 + \frac{t}{8}s_1 - \frac{t^3}{48}s_1 + \frac{t^4}{384}s_1'' - \frac{t^5}{3840}s_1'''$$

whence

$$\alpha_1 - \alpha_0 = \frac{t}{2}(c_1 + c_0) - \frac{t^2}{8}(s_1 - s_0) + \frac{t^3}{48}(s'_1 + s'_0) - \frac{t^4}{384}(s''_1 - s''_0)$$
$$+ \frac{t^5}{3840}(s'''_1 + s'''_0) - \text{etc.}$$

Since each of the quantities c, s, s', etc., is the derivative of the preceding one, their differences are given by a series of the same kind, namely:

$$s_1 - s_0 = \frac{t}{2}(s'_1 + s'_0) - \frac{t^2}{8}(s''_1 - s''_0) + \frac{t^3}{48}(s'''_1 + s'''_0) - \text{etc.}$$
$$s''_1 - s''_0 = \frac{t}{2}(s'''_1 + s'''_0) - \text{etc.}$$

Making these substitutions we find:

$$\alpha_1 - \alpha_0 = \frac{t}{2}(c_1 + c_0) - \frac{t^3}{24}(s'_1 + s'_0) + \frac{t^5}{240}(s'''_1 + s'''_0)$$

From the equations which give the values of the derivatives of s in terms of its differences, putting $n = \frac{1}{t}$, we have by simple reductions:

$$s'_0 + s'_1 = \frac{1}{t}(2\,\Delta'_{\frac{1}{2}} + \tfrac{1}{6}\,\Delta'''_{\frac{1}{2}})$$
$$s'''_1 + s'''_0 = \frac{2}{t^3}\,\Delta'''_{\frac{1}{2}}$$

Making these substitutions in the value of $\alpha_1 - \alpha_0$, it reduces to

$$\alpha_1 - \alpha_0 = \frac{t}{2}(c_1 + c_0) - \frac{t^2}{12}\,\Delta'_{\frac{1}{2}} + \frac{t^2}{720}\,\Delta'''_{\frac{1}{2}}$$

The following are special cases of this formula:

A. Interval, 95 years:

$$\alpha_1 - \alpha_0 = 0.475\,(c_1 + c_0) - 0.075\,\Delta'_{\frac{1}{2}}$$

which may be readily computed when put into the form—

$$\alpha_1 - \alpha_0 = \frac{c_1 + c_0}{2}\left(1 - \frac{1}{20}\right) - \frac{3}{40}\,\Delta'_{\frac{1}{2}}$$

B. Interval, 50 years:

$$\alpha_1 - \alpha_0 = \frac{c_1 + c_0}{4} - \frac{1}{48}\,\Delta'_{\frac{1}{2}} + \frac{1}{2880}\,\Delta'''_{\frac{1}{2}}$$

C. Interval, 25 years:

$$\alpha_1 - \alpha_0 = \frac{c_1 + c_0}{8} - \frac{1}{192}\,\Delta'_{\frac{1}{2}} + \frac{1}{11\,520}\,\Delta'''_{\frac{1}{2}}$$

REDUCTION TO ANY EPOCH.

The following are the principal special forms which will be found useful in reduction to different epochs. They vary with the number and interval of the catalogue epochs, and are therefore classified accordingly.

CLASS A.—*Zodiacal stars.*

Epochs 1755, 1850.

For $1850 + T$

$$\alpha = \alpha_0 + T\,c_0 + \tfrac{1}{2}T^2\,s_0 + \tfrac{1}{6}\tfrac{20}{19}T^3\,\Delta s,$$

Δs being the increment of the secular variation from 1755 to 1850. Especially, to reduce to—

$$\begin{array}{ll}
1860, & \alpha = \alpha_0 + \tfrac{1}{10}c_0 + \tfrac{1}{200}s_0 \\
1870, & \alpha = \alpha_0 + \tfrac{1}{5}c_0 + \tfrac{1}{50}s_0 + \tfrac{1}{712}\Delta s \\
1875, & \alpha = \alpha_0 + \tfrac{1}{4}c_0 + \tfrac{1}{32}s_0 + \tfrac{1}{365}\Delta s \\
1880, & \alpha = \alpha_0 + 0.3\,c_0 + 0.045\,s_0 + 0.0047\,\Delta s \\
1890, & \alpha = \alpha_0 + 0.4\,c_0 + 0.080\,s_0 + 0.0112\,\Delta s \\
1900, & \alpha = \alpha_0 + \tfrac{1}{2}c_0 + \tfrac{1}{8}s_0 + 0.0219\,\Delta s
\end{array}$$

For corresponding epochs before 1850, as far back as 1800, change the signs of the coefficients of c_0 and of Δs.

For epochs between 1755 and 1800 take the values of c and s corresponding to 1755 and count T from this epoch.

CLASS B.—*Time and standard stars.*

Epochs 1755, 1850, 1900.

For epochs previous to 1850, compute as in Class A.

For epochs between 1850 and 1900 put

c_0, centennial variation for 1850.

c_1, centennial variation for 1900.

s_0, secular variation for 1850.

s_1, secular variation for 1900.

$\Delta' s$, secular variation for 1900 *minus* secular variation for 1850

Then in general, we may use either of the forms—

$$\alpha = \alpha_0 + T\,c_0 + \tfrac{1}{2}T^2\,s_0 + \tfrac{1}{3}T^3\,\Delta s \quad \ldots \quad \text{(T from 1850)}$$

$$\alpha = \alpha_1 + T\,c_1 + \tfrac{1}{2}T^2\,s_1 + \tfrac{1}{3}T^3\,\Delta s \quad \ldots \quad \text{(T from 1900)}$$

Especially, to reduce to—

$$1860,\ \alpha = \alpha_0 + \tfrac{1}{10} c_0 + \tfrac{1}{200} s_0$$
$$1870,\ \alpha = \alpha_0 + \tfrac{1}{5} c_0 + \tfrac{1}{50} s_0 + (\tfrac{1}{375} = 0.002\,66)\ \Delta s$$
$$1875,\ \alpha = \alpha_0 + \tfrac{1}{4} c_0 + \tfrac{1}{32} s_0 + (\tfrac{1}{192} = 0.0052)\ \Delta s$$
$$1880,\ \alpha = \alpha_0 + 0.3\, c_0 + 0.045\, s_0 + 0.0090\ \Delta s$$
$$1880,\ \alpha = \alpha_1 - \tfrac{1}{5} c_1 + \tfrac{1}{50} s_1 - 0.002\,66\ \Delta s$$
$$1890,\ \alpha = \alpha_1 - \tfrac{1}{10} c_1 + \tfrac{1}{200} s_1$$

Class C.—*Circumpolar stars.*

Data given for 1755, 1800, 1850, 1900.

Let c_0 be the centennial variation.

s_0, the secular variation.

Δ_0, the mean first difference of s for 50 years for the catalogue epoch nearest that to which the star is to be reduced.

$$\alpha = \alpha_0 + \mathrm{T}\, c_0 + \tfrac{1}{2}\mathrm{T}^2 s_0 + \tfrac{1}{8}\mathrm{T}^3 \Delta_0$$

For 5 years: $\alpha - \alpha_1 = \frac{1}{20} c_0 + \frac{1}{800} s_0 + \frac{1}{24\,000}\Delta$

For 10 years: $\alpha - \alpha_1 = \frac{1}{10} c_0 + \frac{1}{200} s_0 + \frac{1}{3000}\Delta_0$

For 15 years: $\alpha - \alpha_1 = 0.15\, c_0 + 0.011\,25\, s_0 + 0.001\,125\ \Delta_0$

For 20 years: $\alpha - \alpha_1 = \frac{1}{5} c_0 + \frac{1}{50} s_0 + 0.002\,67\ \Delta_0$

For 25 years: $\alpha - \alpha_1 = \frac{1}{4} c_0 + 0.031\,25\, s_0 + 0.005\,20\ \Delta_0$

Class D.—*Data for intervals of twenty-five years.*

Δ_0' the mean first difference for 25 years according to the scheme of differences given at the beginning of this section.

Δ_0'' the second difference for 25 years.

Δ_0''' the mean third difference for 25 years.

Then, in general,

$$\alpha' = \alpha_0 + \mathrm{T}c_0 + \tfrac{1}{2}\mathrm{T}^2 s_0 + \tfrac{2}{3}\mathrm{T}^3 \Delta_0' + \tfrac{2}{3}\mathrm{T}^4 \Delta_0'' + (\tfrac{8}{15}\mathrm{T}^5 - \tfrac{1}{9}\mathrm{T}^3)\ \Delta_0'''$$

Especially,

For — 10 years:

$$\alpha = \alpha_0 - \tfrac{1}{10} c_0 + \tfrac{1}{200} s_0 - \tfrac{1}{1500}\Delta_0' + \tfrac{1}{15\,000}\Delta_0'' - 0.000\,10\ \Delta_0'''$$

For — 5 years:

$$\alpha = \alpha_0 - \tfrac{1}{20} c_0 + \tfrac{1}{800} s_0 - \tfrac{1}{12\,000}\Delta_0' + \tfrac{1}{240\,000}\Delta_0''$$

For 5 years:

$$\alpha - \alpha_0 = \tfrac{1}{20} c_0 + \tfrac{1}{800} s_0 + \tfrac{1}{12\,000}\Delta_0' + \tfrac{1}{240\,000}\Delta_0''$$

For 10 years:

$$\alpha - \alpha_0 = \tfrac{1}{10} c_0 + \tfrac{1}{200} s_0 + \tfrac{1}{1500}\Delta_0' + \tfrac{1}{15\,000}\Delta_0'' - 0.000\,10\ \Delta_0'''$$

HILL'S FORMULÆ FOR THE SECULAR VARIATION OF THE ANNUAL MOTION AND PROPER MOTION OF THE STARS.

μ, the centennial proper motion in R. A., expressed in seconds of time.

μ', the same in declination, expressed in seconds of arc.

p, p', the centennial precessions in R. A. and Dec. at any epoch, expressed in the same units as μ and μ'.

Then:

$$p = m + n \sin \alpha \tan \delta$$

$$p' = n \cos \alpha$$

$$\frac{d\alpha}{dT} = p + \mu$$

$$\frac{d\delta}{dT} = p' + \mu'$$

$$\begin{aligned} \frac{d^2 \alpha}{dT^2} = &+ 0^s.322 \\ &- [6.6338]\, p \\ &+ [7.9878]\, (p + 2\, \mu) \cos \alpha \tan \delta \\ &+ [6.8117]\, (p' + 2\, \mu') \sin \alpha \sec^2 \delta \\ &+ [4.9866]\, \mu\, \mu' \tan \delta \end{aligned}$$

$$\begin{aligned} \frac{d^2 \delta}{dT^2} = &- [6.6338]\, p' \\ &- [9.1640]\, (p + 2\, \mu) \sin \alpha \\ &- [6.7367]\, \mu^2 \sin 2\, \delta \end{aligned}$$

$$\begin{aligned} \frac{d\mu}{dT} = & [7.9878]\, \mu \cos \alpha \tan \delta \\ &+ [6.8117]\, \mu' \sin \alpha \sec^2 \delta \\ &+ [4.9866]\, \mu\, \mu' \tan \delta \end{aligned}$$

$$\begin{aligned} \frac{d\mu'}{dT} = &- [9.1640]\, \mu \sin \alpha \\ &- [6.7367]\, \mu^2 \sin 2\, \delta \end{aligned}$$

Struve's values of m *and* n.

Year.	*m*	*n*	Log. *n*	*n*	Log. *n*
	s.	s.		″	
1750	306.987	133.767	2.126349	2006.50	3.302439
1755	306.997	133.764	2.126339	2006.46	3.302430
1775	307.035	133.753	2.126302	2006.28	3 302392
1800	307.082	133.738	2.126255	2006.07	3.302346
1825	307.130	133.724	2.126209	2005.85	3.302300
1850	307.177	133.710	2.126162	2005.64	3.302253
1875	307.225	133.696	2.126115	2005.42	3.302206
1900	307.272	133.681	2.126069	2005.21	3.302160

CATALOGUE.

RIGHT ASCENSIONS.

No.	Star.	Epoch.	Number of observations.	Right ascension.			Centennial variation.	Secular variation.	Struve's precession.	Proper motion.	Sec. var. of proper motion.
				h.	m.	s.	s.	s.	s.	s.	s.
1	4 Ceti	1755	5	23	55	11.092	+ 307.323	— 0.041	+ 307.188	+ 0.135	
		1850	6	0	0	3.038	307.310	+ 0.013	307.175	0.135	
2	5 Ceti	1755	5	23	55	39.518	+ 307.128	— 0.034	+ 307.165	— 0.037	
		1850	5	0	0	31.283	307.122	+ 0.022	307.160	0.038	
3	α Andromedæ . .	1755	4	23	55	46.413	+ 306.733	+ 1.734	+ 305.701	+ 1.032	+ 0.005
		1850	. .	0	0	38.603	308.417	1.812	307.383	1.034	
		1900	. .	0	3	13.040	309.333	1.853	308.292	1.041	
4	B. A. C. 5 . . .	1850	12	0	1	2.040	+ 307.109	+ 0.037	+ 307.145	— 0.036	
5	B. A. C. 17 . .	1850	11	0	2	38.179	+ 307.088	— 0.113	+ 307.013	+ 0.075	
6	γ Pegasi	1755	10	0	0	38.871	+ 307.069	+ 0.929	+ 307.090	— 0.021	
		1850	. .	0	5	31.016	307.980	0.989	307.999	0.019	
		1900	. .	0	8	5.131	308.482	1.020	308.502	0.020	
7	35 Piscium . . .	1755	5	0	2	22.733	+ 307.869	+ 0.593	+ 307.178	+ 0.691	
		1850	24	0	7	15.485	308.459	0.650	307.771	0.688	
8	36 Piscium . . .	1755	5	0	3	59.888	+ 307.025	+ 0.573	+ 307.277	— 0.252	
		1850	5	0	8	51.829	307.596	0.629	307.849	0.253	
9	38 Piscium . . .	1755	5	0	4	48.507	+ 307.687	+ 0.613	+ 307.366	+ 0.321	
		1850	3	0	9	41.096	308.297	0.670	307.975	0.322	
10	d Piscium . . .	1755	5	0	8	0.488	+ 307.545	+ 0.594	+ 307.556	— 0.011	
		1850	68	0	12	52.932	308.136	0.650	308.147	0.011	
11	44 Piscium . . .	1755	5	0	12	51.247	+ 306.868	+ 0.293	+ 307.073	— 0.205	
		1850	32	0	17	42.912	307.172	0.349	307.377	0.205	
12	β Hydri	1850	. .	0	17	47.12	+ 329.13	—15.88	+ 257.99	+71.14	
		1875	. .	0	19	8.92	325.31	15.30	254.92	70.39	
		1900	. .	0	20	29.77	321.56	—14.75	251.97	69.59	
13	45 Piscium . . .	1755	5	0	13	5.278	+ 307.997	+ 0.597	+ 307.844	+ 0.153	
		1850	44	0	17	58.152	308.590	0.651	308.437	0.153	
14	10 Ceti	1755	5	0	14	3.983	+ 307.214	+ 0.196	+ 306.795	+ 0.419	
		1850	30	0	18	55.933	307.427	0.252	307.007	0.420	
15	11 Ceti	1755	5	0	17	21.136	+ 307.639	+ 0.157	+ 306.559	+ 1.080	
		1850	4	0	22	13.472	307.814	0.211	306.739	1.075	
16	12 Ceti	1755	5	0	17	32.305	+ 306.036	+ 0.013	+ 306.045	— 0.009	
		1850	243	0	22	23.053	306.074	0.068	306.085	0.011	
		1900	. .	0	24	56.100	306.115	0.097	306.127	0.012	
17	51 Piscium . . .	1755	5	0	19	46.562	+ 308.217	+ 0.598	+ 308.127	+ 0.090	
		1850	17	0	24	39.647	308.810	0.651	308.718	0.092	
18	13 Ceti	1755	5	0	22	38.571	+ 308.530	+ 0.056	+ 305.855	+ 2.675	
		1850	46	0	27	31.709	308.610	0.113	305.940	2.670	
19	14 Ceti	1755	5	0	22	58.773	+ 307.457	+ 0.223	+ 306.564	+ 0.893	
		1850	19	0	27	50.965	307.694	0.276	306.801	0.893	

DECLINATIONS.

No.	Star.	Mag.	Epoch.	Declination.	Centennial variation.	Secular variation.	Struve's precession.	Proper motion.	Sec. var. of proper motion.
				° ′ ″	″	″	″	″	″
1	4 Ceti	7.0	1755	− 3 54 49.41	+ 2007.47	+ 0.08	+ 2006.02	+ 1.45	
		6.0	1850	3 23 2.42	2007.09	− 0.87	2005.64	1.45	
2	5 Ceti . . .	7.0	1755	− 3 48 43.44	+ 2005.72	− 0.01	+ 2006.10	− 0.38	
		6.0	1850	3 16 58.15	2005.26	0.96	2005.63	0.37	
3	α Andromedæ . .	1.0	1755	+27 44 13.80	+ 1989.42	− 0.12	+ 2006.11	−16.69	0.00
		2.0	1850	28 15 43.59	1988.94	0.99	2005.63	16.69	
			1900	28 32 17.91	1988.32	1.49	2005.01	16.69	
4	B. A. C. 5 . . .	5.7	1850	− 3 3 27.03	+ 2004.25	− 1.06	+ 2005.62	− 1.37	
5	B. A. C. 17 . .	6.0	1850	6 4 56.50	+ 2002.11	− 1.38	+ 2005.50	− 3.39	
6	γ Pegasi	2.5	1755	+13 49 14.15	+ 2004.61	− 0.99	+ 2006.46	− 1.85	
		2.7	1850	14 20 57.93	2003.21	1.95	2005.06	1.85	
			1900	14 37 39.26	2002.11	2.45	2003.96	1.85	
7	35 Piscium . . .	6.0	1755	+ 7 27 33.39	+ 2003.13	− 1.33	+ 2006.35	− 3.22	
		5.8	1850	7 59 15.62	2001.42	2.28	2004.63	3.21	
8	36 Piscium . . .	6.5	1755	+ 6 52 40.44	+ 2005.22	− 1.64	+ 2006.15	− 0.93	
		6.3	1850	7 24 24.52	2003.21	2.60	2004.14	0.93	
9	38 Piscium . . .	7.5	1755	+ 7 30 27.25	+ 2015.47	− 1.81	+ 2006.01	+ 9.46	
		6.9	1850	8 2 20.98	2013.30	2.76	2003.85	9.45	
10	d Piscium . . .	5.5	1755	+ 6 49 39.28	+ 2006.34	− 2.43	+ 2005.23	+ 1.11	
		5.3	1850	7 21 24.06	2003.58	3.39	2002.47	1.11	
11	44 Piscium . . .	6.0	1755	+ 0 34 52.00	+ 2000.85	− 3.38	+ 2003.32	− 2.47	
		5.9	1850	1 6 31.16	1997.19	4.33	1999.65	2.46	
12	β Hydri	2.7	1850	−78 5 57.53	+ 2030.22	− 4.23	+ 1999.60	+30.62	
			1875	77 57 30.11	2029.12	4.56	1998.42	30.70	
			1900	77 49 2.98	2027.96	4.84	1997.19	30.77	
13	45 Piscium . . .	6.0	1755	+ 6 20 5.18	+ 1997.88	− 3.43	+ 2003.19	− 5.31	
		6.9	1850	6 51 41.47	1994.17	4.39	1999.48	5.31	
14	10 Ceti	6.0	1755	− 1 24 31.65	+ 2002.51	− 3.62	+ 2002.68	− 0.17	
		6.2	1850	0 52 51 02	1998.61	4.58	1998.80	0.19	
15	11 Ceti	7.5	1755	− 2 28 12.11	+ 1993.78	− 4.27	+ 2000.71	− 6.93	
		7.8	1850	1 56 40.10	1989.27	5.22	1996.21	6.94	
16	12 Ceti	6.0	1755	− 5 18 50.80	+ 2000.46	− 4.28	+ 2000.59	− 0.13	
		6.0	1850	4 47 12.42	1995.95	5.22	1996.05	0.10	
			1900	4 30 35.12	1993.22	5.72	1993.35	0.13	
17	51 Piscium . . .	6.5	1755	+ 5 35 57.87	+ 1999.28	− 4.74	+ 1999.00	+ 0.28	
		5.8	1850	6 7 34.91	1994.32	5.70	1994.05	0.27	
18	13 Ceti	6.0	1755	− 4 56 41.41	+ 1994.00	− 5.34	+ 1996.67	− 2.67	
		5.7	1850	4 25 9.67	1988.47	6.30	1991.19	2.72	
19	14 Ceti	. .	1755	− 1 51 17.42	+ 1989.72	− 5.36	+ 1996.38	− 6.66	
		6.0	1850	1 19 49.76	1984.17	6.31	1990.86	6.69	

RIGHT ASCENSIONS.

No.	Star.	Epoch.	Number of observations.	Right ascension.	Centennial variation.	Secular variation.	Struve's precession.	Proper motion.	Sec. var. of proper motion.
				h. m. s.	s.	s.	s.	s.	s.
20	15 Ceti	1755	4	0 25 33.684	+ 306.041	+ 0.238	+ 306.514	− 0.473	
		1850	24	0 30 24.539	306.294	0.294	306.767	0.473	
21	α Cassiopeæ . . .	1755	5	0 26 45.538	330.089	5.253	329.410	+ 0.679	+ 0.010
		1850	449	0 32 1.531	335.194	5.496	334.491	0.703	
		1900	. .	0 34 49.820	337.972	5.621	337.263	0.709	
22	21 Cassiopeæ . . .	1755	5	0 29 56.52	+ 365.28	+14.23	+ 366.38	− 1.10	
		1775	. .	0 31 9.87	368.15	14.51	369.26	1.11	
		1800	. .	0 32 42.36	371.83	14.88	372.95	1.12	
		1825	. .	0 34 15.79	375.60	15.25	376.73	1.13	
		1850	. .	0 35 50.17	379.46	15.64	380.60	1.14	
		1875	. .	0 37 25.52	383.42	16.04	384.57	1.15	
		1900	. .	0 39 1.89	+ 387.48	+16.45	+ 388.64	− 1.16	
23	β Ceti	1755	9	0 31 16.647	+ 302.216	− 0.625	+ 300.611	+ 1.605	− 0.004
		1850	67	0 36 3.478	301.648	0.572	300.041	1.607	
		1900	. .	0 38 34.232	301.370	0.539	299.765	1.605	
24	58 Piscium . . .	1755	5	0 34 16.422	+ 310.984	+ 0.948	+ 310.737	+ 0.247	
		1850	13	0 39 12.293	311.911	1.004	311.665	0.246	
25	60 Piscium . . .	1755	5	0 34 44.622	+ 308.896	+ 0.666	+ 308.905	− 0.009	
		1850	19	0 39 38.382	309.554	0.720	309.564	0.010	
26	62 Piscium . . .	1755	5	0 35 36.165	+ 309.763	+ 0.701	+ 309.157	+ 0.606	
		1850	10	0 40 30.764	310 455	0.756	309.848	0.607	
27	B. A. C. 221 . .	1755	. .	0 35 33.262	+ 313.278	+ 0.580	+ 308.449	+ 4.829	
		1850	29	0 40 31.146	313.852	0.628	309.031	4.821	
28	δ Piscium . . .	1755	5	0 35 59.690	+ 309.718	+ 0.720	+ 309.286	+ 0.432	
		1850	114	0 40 54.256	310.427	0.773	309.994	0.433	
29	B. A. C. 237 . .	1755	. .	0 38 42.313	+ 307.883	+ 0.510	+ 307.802	+ 0.081	
		1850	19	0 43 35.039	308.391	0.561	308.312	0.079	
30	20 Ceti	1755	5	0 40 29.912	+ 305.863	+ 0.289	+ 305.978	− 0.115	
		1850	70	0 45 20.619	306.161	0.340	306.278	0.117	
31	B. A. C. 274 . .	1755	5	0 47 9.005	+ 309.528	+ 0.717	+ 309.467	+ 0.061	
		1850	10	0 52 3.388	310.234	0.769	310.168	0.066	
32	70 Piscium . . .	1755	3	0 49 23.966	+ 310.284	+ 0.807	+ 310.315	− 0.031	
		1850	32	0 54 19.107	311.074	0.858	311.106	0.032	
33	ε Piscium . . .	1755	5	0 50 15.094	+ 309.767	+ 0.810	+ 310.345	− 0.578	
		1850	529	0 55 9.745	310.560	0.860	311.136	0.576	
		1900	. .	0 57 45.134	310.999	+ 0.886	311.574	0.575	
34	26 Ceti	1755	5	0 51 13.425	+ 307.745	+ 0.472	+ 307.021	+ 0.724	
		1850	23	0 56 6.003	308.218	0.523	307.494	0.724	
35	73 Piscium . . .	1755	5	0 52 12.245	+ 309.463	+ 0.698	+ 309.287	+ 0.176	
		1850	5	0 57 6.558	310.151	0.750	309.975	0.176	
36	72 Piscium . . .	1755	3	0 52 11.551	+ 314.263	+ 1.212	+ 314.315	− 0.052	
		1850	8	0 57 10.659	315.440	1.267	315.493	0.053	

DECLINATIONS.

No.	Star.	Mag.	Epoch.	Declination.	Centennial variation.	Secular variation.	Struve's precession.	Proper motion.	Sec. var. of proper motion.
				° ′ ″	″	″	″	″	″
20	15 Ceti	7.0	1755	− 1 51 15.01	+ 1992.13	− 5.82	+ 1994.00	− 1.87	
		6.8	1850	1 19 45.26	1986.16	6.76	1988.02	1.86	
21	α Cassiopeæ . . .	3.0	1755	+55 11 23.66	+ 1988.72	− 6.44	+ 1992.78	− 4.06	− 0.01
		2.5	1850	55 42 49.85	1982.02	7.68	1986.09	4.07	
			1900	55 59 19.88	1978.02	8.32	1982.09	4.07	
22	21 Cassiopeæ . . .	6.0	1755	+73 38 37.07	+ 1987.24	− 7.78	+ 1989.34	− 2.10	
		6.0	1775	73 45 14.38	1985.66	8.11	1987.76	2.10	
			1800	73 53 30 53	1983.59	8.54	1985.68	2.09	
			1825	74 1 46.16	1981.41	8.99	1983.49	2.08	
			1850	74 10 1.21	1979.08	9.45	1981.16	2.08	
			1875	74 18 15.68	1976.67	9.91	1978.74	2.07	
			1900	+74 26 29.53	+ 1974.13	−10.40	+ 1976.20	− 2.07	
23	β Ceti	2.5	1755	−19 20 6.39	+ 1990.32	− 6.88	+ 1987.79	+ 2.53	− 0.03
		2.3	1850	18 48 38.83	1983.36	7.77	1980.86	2.50	
			1900	18 32 8.14	1979.36	8.23	1976.88	2.48	
24	58 Piscium . . .	6.0	1755	+10 37 57.45	+ 1982.52	− 7.62	+ 1984.06	− 1.54	
		5.0	1850	11 9 17.26	1974.82	8.60	1976.36	1.54	
25	60 Piscium . . .	6.0	1755	+ 5 23 56.93	+ 1982.32	− 7.66	+ 1983.44	− 1.12	
		6.2	1850	5 55 16.53	1974.59	8.62	1975.71	1.12	
26	62 Piscium . . .	6.0	1755	+ 5 57 28.89	+ 1982.16	− 7.86	+ 1982.32	− 0.16	
		6.0	1850	6 28 48.23	1974.24	8.83	1974.40	0.16	
27	B. A. C. 221 . .	. .	1755	+ 4 0 59.79	+ 1867.90	− 7.98	+ 1982.36	−114.46	
		5.9	1850	4 30 30.48	1859.84	8.98	1974.37	114.53	
28	δ Piscium . . .	5.0	1755	− 6 14 49.37	+ 1976 97	− 7.94	+ 1981.77	− 4.80	
		4.4	1850	6 46 3.75	1968.95	8.92	1973.78	4.83	
29	B. A. C. 237 . .	7.5	1755	+ 2 3 4.57	+ 1971.04	− 8.40	+ 1977.89	− 6.85	
		6.7	1850	2 34 13.12	1962.60	9.36	1969.47	6.87	
30	20 Ceti	5.0	1755	− 2 28 46.48	+ 1973.87	− 8.69	+ 1975.21	− 1.34	
		5.2	1850	1 57 35.37	1965.17	9 62	1966.51	1.34	
31	B. A. C. 274 . .	6.5	1755	+ 5 9 22.01	+ 1963.69	−10.07	+ 1964.14	− 0.45	
		6.2	1850	5 40 22.83	1953.67	11.03	1954.13	0.46	
32	70 Piscium . . .	8.0	1755	+ 6 36 49.84	+ 1963.35	−10.52	+ 1960.03	+ 3.32	
		8.0	1850	7 7 50.12	1952.89	11.49	1949.57	3.32	
33	ε Piscium . . .	4.0	1755	+ 6 33 55.02	+ 1960.75	−10.67	+ 1958.43	+ 2.32	+ 0.01
		4.2	1850	7 4 52.76	1950.15	11.63	1947.82	2.33	
			1900	7 21 6.36	1944.21	12.13	1941.88	2.33	
34	26 Ceti	6.5	1755	+ 0 2 52.00	+ 1952.26	−10.82	+ 1956.55	− 4.29	
		5.9	1850	0 33 41.62	1941.54	11.76	1945.85	4.31	
35	73 Piscium . . .	6.5	1755	+ 4 20 12.33	+ 1953.58	−11.04	+ 1954.63	− 1.05	
		5.9	1850	4 51 3.10	1942.64	12.00	1943.69	1.05	
36	72 Piscium . . .	6.0	1755	+13 37 21.87	+ 1958.40	−11.19	+ 1954.66	+ 3.74	
		6.0	1850	14 8 17.15	1947.29	12.20	1943.55	3.74	

RIGHT ASCENSIONS.

No.	Star.	Epoch.	Number of observations.	Right ascension.	Centennial variation.	Secular variation.	Struve's precession.	Proper motion.	Sec. var. of proper motion.
				h. m. s.	s.	s.	s.	s.	s.
37	77 Piscium . . .	1755	4	0 53 10.035	+ 308.929	+ 0.662	+ 308.930	− 0.001	
		1850	11	0 58 3.823	309.581	0.711	309.584	0.003	
38	75 Piscium . . .	1755	3	0 53 42.140	+ 313.534	+ 1.105	+ 313.392	+ 0.142	
		1850	4	0 58 40.503	314.608	1.158	314.468	0.140	
39	29 Ceti	1755	5	0 55 22.694	+ 308.220	+ 0.515	+ 307.391	+ 0.829	
		1850	19	1 0 15.742	308.732	0.563	307.910	0.822	
40	ε Piscium . . .	1755	5	0 55 46.266	+ 307.560	+ 0.710	+ 309.449	− 1.889	
		1850	78	1 0 38.776	308.256	0.756	310.147	1.891	
41	β Andromedæ . .	1755	10	0 56 5.522	+ 330.624	+ 2.770	+ 329.112	+ 1.512	
		1850	159	1 1 20.878	333.296	2.856	331.778	1.518	
		1900	. .	1 4 7.885	334.736	2.904	333.214	1.522	
42	33 Ceti	1755	5	0 57 58.143	+ 307.584	+ 0.564	+ 307.660	− 0.076	
		1850	36	1 2 50.611	308.143	0.612	308.217	0.074	
43	35 Ceti	1755	2	0 59 57.839	+ 306.458	+ 0.569	+ 307.701	− 1.243	
		1850	18	1 4 49.239	307.025	0.625	308.269	1.244	
44	α Ursæ Minoris . .	1755	. .	0 43 42.11	+1039.42	+ 483.42	+1031.09	+ 8.33	
		1775	. .	0 47 20.21	1144.48	570.02	1135.66	8.82	
		1800	. .	0 52 25.48	1303.59	709.32	1294.06	9.53	
		1825	. .	0 58 15.32	1503.06	895.82	1492.73	10.33	
		1850	. .	1 5 1.55	1757.17	1150.68	1745.90	11.27	
		1875	. .	1 13 0.16	2086.78	1506.80	2074.40	12.38	
		1900	. .	1 22 33.76	+2523.03	+2016.15	+2509.33	+13.70	
45	ζ Piscium . . .	1755	4	1 0 57.437	+ 311.684	+ 0.844	+ 310.861	+ 0.823	
		1850	52	1 5 53.924	312.509	0.892	311.685	0.824	
46	88 Piscium . . .	1755	5	1 1 59.313	+ 310.359	+ 0.815	+ 310.559	− 0.200	
		1850	6	1 6 54.529	311.157	0.865	311.358	0.201	
47	f Piscium . . .	1755	5	1 5 10.983	+ 308.044	+ 0.657	+ 308.516	− 0.472	
		1850	39	1 10 3.929	308.690	0.703	309.162	0.472	
48	B. A. C. 410 . .	1850	3	1 15 6.529	+ 312.413	+ 0.931	+ 312.175	+ 0.238	
49	θ[1] Ceti	1755	6	1 11 47.032	+ 299.503	+ 0.115	+ 300.135	− 0.632	− 0.004
		1850	656	1 16 31.621	299.636	0.164*	300.269	0.633	
		1900	. .	1 19 1.461	299.724	0.188	300.359	0.635	
50	ρ Piscium . . .	1755	5	1 13 5.870	+ 320.078	+ 1.565	+ 320.535	− 0.457	
		1850	18	1 18 10.658	321.590	1.618	322.048	0.458	
51	94 Piscium . . .	1755	5	1 13 30.517	+ 320.922	+ 1.567	+ 320.668	+ 0.254	
		1850	11	1 18 36.110	322.439	1.628	322.188	0.251	
52	95 Piscium . . .	1755	4	1 14 58.215	+ 309.728	+ 0.784	+ 310.064	− 0.336	
		1850	7	1 19 52.816	310.494	0.828	310.833	0.339	
53	38 Cassiopeæ . . .	1755	5	1 13 24.64	+ 419.11	+13.27	+ 416.67	+ 2.44	
		1800	. .	1 16 34.60	425.18	13.67	422.72	2.46	
		1850	. .	1 20 8.94	432.13	14.13	429.64	2.49	
		1900	. .	1 23 46.79	+ 439.31	+14.60	+ 436.80	+ 2.51	

DECLINATIONS.

No.	Star.	Mag.	Epoch.	Declination.	Centennial variation.	Secular variation.	Struve's precession.	Proper motion.	Sec. var. of proper motion.
				° ′ ″	″	″	″	″	″
37	77 Piscium . . .	7.5	1755	+ 3 35 50.51	+ 1940.64	— 11.20	+ 1952.71	—12.07	
		5.9	1850	4 6 29.92	1929.55	12.16	1941.61	12.06	
38	75 Piscium . . .	6.5	1755	+11 38 10.28	+ 1954.78	— 11.46	+ 1951.63	+ 3.15	
		6.0	1850	12 9 2.00	1943.42	12.46	1940.27	3.15	
39	29 Ceti	7.5	1755	+ 0 42 24.67	+ 1904.23	— 11.63	+ 1948.17	—43.94	
		6.3	1850	1 12 28.30	1892.74	12.57	1936.70	43.96	
40	ε Piscium . . .	5.0	1755	+ 4 20 50.11	+ 1929.08	— 11.60	— 1947.32	—18.24	
		5.5	1850	4 51 17.35	1917.61	12.55	1935.83	18.22	
41	β Andromedæ . .	2.0	1755	+34 18 54.10	+ 1934.49	— 12.56	+ 1946.65	—12.16	— 0.06
		2.2	1850	34 49 26.03	1922.00	13.75	1934.21	12.21	
			1900	35 05 25.28	1914.98	14.37	1927.22	12.24	
42	33 Ceti	6.0	1755	+ 1 8 5.72	+ 1942.09	— 12.09	+ 1942.59	— 0.50	
		6.1	1850	1 38 45.09	1930.18	13.02	1930.71	0.53	
43	35 Ceti	6.5	1755	+ 1 10 16.17	+ 1925.80	— 12.35	+ 1938.16	—12.36	
		6.3	1850	1 40 39.92	1913.49	13.57	1925.95	12.46	
44	α Ursæ Minoris . .	2.5	1755	+87 59 41.11	+ 1970.59	— 29.82	+ 1970.11	+ 0.48	
			1775	88 6 14.60	1964.11	35.35	1963.68	0.43	
			1800	88 14 24.42	1954.19	44.28	1953.84	0.35	
			1825	88 22 31.47	1941.68	55.95	1941.42	0.27	
		2.0	1850	88 30 34.96	1925.60	71.36	1925.44	0.17	
			1875	88 38 33.86	1904.58	96.72	1904.54	+ 0.04	
			1900	+88 46 26.66	+ 1876.41	—131.46	+ 1876.51	— 0.10	
45	ζ Piscium . .	4.0	1755	+ 6 16 22.18	+ 1931.13	— 12.85	+ 1935.91	— 4.78	
		4.8	1850	6 46 50.44	1918.46	13.82	1923.30	4.84	
46	88 Piscium . . .	6.7	1755	+ 5 41 33.28	+ 1930.69	— 12.92	+ 1933.51	— 2.82	
		6.2	1850	6 12 1.45	1917.96	13.88	1920.75	2.79	
47	f Piscium . .	6.0	1755	+ 2 19 2.89	+ 1922.85	— 13.45	+ 1925.83	— 2.98	
		5.2	1850	2 49 23.40	1909.63	14.37	1912.64	3.01	
48	B. A. C. 410 . .	6.0	1850	+ 6 37 23.42	+ 1917.52	— 15.50	+ 1898.88	+18.64	
49	θ¹ Ceti	3.0	1755	— 9 27 17.95	+ 1886.78	— 14.26	+ 1908.85	—22.07	
		3.2	1850	8 57 32.07	1872.83	15.12	1894.87	22.04	
			1900	8 41 57.57	1865.16	15.56	1887.18	22.02	
50	ρ Piscium . . .	5.5	1755	+17 53 18.50	+ 1907.33	— 15.44	+ 1905.26	+ 2.07	
		5.0	1850	18 23 23.34	1892.17	16.48	1890.08	2.09	
51	94 Piscium . . .	6.5	1755	+17 57 43.89	+ 1899.37	— 15.60	+ 1904.08	— 4.71	
		6.3	1850	18 27 41.02	1884.05	16.66	1888.83	4.78	
52	95 Piscium . . .	7.0	1755	+ 4 4 57.01	+ 1885.18	— 15.32	+ 1900.06	—14.88	
		8.0	1850	4 34 40.88	1870.18	16.26	1885.04	14.86	
53	38 Cassiopeæ . . .	6.0	1755	+68 59 31.75	+ 1896.98	— 20.18	+ 1904.40	— 7.42	
			1800	69 13 43.30	1887.67	21.27	1895.14	7.47	
		6.7	1850	69 29 24.42	1876.71	22.53	1884.24	7.53	
			1900	+69 44 59.91	+ 1865.12	— 23.86	+ 1872.71	— 7.59	

RIGHT ASCENSIONS.

No.	Star.	Epoch.	Number of observations.	Right ascension.	Centennial variation.	Secular variation.	Struve's precession.	Proper motion.	Sec. var. of proper motion.
				h. m. s.	s.	s.	s.	s.	s.
54	96 Piscium . . .	1755	5	1 16 17.432	+ 311.333	+ 0.893	+ 311.607	— 0.274	
		1850	13	1 21 13.608	312.203	0.939	312.479	0.276	
55	μ Piscium . . .	1755	5	1 17 22.463	+ 312.572	+ 0.842	+ 310.775	+ 1.797	
		1850	63	1 22 19.793	313.394	0.888	311.594	1.800	
56	η Piscium . . .	1755	5	1 18 24.745	+ 318.388	+ 1.354	+ 318.248	+ 0.140	
		1850	485	1 23 27.832	319.698	1.403	319.557	0.141	
		1900	. .	1 26 7.857	320.406	1.428	320.263	0.143	
57	B. A. C. 455 . .	1755	1	1 18 53.546	+ 320.407	+ 1.456	+ 319.671	+ 0.736	
		1850	3	1 23 58.597	321.813	1.505	321.079	0.734	
58	100 Piscium . . .	1755	5	1 21 53.046	+ 316.113	+ 1.204	+ 316.340	— 0.227	
		1850	4	1 26 53.902	317.278	1.249	317.507	0.229	
59	101 Piscium . . .	1755	5	1 22 42.705	+ 318.200	+ 1.330	+ 318.253	— 0.053	
		1850	18	1 27 45.601	319.485	1.376	319.535	0.050	
60	π Piscium . . .	1755	5	1 24 8.649	+ 315.766	+ 1.193	+ 316.226	— 0.460	
		1850	44	1 29 9.173	316.920	1.237	317.377	0.457	
61	B. A. C. 490 . .	1755	4	1 24 40.328	+ 317.139	+ 1.189	+ 316.230	+ 0.909	
		1850	6	1 29 42.153	318.292	1.239	317.382	0.910	
62	103 Piscium . . .	1755	5	1 26 5.699	+ 320.347	+ 1.456	+ 320.489	— 0.142	
		1850	7	1 31 10.692	321.752	1.503	321.897	0.145	
63	104 Piscium . . .	1755	4	1 26 9.951	+ 319.026	+ 1.320	+ 318.365	+ 0.661	
		1850	3	1 31 13.628	320.302	1.366	319.642	0.660	
64	105 Piscium . . .	1755	5	1 26 30.367	+ 320.745	+ 1.448	+ 320.347	+ 0.398	
		1850	17	1 31 35.734	322.142	1.495	321.745	0.397	
65	α Eridani	1850	. .	1 32 7.30	+ 223.74	— 1.32	+ 223.46	+ 0.28	
		1875	. .	1 33 3.20	223.42	1.30	223.14	0.28	
		1900	. .	1 33 59.01	223.10	1.27	222.82	0.28	
66	ν Piscium . . .	1755	5	1 28 42.308	+ 310.596	+ 0.852	+ 310.737	— 0.141	
		1850	316	1 33 37.766	311.425	0.894	311.569	0.144	
		1900	. .	1 36 13.591	311.877	0.916	312.021	0.144	
67	o Piscium . . .	1755	5	1 32 29.178	+ 314.765	+ 1.062	+ 314.300	+ 0.465	0.000
		1850	267	1 37 28.690	315.793	1.102	315.325	0.468	
		1900	. .	1 40 6.725	316.349	1.124	315.881	0.468	
68	3 Arietis	1755	5	1 33 20.125	+ 322.507	+ 1.531	+ 322.364	+ 0.143	
		1850	5	1 38 27.204	323.982	1.575	323.841	0.141	
69	4 Arietis	1755	5	1 34 56.282	+ 322.394	+ 1.508	+ 322.155	+ 0.239	
		1850	7	1 40 3.246	323.848	1.552	323.611	0.237	
70	B. A. C. 549 . .	1755	2	1 35 7.254	+ 321.935	+ 1.510	+ 322.244	— 0.309	
		1850	2	1 40 13.780	323.390	1.554	323.707	0.317	
71	54 Ceti	1755	5	1 37 53.975	+ 316.052	+ 1.172	+ 316.589	— 0.537	
		1850	27	1 42 54.759	+ 317.184	1.212	317.723	0.539	

DECLINATIONS.

No.	Star.	Mag.	Epoch.	Declination.	Centennial variation.	Secular variation.	Struve's precession.	Proper motion.	Sec. var. of proper motion.
				° ′ ″	″	″	″	″	″
54	96 Piscium . . .	6.5	1755	+ 6 1 16.62	+ 1889.80	− 15.64	+ 1896.35	− 6.55	
		6.6	1850	6 31 4.72	1874.48	16.60	1880.98	6.50	
55	μ Piscium . . .	5.0	1755	+ 4 52 20.08	+ 1889.61	− 16.00	+ 1893.19	− 3.58	
		5.0	1850	5 22 7.84	1873.95	16.97	1877.61	3.66	
56	η Piscium . . .	4.0	1775	+ 14 4 27.45	+ 1889.43	− 16.40	+ 1890.17	− 0.74	
		3.7	1850	14 34 14.85	1873.37	17.41	1874.11	0.74	
			1900	14 49 49.34	1864.53	17.95	1865.27	0.74	
57	B. A. C. 455 . .	8.0	1755	. . . . .	. . . .	− 16.62	+ 1888.75	. . .	
		7.0	1850	+ 16 11 7	. . . .	17.66	1872.54	. . .	
58	100 Piscium . . .	7.0	1755	+ 11 17 41.73	+ 1879.14	− 16.92	+ 1879.75	− 0.61	
		6.8	1850	11 47 19.13	1862.59	17.92	1863.19	0.60	
59	101 Piscium . . .	6.0	1755	+ 13 23 59.57	+ 1875.76	− 17.22	+ 1877.22	− 1.46	
		6.3	1850	13 53 33.62	1858.92	18.22	1860.38	1.46	
60	π Piscium . . .	6.0	1755	+ 10 52 45.18	+ 1877.56	− 17.36	+ 1872.69	+ 4.87	
		5.7	1850	11 22 20.86	1860.60	18.34	1855.79	4.81	
61	B. A. C. 490 . .	. .	1755	+ 10 49 16.61	+ 1866.76	− 17.56	+ 1871.04	− 4.28	
		7.5	1850	11 18 41.93	1849.52	18.73	1853.94	4.42	
62	103 Piscium . . .	7.5	1755	+ 15 22 21.89	+ 1863.94	− 17.94	+ 1866.54	− 2.60	
		6.8	1850	15 51 44.38	1846.41	18.97	1849.00	2.59	
63	104 Piscium . . .	6.5	1755	+ 13 1 59.31	+ 1862.77	− 17.93	+ 1866.36	− 3.59	
		7.5	1850	13 31 20.70	1845.26	18.94	1848.84	3.58	
64	105 Piscium . . .	6.0	1755	+ 15 9 11.36	+ 1864.14	− 18.07	+ 1865.22	− 1.08	
		6.3	1850	15 38 33.98	1846.49	19.10	1847.58	1.09	
65	α Eridani	1.0	1850	− 58 0 0.14	+ 1841.07	− 13.58	+ 1845.79	− 4.72	
			1875	57 52 20.30	1837.63	13.68	1842.38	4.75	
			1900	57 44 41.33	1834.18	13.78	1838.95	4.77	
66	ν Piscium . . .	5.0	1755	+ 4 14 18.86	+ 1858.20	− 17.93	+ 1858.00	+ 0.20	
		4.5	1850	4 43 35.92	1840.72	18.87	1840.54	0.18	
			1900	4 58 53.79	1830.90	19.36	1830.80	0.10	
67	ο Piscium . . .	5.0	1755	+ 7 54 56.13	+ 1847.97	− 18.87	+ 1845.30	+ 2.67	
		4.3	1850	8 24 3.04	1829.58	19.84	1826.94	2.64	
			1900	8 39 15.32	1819.54	20.34	1816.92	2.62	
68	3 Arietis	6.5	1755	+ 16 10 33.28	+ 1840.95	− 19.44	+ 1842.35	− 1.40	
		6.0	1850	16 39 33.26	1821.99	20.48	1823.40	1.41	
69	4 Arietis	6.5	1755	+ 15 43 29.43	+ 1834.64	− 19.73	+ 1836.75	− 2.11	
		5.7	1850	16 12 23.28	1815.40	20.77	1817.53	2.13	
70	B. A. C. 529 . .	8.0	1755	+ 15 47 15.55	+ 1838.60	− 19.71	+ 1836.12	+ 2.48	
		8.2	1850	16 16 13.17	1819.39	20.74	1816.87	2.52	
71	54 Ceti	6.0	1755	+ 9 49 11.68	+ 1822.91	− 19.85	+ 1826.16	− 3.25	
		5.5	1850	10 17 54.34	1803.59	20.83	1806.81	3.22	

RIGHT ASCENSIONS.

No.	Star.	Epoch.	Number of observations.	Right ascension.			Centennial variation.	Secular variation.	Struve's precession.	Proper motion.	Sec. var. of proper motion.
				h.	m.	s.	s.	s.	s.	s.	s.
72	γ^1 Arietis	1755	9	1	40	8.085	+ 325.918	+ 1.660	+ 325.477	+ 0.441	
		1850	26	1	45	18.463	327.518	1.709	327.083	0.435	
73	γ^2 Arietis	1755	0	1	40	8.085	+ 325.918	+ 1.660	+ 325.477	+ 0.441	
		1850	35	1	45	18.463	327.518	1.709	327.083	0.435	
74	β Arietis	1755	10	1	41	9.443	+ 327.947	+ 1.744	+ 327.345	+ 0.602	
		1850	408	1	46	21.789	329.636	1.811	329.045	0.591	—0.002
		1900	. .	1	49	6.835	330.548	1.836	329.961	0.587	
75	ι Arietis	1755	5	1	44	0.668	+ 324.693	+ 1.585	+ 324.491	+ 0.202	
		1850	33	1	49	9.848	326.219	1.627	326.019	0.200	
76	50 Cassiopeæ . . .	1755	5	1	43	2.90	+ 476.85	+17.29	+ 477.96	— 1.11	
		1800	. .	1	46	39.24	484.74	17.79	485.86	1.12	
		1850	. .	1	50	43.85	493.78	18.36	494.91	1.13	
		1900	. .	1	54	53.06	503.10	18.93	504.26	1.16	
77	B. A. C. 609 . .	1850	20	1	51	24.494	+ 319.911	+ 1.302	+ 319.960	— 0.049	
78	α Arietis	1755	10	1	53	25.083	+ 334.403	+ 1.982	+ 333.040	+ 1.363	0.000
		1850	. .	1	58	43.665	336.302	2.020	334.944	1.358	
		1900	. .	2	1	32.069	337.317	2.042	335.960	1.357	
79	15 Arietis	1755	5	1	57	5.725	+ 329.236	+ 1.721	+ 328.669	+ 0.567	
		1850	12	2	2	19.281	330.888	1.757	330.323	0.565	
80	64 Ceti	1755	5	1	58	26.896	+ 314.602	+ 1.092	+ 315.603	— 1.001	
		1850	6	2	3	26.267	315.656	1.126	316.652	0.996	
81	η Arietis	1755	5	1	59	8.302	+ 332.269	+ 1.842	+ 331.242	+ 1.027	
		1850	15	2	4	24.793	334.035	1.877	333.006	1.029	
82	19 Arietis	1755	5	1	59	44.054	+ 324.375	+ 1.468	+ 323.786	+ 0.589	
		1850	14	2	4	52.877	325.786	1.502	325.196	0.590	
83	ξ^1 Ceti	1755	5	2	0	2.724	+ 315.875	+ 1.119	+ 316.026	— 0.151	
		1850	164	2	5	3.314	316.952	1.151	317.102	0.150	
		1900	. .	2	7	41.935	317.531	1.165	317.681	0.150	
84	B. A. C. 686 . .	1755	3	2	0	19.168	+ 329.479	+ 1.732	+ 329.363	+ 0.116	
		1850	3	2	5	32.960	331.142	1.768	331.033	0.109	
85	θ Arietis	1755	5	2	4	32.800	+ 330.363	+ 1.754	+ 330.481	— 0.118	
		1850	34	2	9	47.442	332.045	1.787	332.166	0.121	
86	23 Arietis	1755	4	2	5	34.405	+ 330.205	+ 1.734	+ 330.382	— 0.177	
		1850	3	2	10	48.886	331.867	1.765	332.050	0.183	
87	B. A. C. 738 . .	1755	1	2	11	5.864	+ 318.456	+ 1.211	+ 318.659	— 0.203	
		1850	3	2	16	8.948	319.620	1.239	319.822	0.202	
88	ι Cassiopeæ . . .	1755	. .	2	9	15.46	+ 468.96	+12.51	+ 469.74	— 0.78	
		1800	. .	2	12	47.76	474.65	12.73	475.43	0.78	
		1850	. .	2	16	46.68	481.07	12.97	481.86	0.79	
		1900	. .	2	20	48.85	+ 487.61	+13.19	+ 488.41	— 0.80	

DECLINATIONS.

No.	Star.	Mag.	Epoch.	Declination.	Centennial variation.	Secular variation.	Struve's precession.	Proper motion.	Sec. var. of proper motion.
				° ′ ″	″	″	″	″	″
72	γ^1 Arietis	4.5	1755	+ 18 4 54.44	+ 1807.98	− 20.93	+ 1817.97	− 9.99	
		3.7	1850	18 33 22.41	1787.59	21.99	1797.61	10.02	
73	γ^2 Arietis	4.5	1755	+ 18 5 3.02	+ 1807.97	− 20.93	+ 1817.97	− 10.00	
		3.7	1850	18 33 30.98	1787.58	21.99	1797.61	10.03	
74	β Arietis	3.0	1755	+ 19 35 58.67	+ 1802.42	− 21.26	+ 1814.17	− 11.75	− 0.04
		3.0	1850	20 4 21.21	1781.71	22.34	1793.50	11.79	
			1900	20 19 9.25	1770.41	22.90	1782.22	11.81	
75	ι Arietis	6.0	1755	+ 16 36 38.17	+ 1800.12	− 21.55	+ 1803.34	− 3.22	
		5.7	1850	17 4 58.40	1779.15	22.60	1782.38	3.23	
76	50 Cassiopeæ . . .	4.5	1755	+ 71 13 5.95	+ 1809.28	− 30.94	+ 1807.02	+ 2.26	
			1800	71 26 36.93	1795.04	32.43	1792.75	2.29	
		4.0	1850	71 41 30.32	1778.39	34.15	1776.06	2.33	
			1900	71 56 15.17	1760.88	35.96	1758.49	2.39	
77	B. A. C. 609 . .	6.0	1850	+ 11 33 53.54	+ 1766.98	− 22.56	+ 1773.31	− 6.33	
78	α Arietis	3.0	1755	+ 22 17 29.96	+ 1750.71	− 24.04	+ 1765.74	−15.03	− 0.10
		2.0	1850	22 45 2.12	1727.35	25.14	1742.47	15.12	
			1900	22 59 22.63	1714.63	25.72	1729.80	15.17	
79	15 Arietis	6.0	1755	+ 18 19 56.14	+ 1746.48	− 24.30	+ 1750.22	− 3.74	
		5.7	1850	18 47 24.18	1722.90	25.34	1726.67	3.77	
80	64 Ceti	6.5	1755	+ 7 24 37.90	+ 1733.83	− 23.36	+ 1744.36	−10.53	
		5.7	1850	7 51 54.35	1711.20	24.28	1721.69	10.49	
81	η Arietis	6.0	1755	+ 20 2 48.92	+ 1742.69	− 24.90	+ 1741.40	+ 1.29	
		5.3	1850	20 30 13.07	1718.52	25.99	1717.31	1.21	
82	19 Arietis	7.0	1755	+ 14 7 8.77	+ 1736.66	− 24.42	+ 1738.81	− 2.15	
		5.7	1850	14 34 27.44	1713.00	25.42	1715.19	2.19	
83	ξ^1 Ceti	5.0	1755	+ 7 41 7.89	+ 1735.86	− 23.76	+ 1737.43	− 1.57	+ 0.03
		4.3	1850	8 8 26.11	1712.86	24.68	1714.40	1.54	
			1900	8 22 39.42	1700.40	25.16	1701.92	1.52	
84	B. A. C. 686 . .	8.0	1755	+ 18 27 14.83	+ 1736.25	− 24.84	+ 1736.26	− 0.01	
		7.2	1850	18 54 32.90	1712.15	25.90	1712.15	0.00	
85	θ Arietis	6.0	1755	+ 18 45 17.21	+ 1716.55	− 25.65	+ 1717.49	− 0.84	
		5.7	1850	19 12 16.19	1691.68	26.71	1692.52	0.84	
86	23 Arietis	7.5	1755	+ 18 33 9.72	+ 1701.07	− 25.82	+ 1712.73	−11.66	
		7.5	1850	18 59 53.93	1676.05	26.87	1687.70	11.65	
87	B. A. C. 738 . .	8.0	1755	. . . . . .	. . . .	− 25.86	+ 1687 05	. . .	
		7.7	1850	+ 9 35 25.6	. . . .	26.80	1662.08	. . .	
88	ι Cassiopeæ . . .	4.5	1755	+ 66 16 51.55	+ 1696.10	− 37.24	+ 1695.71	+ 0.39	
			1800	66 29 31.02	1679.06	38.57	1678.64	0.42	
		4.0	1850	66 43 25.68	1659.39	40.09	1658.93	0.46	
			1900	+ 66 57 10.30	+ 1638.96	− 41.65	+ 1638.48	+ 0.48	

RIGHT ASCENSIONS.

No.	Star.	Epoch.	Number of observations.	Right ascension.			Centennial variation.	Secular variation.	Struve's precession.	Proper motion.	Sec. var. of proper motion.
				h.	m.	s.	s.	s.	s.	s.	s.
89	ξ Arietis	1755	5	2	11	43.259	+ 319.150	+ 1.228	+ 319.150	— 0.000	
		1850	22	2	16	47.010	320.328	1.254	320.330	0.002	
90	B. A. C. 755	1755	1	2	13	39.213	+ 319.477	+ 1.230	+ 319.253	+ 0.224	
		1850	10	2	18	43.275	320.658	1.256	320.436	0.222	
91	25 Arietis	1755	5	2	14	23.403	+ 316.922	+ 1.191	+ 318.849	— 1.927	
		1850	14	2	19	25.020	318.066	1.217	320.005	1.939	
92	ξ² Ceti	1755	5	2	15	9.875	+ 316.832	+ 1.129	+ 316.590	+ 0.242	
		1850	232	2	20	11.379	317.917	1.155	317.674	0.243	
		1900	. .	2	22	50.483	318.498	1.168	318.255	0.243	
93	26 Arietis	1755	5	2	16	57.060	+ 333.019	+ 1.761	+ 332.548	+ 0.471	
		1850	5	2	22	14.226	334.704	1.788	334.235	0.469	
94	27 Arietis	1755	5	2	17	21.658	+ 329.764	+ 1.622	+ 329.500	+ 0.264	
		1850	42	2	22	35.669	331.317	1.649	331.058	0.259	
95	29 Arietis	1755	5	2	19	31.426	+ 325.817	+ 1.477	+ 325.978	— 0.161	
		1850	9	2	24	41.622	327.232	1.501	327.392	0.160	
96	B. A. C. 782	1755	5	2	19	57.545	+ 332.102	+ 1.704	+ 331.601	+ 0.501	
		1850	7	2	25	13.814	333.732	1.729	333.232	0.500	
97	31 Arietis	1755	5	2	23	18.539	+ 324.624	+ 1.337	+ 322.742	+ 1.882	
		1850	17	2	28	27.538	325.905	1.360	324.024	1.881	
98	ν Arietis	1755	5	2	24	57.378	+ 337.095	+ 1.894	+ 337.172	— 0.077	
		1850	25	2	30	18.476	338.905	1.917	338.984	0.079	
99	μ Arietis	1755	5	2	28	36.095	+ 334.908	+ 1.771	+ 334.735	+ 0.173	
		1850	21	2	33	55.060	336.601	1.793	336.425	0.176	
100	B. A. C. 826	1755	1	2	28	51.425	+ 320.418	+ 1.243	+ 320.510	— 0.092	
		1850	6	2	33	56.385	321.606	1.260	321.698	0.092	
101	85 Ceti	1755	5	2	29	19.681	+ 320.570	+ 1.248	+ 320.837	— 0.267	
		1850	7	2	34	24.785	321.766	1.269	322.035	0.269	
102	γ Ceti	1755	5	2	30	37.916	+ 309.099	+ 0.898	+ 310.122	— 1.023	—0.006
		1850	495	2	35	31.968	309.961	0.916	310.992	1.031	
		1900	. .	2	38	7.064	310.422	0.929	311.459	1.037	
103	36 Arietis	1755	5	2	30	41.235	+ 331.913	+ 1.630	+ 331.540	+ 0.373	
		1850	3	2	35	57.290	333.470	1.649	333.102	0.368	
104	ο Arietis	1755	5	2	31	5.377	+ 327.767	+ 1.489	+ 327.819	— 0.052	
		1850	11	2	36	17.430	329.190	1.508	329.243	0.053	
105	38 Arietis	1755	5	2	31	38.836	+ 324.388	+ 1.334	+ 323.568	+ 0.820	
		1850	17	2	36	47.607	325.661	1.346	324.848	0.813	
106	μ Ceti	1755	5	2	31	44.073	+ 321.842	+ 1.225	+ 320.116	+ 1.726	
		1850	52	2	36	50.378	323.013	1.241	321.288	1.725	
107	40 Arietis	1755	5	2	34	50.749	+ 333.208	+ 1.670	+ 332.967	+ 0.241	
		1850	11	2	40	8.051	334.800	1.682	334.552	0.248	

DECLINATIONS.

No.	Star.	Mag.	Epoch.	Declination.	Centennial variation.	Secular variation.	Struve's precession.	Proper motion.	Sec. var. of proper motion.
				° ′ ″	″	″	″	″	″
89	ξ Arietis	6.0	1755	+ 9 29 16.62	+ 1682.27	− 26.08	+ 1684.11	− 1.84	
		5.3	1850	9 55 42.85	1657.06	27.01	1658.91	1.85	
90	B. A. C. 755 . .	6.0	1755	+ 9 26 57.59	+ 1672.83	− 26.40	+ 1674.83	− 2.00	
		7.0	1850	9 53 14.72	1647.30	27.35	1649.22	1.92	
91	25 Arietis	7.5	1755	+ 9 5 52.62	+ 1648.70	− 26.14	+ 1671.27	−22.57	
		7.3	1850	9 31 46.95	1623.43	27.06	1645.83	22.40	
92	ξ² Ceti	5.0	1755	+ 7 20 55.12	+ 1666.02	− 26.44	+ 1667.50	− 1.48	
		4.4	1850	7 47 5.77	1640.43	27.44	1641.97	1.54	
			1900	8 0 42.53	1626.58	27.96	1628.19	1.61	
93	26 Arietis	6.5	1755	+ 18 45 10.77	+ 1655.58	− 28.08	+ 1658.76	− 3.18	
		6.0	1850	19 11 10.74	1628.40	29.15	1631.62	3.22	
94	27 Arietis	6.0	1755	+ 16 36 22.99	+ 1647.16	− 27.87	+ 1656.74	− 9.58	
		6.3	1850	17 2 15.07	1620.20	28.89	1629.80	9.60	
95	29 Arietis	6.5	1755	+ 13 56 9.59	+ 1648.89	− 27.88	+ 1645.98	+ 2.91	
		6.3	1850	14 22 3.31	1621.95	28.85	1619.04	2.91	
96	B. A. C. 782 . .	6.5	1755	+ 17 47 6.66	+ 1644.61	− 28.53	+ 1643.80	+ 0.81	
		7.0	1850	18 12 56.01	1617.01	29.57	1616.23	0.78	
97	31 Arietis	6.0	1755	+ 11 22 14.26	+ 1618.88	− 28.58	+ 1626.81	− 7.93	
		5.7	1850	11 47 39.15	1591.27	29.54	1599.36	8.09	
98	ν Arietis	5.5	1755	+ 20 53 12.36	+ 1617.17	− 29.76	+ 1618.32	− 1.15	
		5.7	1850	21 18 35.08	1588.39	30.82	1589.55	1.16	
99	μ Arietis	6.0	1755	+ 18 57 8.34	+ 1593.87	− 30.26	+ 1599.26	− 5.39	
		6.0	1850	19 22 8.72	1564.63	31.30	1570.09	5.46	
100	B. A. C. 826 . .	7.0	1755	+ 9 29 3.98	+ 1593.97	− 28.95	+ 1597.89	− 3.92	
		7.1	1850	9 54 5.04	1566.01	29.91	1569.95	3.94	
101	85 Ceti	6.0	1755	+ 9 40 58.18	+ 1591.46	− 29.05	+ 1595.41	− 3.95	
		6.0	1850	10 5 56.87	1563.45	29.91	1567.37	3.92	
102	γ Ceti	3.0	1755	+ 2 11 21.55	+ 1572.62	− 28.17	+ 1588.46	−15.84	+ 0.08
		3.2	1850	2 36 2.70	1545.47	28.98	1561.24	15.77	
			1900	2 48 51.80	1530.87	29.40	1546.60	15.73	
103	36 Arietis	7.0	1755	+ 16 42 41.95	+ 1584 41	− 30.31	+ 1588.15	− 3.74	
		6.5	1850	17 7 33.31	1555.14	31.31	1558.92	3.78	
104	ο Arietis	6.5	1755	+ 14 15 34.78	+ 1582.89	− 29.96	+ 1586.00	− 3.11	
		6.0	1850	14 40 24.85	1553.96	30.94	1557.07	3.11	
105	38 Arietis	5.5	1755	+ 11 23 58.27	+ 1575.22	− 29.90	+ 1583.04	− 7.82	
		5.0	1850	11 48 41.09	1546.39	30.80	1554.30	7.91	
106	μ Ceti	4.0	1755	+ 9 3 52.84	+ 1579.62	− 29.75	+ 1582.56	− 2.94	
		4.3	1850	9 28 39.98	1551.10	30.32	1554.04	2.94	
107	40 Arietis	6.0	1755	+ 17 14 50.54	+ 1561.96	− 31.14	+ 1565.65	− 3.69	
		6.3	1850	17 39 20.22	1531.97	32.00	1535.66	3.69	

RIGHT ASCENSIONS.

No.	Star.	Epoch.	Number of observations.	Right ascension.	Centennial variation.	Secular variation.	Struve's precession.	Proper motion.	Sec. var. of proper motion.
				h. m. s.	s.	s.	s.	s.	s.
108	π Arietis	1755	5	2 35 39.856	+ 331.772	+ 1.609	+ 331.774	− 0.002	
		1850	43	2 40 55.768	333.309	1.627	333.313	0.004	
109	σ Arietis	1755	5	2 38 0.401	+ 328.450	+ 1.475	+ 328.301	+ 0.149	
		1850	108	2 43 13.097	329.859	1.491	329.710	0.149	
110	ρ^1 Arietis	1755	5	2 41 13.977	+ 333.211	+ 1.619	+ 332.993	+ 0.218	
		1850	3	2 46 31.260	334.755	1.632	334.544	0.211	
111	ρ^2 Arietis	1755	5	2 42 5.265	+ 333.954	+ 1.650	+ 334.103	− 0.149	
		1850	15	2 47 23.268	335.528	1.665	335.681	0.153	
112	ρ^3 Arietis	1755	5	2 42 39.036	+ 335.604	+ 1.610	+ 333.694	+ 1.910	
		1850	39	2 47 58.591	337.150	1.644	335.249	1.901	
113	47 Arietis	1755	4	2 44 6.923	+ 339.907	+ 1.794	+ 338.383	+ 1.524	
		1850	14	2 49 30.646	341.617	1.806	340.092	1.525	
114	B. A. C. 920	1755	2	2 44 53.42	+ 340.45	+ 1.852	+ 340.165	+ 0.285	
		1850	4	2 50 17.67	342.19	1.867	341.928	0.262	
115	ε Arietis	1755	5	2 45 15.152	+ 339.632	+ 1.834	+ 339.740	− 0.108	
		1850	71	2 50 38.630	341.378	1.843	341.482	0.104	
116	50 Arietis	1755	3	2 46 47.920	+ 334.036	+ 1.624	+ 334.229	− 0.193	
		1850	3	2 52 5.989	335.584	1.635	335.782	0.198	
117	α Ceti	1755	10	2 49 30.009	+ 311.745	+ 0.952	+ 311.901	− 0.156	− 0.004
		1850	. .	2 54 26.599	312.657	0.965	312.815	0.158	
		1900	. .	2 57 3.048	313.141	0.972	313.301	0.160	
118	52 Arietis	1755	5	2 51 8.024	+ 347.853	+ 2.070	+ 347.989	− 0.136	
		1850	3	2 56 39.419	349.822	2.076	349.960	0.138	
119	Lal. 5725	1850	. .	2 58 9.9	. . . .	+ 1.372	+ 328.155	. . .	
120	53 Arietis	1755	5	2 53 40.810	+ 334.643	+ 1.650	+ 334.961	− 0.318	
		1850	21	2 58 59.461	336.193	1.613	336.490	0.297	
121	54 Arietis	1755	5	2 54 30.821	+ 336.713	+ 1.655	+ 336.695	+ 0.018	
		1850	5	2 59 51.447	338.289	1.662	338.271	0.018	
122	48 Cephei	1755	. .	2 50 14.26	+ 694.24	+33.57	+ 692.59	+ 1.65	
		1775	. .	2 52 33.79	700.99	33.88	699.34	1.65	
		1800	. .	2 55 30.10	709.51	34.25	707.86	1.65	
		1825	. .	2 58 28.55	718.12	34.60	716.48	1.64	
		1850	. .	3 1 29.16	726.81	34.94	725.17	1.64	
		1875	. .	3 4 31.97	735.59	35.25	733.95	1.64	
		1900	. .	3 7 36.97	+ 744.43	+35.55	+ 742.80	+ 1.63	
123	δ Arietis	1755	5	2 57 40.006	+ 339.816	+ 1.710	+ 338.832	+ 0.984	
		1850	233	3 3 3.603	341.442	1.714	340.455	0.987	
		1900	. .	3 5 54.538	342.299	1.715	341.309	0.990	
124	ζ Arietis	1755	5	3 0 51.999	+ 341.559	+ 1.766	+ 341.775	− 0.216	− 0.003
		1850	114	3 6 17.278	343.238	1.764	343.454	0.216	
		1900	. .	3 9 9.117	344.120	1.764	344.339	− 0.219	

DECLINATIONS.

No.	Star.	Mag.	Epoch.	Declination.	Centennial variation.	Secular variation.	Struve's precession.	Proper motion.	Sec. var. of proper motion.
				° ′ ″	″	″	″	″	″
108	π Arietis	5.0	1755	+ 16 25 45.62	+ 1560.34	− 31.07	+ 1561.16	− 0.82	
		5.7	1850	16 50 13.77	1530.35	32.06	1531.17	0.82	
109	σ Arietis	6.0	1755	+ 14 3 26.06	+ 1544.22	− 31.19	+ 1548.19	− 3.97	
		5.7	1850	14 27 38.85	1514.14	32.13	1518.16	4.02	
110	ρ^1 Arietis	7.5	1755	+ 16 43 20.87	+ 1527.05	− 32.12	+ 1530.07	− 3.02	
		7.0	1850	17 7 16.92	1496.07	33.11	1499.10	3.03	
111	ρ^2 Arietis	6.0	1755	+ 17 19 20.80	+ 1523.75	− 32.30	+ 1525.22	− 1.47	
		6.0	1850	17 43 13.63	1492.60	33.28	1494.06	1.46	
112	ρ^3 Arietis	6.0	1755	+ 17 1 44.83	+ 1502.80	− 32.81	+ 1522.00	−19.20	
		6.0	1850	17 25 17.54	1471.15	33.81	1490.62	19.47	
113	47 Arietis	6.0	1755	+ 19 40 8.78	+ 1510.95	− 33.42	+ 1513.62	− 2.67	
		6.0	1850	20 3 48.95	1478.74	34.40	1481.61	2.87	
114	B. A. C. 920 . .	7.0	1755	. . . . .	. . . .	. . .	+ 1509.21	. . .	
		7.0	1850	+ 21 0 53.9	. . . .	. . .	1476.96	. . .	
115	ε Arietis	5.0	1755	+ 20 20 36.47	+ 1507.06	− 33.45	+ 1507.07	− 0.01	
		4.3	1850	20 44 12.95	1474.80	34.47	1474.91	0.11	
116	50 Arietis	7.5	1755	+ 17 0 54.89	+ 1495.91	− 33.04	+ 1498.10	− 2.19	
		6.8	1850	17 24 20.94	1464.05	34.02	1466.17	2.12	
117	α Ceti	2.5	1755	+ 3 6 47.30	+ 1473.65	− 31.28	+ 1482.25	− 8.60	+ 0.01
		2.4	1850	3 29 53.00	1443.56	32.06	1452.15	8.59	
			1900	3 41 50.76	1427.44	32.46	1436.03	8.59	
118	52 Arietis	6.5	1755	+ 24 17 2.66	+ 1470.29	− 35.09	+ 1472.59	− 2.30	
		5.7	1850	24 40 3.44	1436.45	36.16	1438.73	2.28	
119	Lal. 5725 . . .	6.0	1850	+ 12 36 28.8	. . . .	− 34.19	+ 1429.51	. . .	
120	53 Arietis	6.0	1755	+ 16 55 1.43	+ 1458.00	− 34.17	+ 1457.40	+ 0.60	
		6.3	1850	17 17 50.97	1425.08	35.12	1424.43	0.65	
121	54 Arietis	6.5	1755	+ 17 50 14.33	+ 1450.74	− 34.52	+ 1452.33	− 1.59	
		6.3	1850	18 12 56.82	1417.52	35.41	1419.08	1.56	
122	48 Cephei	6.0	1755	+ 76 47 43.66	+ 1472.88	− 69.34	+ 1477.88	− 5.00	
			1775	76 52 36.85	1458.93	70.75	1463.96	5.03	
			1800	76 58 39.34	1441.00	72.57	1446.07	5.07	
			1825	77 4 37.31	1422.63	74.37	1427.74	5.11	
		6.3	1850	77 10 30.63	1403.81	76.27	1408.96	5.15	
			1875	77 16 19.19	1384.50	78.19	1389.69	5.19	
			1900	+ 77 22 2.86	+ 1364.74	− 80.08	+ 1370.00	− 5.24	
123	δ Arietis	4.0	1755	+ 18 46 54.86	+ 1433.19	− 35.51	+ 1433.13	+ 0.06	
		4.0	1850	19 9 20.22	1398.99	36.50	1399.13	− 0.14	
			1900	19 20 55.13	1380.61	37.00	1380.87	0.26	
124	ζ Arietis	5.0	1755	+ 20 7 7.11	+ 1404.93	− 35.96	+ 1413.37	− 8.44	+ 0.02
		4.7	1850	20 29 5.43	1370.33	36.91	1378.76	8.43	
			1900	20 40 25.97	1351.75	37.41	1360.16	8.41	

RIGHT ASCENSIONS.

No.	Star.	Epoch.	Number of observations.	Right ascension.	Centennial variation.	Secular variation.	Struve's precession.	Proper motion.	Sec. var. of proper motion.
				h. m. s.	s.	s.	s.	s.	s.
125	60 Arietis	1755	5	3 5 57.568	+ 351.809	+ 2.033	+ 351.743	+ 0.066	
		1850	6	3 11 32.702	353.738	2.028	353.673	0.065	
126	τ^1 Arietis	1755	1	3 7 7.646	+ 343.235	+ 1.751	+ 342.954	+ 0.281	
		1850	45	3 12 34.508	344.896	1.746	344.614	0.282	
127	α Persei	1755	8	3 6 57.864	+ 419.245	+ 4.857	+ 418.930	+ 0.315	—0.005
		1850	295	3 13 38.335	423.848	4.833	423.537	0.311	
		1900	. .	3 17 10.862	426.261	4.819	425.952	0.309	
128	τ^2 Arietis	1755	5	3 8 42.069	+ 342.050	+ 1.722	+ 342.408	— 0.358	
		1850	9	3 14 7.793	343.686	1.722	344.044	0.358	
129	64 Arietis	1755	5	3 9 53.618	+ 350.586	+ 1.957	+ 350.552	+ 0.034	
		1850	24	3 15 27.556	352.443	1.952	352.411	0.032	
130	65 Arietis	1755	5	3 10 21.271	+ 342.759	+ 1.721	+ 342.778	— 0.019	
		1850	13	3 15 47.667	344.390	1.714	344.411	0.021	
131	B. A. C. 1055 . .	1755	1	3 10 22.605	+ 345.693	+ 1.797	+ 345.208	+ 0.485	
		1850	6	3 15 51.823	347.398	1.792	346.921	0.477	
132	66 Arietis	1755	5	3 14 10.132	+ 347.370	+ 1.810	+ 347.379	— 0.009	
		1850	8	3 19 40.949	349.085	1.801	349.101	0.016	
133	*s* Tauri . . .	1755	5	3 17 3.022	+ 325.606	+ 1.227	+ 325.740	— 0.134	
		1850	11	3 22 12.902	326.771	1.226	326.906	0.135	
134	*f* Tauri	1755	5	3 17 22.971	+ 328.748	+ 1.300	+ 328.712	+ 0.036	
		1850	113	3 22 35.868	329.983	1.300	329.946	0.036	
135	7 Tauri	1755	5	3 19 58.979	+ 351.871	+ 1.891	+ 351.811	+ 0.060	
		1850	13	3 25 34.108	353.661	1.878	353.603	0.058	
136	ε Eridani . . .	1755	5	3 21 24.167	+ 281.627	+ 0.540	+ 288.275	— 6.648	
		1850	115	3 25 51.958	282.145	0.550	288.790	6.645	
		1900	. .	3 28 13.099	282.421	0.555	289.065	6.644	
137	9 Tauri	1755	5	3 22 36.464	+ 349.501	+ 1.767	+ 349.594	— 0.093	
		1850	36	3 28 9.290	351.190	1.788	351.298	0.108	
138	B. A. C. 1119 . .	1850	10	3 30 57.153	+ 338.108	+ 1.423	+ 337.781	+ 0.327	
139	11 Tauri	1755	5	3 26 11.362	+ 354.811	+ 1.911	+ 354.771	+ 0.040	
		1850	61	3 31 49.291	356.617	1.892	356.581	0.036	
140	δ Persei	1755	5	3 25 35.668	+ 419.380	+ 4.217	+ 419.019	+ 0.361	—0.006
		1850	128	3 32 15.974	423.361	4.165	423.012	0.349	
		1900	. .	3 35 48.174	425.435	4.132	425.089	0.346	
141	13 Tauri	1755	5	3 28 13.677	+ 343.105	+ 1.578	+ 343.105	0.000	
		1850	13	3 33 40.338	344.603	1.569	344.603	0.000	
142	14 Tauri	1755	5	3 29 39.727	+ 344.039	+ 1.570	+ 343.226	+ 0.813	
		1850	5	3 35 7.270	345.524	1.556	344.714	0.810	
143	B. A. C. 1143 . .	1850	7	3 35 45.078	+ 347.297	+ 1.618	+ 347.480	— 0.183	
144	*g* Pleiadum . . .	1755	4	3 30 17.299	+ 353.268	+ 1.822	+ 353.181	+ 0.087	
		1850	19	3 35 53.722	354.988	1.801	354.900	0.088	

DECLINATIONS.

No.	Star.	Mag.	Epoch.	Declination.	Centennial variation.	Secular variation.	Struve's precession.	Proper motion.	Sec. var. of proper motion.
				° ′ ″	″	″	″	″	″
125	60 Arietis	7.5	1755	+ 24 45 40.07	+ 1371.63	− 37.82	+ 1381.41	− 9.78	
		6.3	1850	25 7 5.89	1335.20	38.86	1345.11	9.91	
126	τ^1 Arietis	6.0	1755	+ 20 14 45.16	+ 1369.61	− 37.13	+ 1373.97	− 4.36	
		5.0	1850	20 36 9.39	1333.88	38.10	1338.29	4.41	
127	α Persei	2.5	1755	+ 48 57 58.91	+ 1371.69	− 45.15	+ 1375.01	− 3.32	− 0.02
		2.0	1850	49 19 20.89	1328.01	46.81	1331.34	3.33	
			1900	49 30 19.01	1304.39	47.69	1307.73	3.34	
128	τ^2 Arietis	7.0	1755	+ 19 50 48.35	+ 1362.20	− 37.20	+ 1363.91	− 1.71	
		5.3	1850	20 12 5.48	1326.38	38.20	1328.12	1.74	
129	64 Arietis	5.5	1755	+ 23 50 13.65	+ 1350.68	− 38.28	+ 1356.32	− 5.64	
		5.7	1850	24 11 19.37	1313.83	39.29	1319.40	5.57	
130	65 Arietis	6.0	1755	+ 19 54 53.88	+ 1352.59	− 37.52	+ 1353.22	− 0.63	
		6.0	1850	20 16 1.76	1316.53	38.41	1317.17	0.64	
131	B. A. C. 1055 . .	7.5	1755	. . . . . .	. . . .	− 37.87	+ 1353.10	. . .	
		6.8	1850	+ 21 30 23.0	. . . .	38.85	1316.69	. . .	
132	66 Arietis	6.5	1755	+ 21 56 26.35	+ 1316.03	− 38.55	+ 1328.40	− 12.37	
		6.0	1850	22 16 59.04	1278.95	39.52	1291.33	12.38	
133	s Tauri	6.0	1755	+ 10 28 39.78	+ 1306.99	− 36.54	+ 1309.40	− 2.41	
		5.0	1850	10 49 4.81	1271.90	37.34	1274.28	2.38	
134	*f* Tauri	5.5	1755	+ 12 4 42.32	+ 1307.58	− 37.07	+ 1307.21	+ 0.37	
		4.0	1850	12 25 7.65	1271.97	37.90	1271.69	0.28	
135	7 Tauri	6.0	1755	+ 23 37 22.24	+ 1285.57	− 39.88	+ 1289.84	− 4.27	
		6.0	1850	23 57 25.38	1247.22	40.86	1251.49	4.27	
136	ε Eridani	4.0	1755	− 10 18 13.12	+ 1282.31	− 31.56	+ 1280.30	+ 2.01	
		3.6	1850	9 58 9.25	1252.07	32.10	1249.43	2.64	
			1900	9 47 47.24	1235.95	32.36	1233.02	2.93	
137	9 Tauri	6.0	1755	+ 22 21 53.65	+ 1266.79	− 39.94	+ 1272.15	− 5.36	
		7.0	1850	22 42 38.93	1228.37	40.94	1233.72	5.35	
138	B. A. C. 1119 . .	6.0	1850	+ 16 2 41.21	+ 1209.52	− 39.82	+ 1214.34	− 4.82	
139	11 Tauri	6.0	1755	+ 24 30 59.33	+ 1245.66	− 41.08	+ 1247.75	− 2.09	
		6.7	1850	24 50 24.02	1206.18	42.05	1208.28	2.10	
140	δ Persei	3.5	1755	+ 46 58 46.37	+ 1248.43	− 48.34	+ 1252.81	− 4.38	
		3.3	1850	47 18 9.86	1200.79	49.85	1205.17	4.38	
			1900	47 28 3.99	1175.67	50.63	1180.05	4.38	
141	13 Tauri	6.5	1755	+ 18 53 44.74	+ 1231.43	− 40.03	+ 1233.70	− 2.27	
		5.7	1850	19 12 56.38	1192.95	40.97	1195.31	2.36	
142	14 Tauri	7.0	1755	+ 18 52 11.26	+ 1219.68	− 40.39	+ 1223.77	− 4.09	
		6.3	1850	19 11 11.60	1180.89	41.27	1185.10	4.21	
143	B. A. C. 1143 . .	6.0	1850	+ 20 27 2.63	+ 1180.53	− 41.44	+ 1180.67	− 0.14	
144	*g* Pleiadum . . .	5.5	1755	+ 23 29 53.71	+ 1213.71	− 41.58	+ 1219.47	− 5.76	
		6.3	1850	23 48 47.83	1173.76	42.52	1179.63	5.87	

RIGHT ASCENSIONS.

No.	Star.	Epoch.	Number of observations.	Right ascension.	Centennial variation.	Secular variation.	Struve's precession.	Proper motion.	Sec. var. of proper motion.
				h. m. s.	s.	s.	s.	s.	s.
145	17 Tauri	1755	5	3 30 22.533	+ 352.896	+ 1.808	+ 352.805	+ 0.091	
		1850	68	3 35 58.598	354.606	1.791	354.518	0.088	
146	18 Tauri	1755	1	3 30 35.668	+ 354.545	+ 1.845	+ 354.454	+ 0.091	
		1850	15	3 36 13.315	356.286	1.820	356.198	0.088	
147	19 Tauri	1755	5	3 30 40.402	+ 353.721	+ 1.833	+ 353.635	+ 0.086	
		1850	22	3 36 17.261	355.452	1.811	355.364	0.088	
148	20 Tauri	1755	5	3 31 17.817	+ 353.608	+ 1.820	+ 353.520	+ 0.088	
		1850	20	3 36 54.563	355.327	1.798	355.239	0.088	
149	21 Tauri	1755	3	3 31 21.659	+ 354.039	+ 1.826	+ 353.943	+ 0.096	
		1850	13	3 36 58.817	355.763	1.805	355.675	0.088	
150	22 Tauri	1755	2	3 31 30.251	+ 354.003	+ 1.824	+ 353.909	+ 0.094	
		1850	9	3 37 7.372	355.725	1.803	355.637	0.088	
151	23 Tauri	1755	5	3 31 49.954	+ 352.764	+ 1.795	+ 352.676	+ 0.088	
		1850	13	3 37 25.887	354.459	1.773	354.371	0.088	
152	24 Tauri	1755	3	3 32 50.142	+ 353.306	+ 1.791	+ 353.213	+ 0.093	
		1850	33	3 38 26.586	354.996	1.769	354.908	0.088	
153	η Tauri	1755	10	3 32 58.132	+ 353.299	+ 1.795	+ 353.210	+ 0.089	−0.004
		1850	643	3 38 34.572	354.992	1.769	354.904	0.088	
		1900	. .	3 41 32.288	355.873	1.756	355.788	0.085	
154	B. A. C. 1170 . .	1755	2	3 33 53.511	+ 351.848	+ 1.745	+ 351.845	+ 0.003	
		1850	3	3 39 28.550	353.495	1.724	353.495	0.000	
155	B. A. C. 1171 . .	1755	1	3 33 57.516	+ 353.918	+ 1.794	+ 353.907	+ 0.011	
		1850	10	3 39 34.545	355.612	1.773	355.608	0.004	
156	26 Tauri	1755	3	3 34 26.656	+ 352.989	+ 1.765	+ 352.894	+ 0.095	
		1850	9	3 40 2.789	354.655	1.744	354.567	0.088	
157	27 Tauri	1755	5	3 34 38.477	+ 353.453	+ 1.780	+ 353.365	+ 0.088	
		1850	134	3 40 15.057	355.132	1.754	355.044	0.088	
158	28 Tauri	1755	5	3 34 39.460	+ 353.645	+ 1.782	+ 353.554	+ 0.091	
		1850	40	3 40 16.223	355.326	1.757	355.238	0.088	
159	B. A. C. 1189 . .	1850	. .	3 41 6.4	. . . .	+ 1.650	+ 351.097	. . .	
160	B. A. C. 1192 . .	1850	10	3 41 18.419	+ 358.527	+ 1.819	+ 358.737	− 0.210	
161	Lal. 7110 . . .	1850	. .	3 42 55.4	. . . .	+ 1.220	+ 331.858	. . .	
162	B. A. C. 1206 . .	1850	7	3 44 35.797	+ 342.053	+ 1.395	+ 340.865	+ 1.188	
163	ζ Persei	1755	5	3 38 47.435	+ 373.017	+ 2.264	+ 372.942	+ 0.075	−0.002
		1850	92	3 44 42.817	375.149	2.224	375.072	0.077	
		1900	. .	3 47 50.668	376.255	2.202	376.181	0.074	
164	32 Tauri	1850	14	3 48 0.759	+ 352.880	+ 1.604	+ 352.582	+ 0.298	
165	33 Tauri	1755	1	3 42 34.470	+ 353.051	+ 1.683	+ 352.610	+ 0.441	
		1850	19	3 48 10.625	354.638	1.657	354.195	0.443	

DECLINATIONS.

No.	Star.	Mag.	Epoch.	Declination.	Centennial variation.	Secular variation.	Struve's precession.	Proper motion.	Sec. var. of proper motion.
				° ′ ″	″	″	″	″	″
145	17 Tauri	4.5	1755	+ 23 19 21.06	+ 1213.03	− 41.49	+ 1218.86	− 5.83	
		4.3	1850	23 38 14.57	1173.16	42.44	1179.03	5.87	
146	18 Tauri	7.0	1755	+ 24 2 58.78	+ 1211.46	− 41.65	+ 1217.31	− 5.85	
		6.3	1850	24 21 50.73	1171.44	42.60	1177.31	5.87	
147	19 Tauri	5.0	1755	+ 23 40 40.61	+ 1210.97	− 41.63	+ 1216.78	− 5.81	
		5.0	1850	23 59 32.10	1170.97	42.58	1176.84	5.87	
148	20 Tauri	5.0	1755	+ 23 34 53.39	+ 1206.63	− 41.70	+ 1212.41	− 5.78	
		5.0	1850	23 53 40.73	1166.56	42.65	1172.43	5.87	
149	21 Tauri	7.5	1755	+ 23 46 7.23	+ 1206.07	− 41.69	+ 1211.98	− 5.91	
		7.0	1850	24 4 54.04	1166.02	42.63	1171.89	5.87	
150	22 Tauri	7.5	1755	+ 23 44 32.81	+ 1205.12	− 41.71	+ 1210.98	− 5.86	
		7.0	1850	24 3 18.72	1165.05	42.65	1170.92	5.87	
151	23 Tauri	5.0	1755	+ 23 9 52.52	+ 1202.94	− 41.72	+ 1208.72	− 5.78	
		4.7	1850	23 28 36.35	1162.86	42.66	1168.73	5.87	
152	24 Tauri	7.5	1755	+ 23 20 14.82	+ 1195.82	− 41.80	+ 1201.63	− 5.81	
		8.0	1850	23 38 51.85	1155.67	42.73	1161.54	5.87	
153	η Tauri	3.0	1755	+ 23 19 37.09	+ 1194.90	− 41.85	+ 1200.76	− 5.86	− 0.01
		3.0	1850	23 38 13.22	1154.71	42.75	1160.58	5.87	
			1900	23 47 45.21	1133.22	43.21	1139.10	5.88	
154	B. A. C. 1170 . .	7.0	1755	+ 22 38 49.93	+ 1189.49	− 41.76	+ 1194.24	− 4.75	
		6.3	1850	22 57 20.97	1149.39	42.67	1154.16	4.77	
155	B. A. C. 1171 . .	7.5	1755	+ 23 34 21.75	+ 1186.77	− 42.01	+ 1193.79	− 7.02	
		7.8	1850	23 52 50.08	1146.42	42.94	1153.43	7.01	
156	26 Tauri	7.5	1755	+ 23 5 8.98	+ 1184.51	− 41.97	+ 1190.38	− 5.87	
		7.0	1850	23 23 35.18	1144.20	42.89	1150.07	5.87	
157	27 Tauri	5.0	1755	+ 23 17 0.62	+ 1183.16	− 42.11	+ 1188.97	− 5.81	
		4.0	1850	23 35 25.48	1142.73	43.02	1148.60	5.87	
158	28 Tauri	5.5	1755	+ 23 22 1.01	+ 1183.04	− 42.15	+ 1188.86	− 5.82	
		6.2	1850	23 40 25.73	1142.56	43.08	1148.43	5.87	
159	B. A. C. 1189 . .	6.0	1850	+ 21 47 2.2	. . . .	− 42.66	+ 1142.45	. . .	
160	B. A. C. 1192 . .	6.0	1850	+ 25 7 19.70	+ 1125.70	− 43.48	+ 1140.98	−15.28	
161	Lal. 7110 . . .	6.0	1850	+ 12 35 28.4	. . . .	− 40.49	+ 1129.34	. . .	
162	B. A. C. 1206 . .	6.0	1850	+ 16 52 34.51	+ 1116.18	− 42.06	+ 1117.23	− 1.05	
163	ζ Persei	3.5	1755	+ 31 8 2.55	+ 1156.40	− 44.94	+ 1159.52	− 3.12	− 0.01
		3.0	1850	31 26 0.69	1113.23	45.95	1116.36	3.13	
			1900	31 35 11.54	1090.13	46.47	1093.26	3.13	
164	32 Tauri	6.0	1850	+ 22 2 28.99	+ 1081.13	− 43.67	+ 1092.26	−11.13	
165	33 Tauri	5.5	1755	+ 22 26 32.93	+ 1130.48	− 43.12	+ 1132.32	− 1.84	
		6.3	1850	22 44 7.30	1089.10	44.02	1091.06	1.96	

RIGHT ASCENSIONS.

No.	Star.	Epoch.	Number of observations.	Right ascension.			Centennial variation.	Secular variation.	Struve's precession.	Proper motion.	Sec. var. of proper motion.
				h.	m.	s.	s.	s.	s.	s.	s.
166	γ^1 Eridani	1755	5	3	46	36.483	+ 279.238	+ 0.459	+ 278.674	+ 0.564	—0.005
		1850	397	3	51	1.965	279.672	0.454	279.109	0.563	
		1900	. .	3	53	21.858	279.900	0.456	279.341	0.559	
167	B. A. C. 1238	1755	5	3	46	26.485	+ 353.210	+ 1.649	+ 353.233	— 0.023	
		1850	3	3	52	2.774	354.762	1.620	354.790	0.028	
168	B. A. C. 1240	1850	11	3	52	10.676	+ 344.498	+ 1.383	+ 343.528	+ 0.970	
169	λ Tauri	1755	5	3	47	8.259	+ 330.268	+ 1.167	+ 330.330	— 0.062	
		1850	57	3	52	22.537	331.369	1.151	331.433	0.064	
170	B. A. C. 1242	1755	5	3	46	53.127	+ 346.627	+ 1.492	+ 346.576	+ 0.051	
		1850	15	3	52	23.092	348.032	1.466	347.986	0.046	
171	36 Tauri	1755	2	3	49	45.103	+ 355.807	+ 1.672	+ 355.805	+ 0.002	
		1850	11	3	55	23.869	357.381	1.642	357.382	— 0.001	
172	A[1] Tauri	1755	5	3	50	15.040	+ 351.913	+ 1.566	+ 351.219	+ 0.694	
		1850	86	3	55	50.059	353.386	1.534	352.691	0.695	
173	A[2] Tauri	1755	4	3	50	52.400	+ 352.409	+ 1.549	+ 351.147	+ 1.262	
		1850	19	3	56	27.882	353.865	1.518	352.608	1.257	
174	41 Tauri	1755	2	3	51	37.531	+ 364.728	+ 1.849	+ 364.531	+ 0.197	
		1850	5	3	57	24.851	366.466	1.810	366.276	0.190	
175	ψ Tauri	1755	5	3	51	54.527	+ 367.535	+ 1.938	+ 368.102	— 0.567	
		1850	15	3	57	44.553	369.355	1.895	369.927	0.572	
176	B. A. C. 1272	1755	. .	3	53	42.400	+ 341.311	+ 1.325	+ 341.134	+ 0.177	
		1850	13	3	59	24.336	342.560	1.305	342.382	0.178	
177	ω^1 Tauri	1755	5	3	54	55.849	+ 346.913	+ 1.415	+ 346.231	+ 0.682	
		1850	87	4	0	26.050	348.244	1.387	347.564	0.680	
178	p Tauri	1755	5	3	55	57.369	+ 362.201	+ 1.745	+ 362.464	— 0.263	
		1850	9	4	1	42.241	363.839	1.704	364.110	0.271	
179	48 Tauri	1755	5	4	1	53.483	+ 338.562	+ 1.199	+ 337.676	+ 0.886	
		1850	18	4	7	15.654	339.687	1.169	338.802	0.885	
180	ω^2 Tauri	1755	5	4	2	56.479	+ 349.012	+ 1.397	+ 349.361	— 0.349	
		1850	36	4	8	28.664	350.322	1.361	350.672	0.350	
181	51 Tauri	1755	5	4	3	55.488	+ 352.464	+ 1.445	+ 351.800	+ 0.664	
		1850	10	4	9	30.975	353.818	1.406	353.147	0.671	
182	53 Tauri	1755	4	4	5	1.793	+ 351.121	+ 1.409	+ 350.909	+ 0.212	
		1850	9	4	10	35.990	352.446	1.375	352.231	0.215	
183	56 Tauri	1755	4	4	5	8.688	+ 352.559	+ 1.440	+ 352.405	+ 0.154	
		1850	12	4	10	44.263	353.909	1.402	353.749	0.160	
184	ϕ Tauri	1755	5	4	5	19.714	+ 366.025	+ 1.700	+ 366.130	— 0.105	
		1850	10	4	11	8.197	367.616	1.650	367.731	0.115	
185	γ Tauri	1755	6	4	5	52.836	+ 339.341	+ 1.178	+ 338.520	+ 0.821	+0.001
		1850	216	4	11	15.738	340.449	1.154	339.628	0.821	
		1900	. .	4	14	6.105	341.023	1.141	340.202	0.821	

DECLINATIONS.

No.	Star.	Mag.	Epoch.	Declination.	Centennial variation.	Secular variation.	Struve's precession.	Proper motion.	Sec. var. of proper motion.
				° ′ ″	″	″	″	″	″
166	γ^1 Eridani	2.5	1755	− 14 13 20.73	+ 1091.40	− 34.52	+ 1102.98	−11.58	− 0.02
		2.8	1850	13 56 19.54	1058.41	34.96	1070.00	11.59	
			1900	13 47 34.72	1040.87	35.18	1052.47	11.60	
167	B. A. C. 1238 . .	7.5	1755	+ 22 29 20.26	+ 1101.73	− 43.49	+ 1104.21	− 2.48	
		6.3	1850	22 46 27.15	1060.01	44.35	1062.49	2.48	
168	B. A. C. 1240 . .	6.0	1850	+ 17 45 59.63	+ 1057.84	− 43.22	+ 1061.54	− 3.70	
169	λ Tauri	4.0	1755	+ 11 46 40.48	+ 1097.05	− 40.77	+ 1099.12	− 2.07	
		3.7	1850	12 3 44.16	1057.97	41.48	1060.04	2.07	
170	* B. A. C. 1242 . .	. .	1755	+ 19 29 27.42	+ 1095.05	− 42.74	+ 1100.96	− 5.91	
		6.3	1850	19 46 28.30	1054.06	43.56	1059.98	5.92	
171	36 Tauri	6.5	1755	+ 23 24 34.46	+ 1077.55	− 44.21	+ 1079.90	− 2.35	
		6.0	1850	23 41 18.05	1035.15	45.06	1037.52	2.37	
172	A[1] Tauri	5.0	1755	+ 21 23 28.06	+ 1068.24	− 43.87	+ 1076.19	− 7.95	
		4.7	1850	21 40 2.99	1026.19	44.70	1034.23	8.04	
173	A[2] Tauri	6.5	1755	+ 21 19 32.67	+ 1058.97	− 44.08	+ 1071.62	−12.65	
		6.3	1850	21 35 58.67	1016.70	44.92	1029.55	12.85	
174	41 Tauri	6.0	1755	+ 26 55 1.50	+ 1059.05	− 45.56	+ 1066.04	− 6.99	
		5.3	1850	27 11 26.87	1015.25	46.66	1022.34	7.09	
175	ψ Tauri	5.5	1755	+ 28 18 57.50	+ 1063.91	− 45.84	+ 1063.94	− 0.03	
		5.7	1850	28 35 27.38	1019.92	46.76	1019.85	+ 0.07	
176	B. A. C. 1272 . .	. .	1755	+ 16 39 49.81	+ 1046.34	− 43.00	+ 1048.48	− 2.14	
		6.0	1850	16 56 4.32	1005.12	43.78	1007.30	2.18	
177	ω^1 Tauri	6.0	1755	+ 18 56 23.51	+ 1037.56	− 43.79	+ 1041.42	− 3.86	
		6.0	1850	19 12 29.31	995.59	44.56	999.53	3.94	
178	p Tauri	6.5	1755	+ 25 49 8.58	+ 1028.85	− 45.70	+ 1033.73	− 4.88	
		6.0	1850	26 5 5.22	985.03	46.55	989.85	4.82	
179	48 Tauri	6.0	1755	+ 14 45 58.92	+ 985.40	− 43.54	+ 988.85	− 3.45	
		6.0	1850	15 1 15.31	943.74	44.16	947.30	3.56	
180	ω^2 Tauri	5.6	1755	+ 19 57 9.84	+ 976.15	− 44.91	+ 980.87	− 4.72	
		5.7	1850	20 12 16.81	933.15	45.60	937.90	4.75	
181	51 Tauri	7.0	1755	+ 20 57 27.84	+ 969.81	− 45.60	+ 973.35	− 3.54	
		6.0	1850	21 12 28.46	926.15	46.32	929.85	3.70	
182	53 Tauri	6.5	1755	+ 20 31 37.91	+ 959.62	− 45.40	+ 964.82	− 5.20	
		6.0	1850	20 46 28.97	916.15	46.10	921.44	5.29	
183	56 Tauri	6.5	1755	+ 21 9 32.48	+ 958.88	− 45.63	+ 963.94	− 5.06	
		6.0	1850	21 24 22.70	915.15	46.43	920.36	5.21	
184	φ Tauri	6.0	1755	+ 26 44 28.09	+ 954.18	− 47.25	+ 962.57	− 8.39	
		5.3	1850	26 59 13.11	908.90	48.07	917.27	8.37	
185	γ Tauri	3.5	1755	+ 15 0 51.27	+ 955.73	− 44.06	+ 958.35	− 2.62	− 0.11
		4.0	1850	15 15 39.23	913.58	44.69	916.30	2.72	
			1900	15 23 10.42	891.14	45.03	893.92	2.78	

RIGHT ASCENSIONS.

No.	Star.	Epoch.	Number of observations.	Right ascension.			Centennial variation.	Secular variation.	Struve's precession.	Proper motion.	Sec. var. of proper motion.
				h.	m.	s.	s.	s.	s.	s.	s.
186	55 Tauri	1755	2	4	5	55.215	+ 341.237	+ 1.212	+ 340.499	+ 0.738	
		1850	5	4	11	19.933	342.376	1.186	341.641	0.735	
187	*h* Tauri	1755	5	4	6	11.702	+ 335.757	+ 1.122	+ 335.063	+ 0.694	
		1850	8	4	11	31.172	336.811	1.096	336.120	0.691	
188	58 Tauri	1755	3	4	6	44.396	+ 338.230	+ 1.156	+ 337.423	+ 0.807	
		1850	12	4	12	6.231	339.314	1.126	338.507	0.807	
189	B. A. C. 1335 . .	1755	3	4	7	7.507	+ 335.478	+ 1.111	+ 334.766	+ 0.712	
		1850	6	4	12	26.710	336.521	1.085	335.811	0.710	
190	χ Tauri	1755	5	4	7	42.650	+ 362.424	+ 1.591	+ 362.105	+ 0.319	
		1850	18	4	13	27.664	363.915	1.547	363.598	0.317	
191	60 Tauri	1755	5	4	8	16.978	+ 336.001	+ 1.110	+ 335.318	+ 0.683	
		1850	3	4	13	36.675	337.042	1.083	336.363	0.679	
192	δ^1 Tauri	1755	5	4	8	50.204	+ 343.826	+ 1.237	+ 343.051	+ 0.775	
		1850	59	4	14	17.390	344.984	1.202	344.209	0.775	
193	P. A. C. 1347 . .	1755	2	4	8	44.427	+ 359.729	+ 1.530	+ 359.201	+ 0.528	
		1850	11	4	14	26.853	361.162	1.486	360.633	0.529	
194	63 Tauri	1755	5	4	9	23.511	+ 342.043	+ 1.206	+ 341.383	+ 0.660	
		1850	18	4	14	48.991	343.174	1.176	342.509	0.665	
195	62 Tauri	1755	4	4	9	15.689	+ 359.118	+ 1.514	+ 359.011	+ 0.107	
		1850	7	4	14	57.527	360.535	1.470	360.432	0.103	
196	δ^2 Tauri	1755	5	4	10	0.116	+ 343.753	+ 1.226	+ 342.940	+ 0.813	
		1850	26	4	15	27.228	344.902	1.192	344.085	0.817	
197	χ^1 Tauri	1755	6	4	10	48.289	+ 354.962	+ 1.409	+ 354.325	+ 0.637	
		1850	14	4	16	26.132	356.280	1.366	355.643	0.637	
198	χ^2 Tauri	1755	5	4	10	51.428	+ 354.968	+ 1.411	+ 354.105	+ 0.863	
		1850	10	4	16	29.277	356.288	1.368	355.420	0.868	
199	δ^3 Tauri	1755	5	4	11	20.772	+ 344.895	+ 1.236	+ 344.166	+ 0.729	
		1850	13	4	16	48.974	346.053	1.203	345.319	0.734	
200	70 Tauri	1755	5	4	11	40.046	+ 340.273	+ 1.156	+ 339.708	+ 0.565	
		1850	6	4	17	3.822	341.357	1.125	340 791	0.566	
201	υ^1 Tauri	1755	3	4	11	40.690	+ 356.775	+ 1.430	+ 355.681	+ 1.094	
		1850	45	4	17	20.266	358.113	1.388	357.019	1.094	
202	71 Tauri	1755	5	4	12	24.997	+ 339.740	+ 1.137	+ 339.052	+ 0.688	
		1850	8	4	17	48.258	340.805	1.106	340.117	0.688	
203	π Tauri	1755	4	4	12	47.490	+ 337.053	+ 1.102	+ 337.069	− 0.016	
		1850	3	4	18	8.183	338.086	1.073	338.104	0.018	
204	υ^2 Tauri	1755	5	4	12	40.570	+ 356.210	+ 1.430	+ 356.244	− 0.034	
		1850	9	4	18	19.608	357.548	1.387	357.578	0.030	
205	B. A. C. 1373 . .	1755	. .	4	13	30.143	+ 353.872	+ 1.294	+ 353.018	+ 0.854	
		1850	5	4	19	6.909	355.114	1.320	354.296	0.818	

DECLINATIONS.

No.	Star.	Mag.	Epoch.	Declination.	Centennial variation.	Secular variation.	Struve's precession.	Proper motion.	Sec. var. of proper motion.
				° ′ ″	″	″	″	″	″
186	55 Tauri	7.5	1755	+ 15 54 37.69	+ 953 08	− 44.24	+ 958.01	− 4.93	
		7.3	1850	16 9 23.05	910.72	44.92	915.74	5.02	
187	*h* Tauri	6.0	1755	+ 13 25 22.88	+ 952.75	− 43.56	+ 955.90	− 3.15	
		6.0	1850	13 40 8.24	911.05	44.22	914.28	3.23	
188	58 Tauri	6.0	1755	+ 14 29 10.95	+ 949.11	− 44.00	+ 951.68	− 2.57	
		6.3	1850	14 43 52.66	907.01	44.65	909.73	2.72	
189	B. A. C. 1335 . .	6.5	1755	+ 13 15 26.00	+ 946.11	− 43.62	+ 948.74	− 2.63	
		6.5	1850	13 30 5.02	904.36	44.27	907.05	2.69	
190	χ Tauri	6.0	1755	+ 25 1 42.08	+ 940.62	− 47.15	+ 944.20	− 3.58	
		5.7	1850	25 16 14.29	895.46	47.93	899.14	3.68	
191	60 Tauri	7.0	1755	+ 13 28 34.69	+ 936.29	− 43.80	+ 939.79	− 3.50	
		6.0	1850	13 43 4.30	894.38	44.44	897.96	3.58	
192	δ¹ Tauri	4.0	1755	+ 16 56 44.33	+ 932.62	− 44.91	+ 935.50	− 2.88	
		4.0	1850	17 11 9.96	889.63	45.60	892.64	3.01	
193	B. A. C. 1347 . .	. .	1755	. . . . . .	. . . .	− 46.89	+ 936.25	. . .	
		7.3	1850	+ 24 3 3.9	. . . .	47.69	891.40	. . .	
194	63 Tauri	6.0	1755	+ 16 11 0.00	+ 927.33	− 44.77	+ 931.20	− 3.87	
		6.0	1850	16 25 20.65	884.46	45.46	888.51	4.05	
195	62 Tauri	7.0	1755	+ 23 42 26.42	+ 929.40	− 46.81	+ 932.21	− 2.81	
		6.0	1850	23 56 48.11	884.57	47.56	887.40	2.83	
196	δ² Tauri	4.5	1755	+ 16 51 15.09	+ 921.96	− 45.06	+ 926.54	− 4.58	
		5.7	1850	17 5 30.53	878.87	45.67	883.51	4.64	
197	χ¹ Tauri	5.5	1755	+ 21 42 36.64	+ 915.05	− 46.43	+ 920.17	− 5.12	
		4.7	1850	21 56 44.93	870.61	47.14	875.80	5.19	
198	χ² Tauri	6.5	1755	+ 21 37 0.00	+ 914.33	− 46.56	+ 919.80	− 5.47	
		6.3	1850	21 51 7.51	869.76	47.25	875.37	5.61	
199	δ³ Tauri	5.0	1755	+ 17 20 42.56	+ 912.06	− 45.37	+ 916.17	− 4.11	
		5.0	1850	17 34 48.46	868.68	45.97	872.78	4.10	
200	70 Tauri	7.0	1755	+ 15 21 31.51	+ 911.12	− 44.75	+ 913.50	− 2.38	
		6.3	1850	15 35 36.78	868.30	45.40	870.84	2.54	
201	υ¹ Tauri	5.0	1755	+ 22 14 4.21	+ 909.00	− 46.85	+ 913.42	− 4.42	
		4.7	1850	22 28 6.51	864.15	47.58	868.68	4.53	
202	71 Tauri	5.5	1755	+ 15 2 23.77	+ 904.45	− 44.76	+ 907.66	− 3.21	
		6.0	1850	15 16 22.73	861.65	45.35	864.98	3.33	
203	π Tauri	5.0	1755	+ 14 8 15.86	+ 901.44	− 44.27	+ 904.73	− 3.29	
		5.0	1850	14 22 12.16	859.08	44.90	862.38	3.30	
204	υ² Tauri	6.0	1755	+ 22 25 13.97	+ 904.17	− 46.87	+ 905.75	− 1.58	
		6.0	1850	22 39 11.68	859.30	47.58	860.86	1.56	
205	B. A. C. 1373 . .	. .	1755	+ 21 3 3.79	+ 892.71	− 46.76	+ 899.20	− 6.49	
		6.0	1850	21 16 50.64	847.94	47.50	854.62	6.68	

RIGHT ASCENSIONS.

No.	Star.	Epoch.	Number of observations.	Right ascension.	Centennial variation.	Secular variation.	Struve's precession.	Proper motion.	Sec. var. of proper motion.
				h. m. s.	s.	s.	s.	s.	s.
206	ε Tauri	1755	5	4 14 20.531	+ 348.094	+ 1.258	+ 347.346	+ 0.748	
		1850	321	4 19 51.783	349.271	1.219	348.521	0.750	
		1900	- -	4 22 46.570	349.875	1.198	349.126	0.749	
207	75 Tauri	1755	5	4 14 27.949	+ 340.790	+ 1.156	+ 340.880	− 0.090	
		1850	12	4 19 52.217	341.873	1.125	341.962	0.089	
208	76 Tauri	1755	5	4 14 32.229	+ 337.990	+ 1.093	+ 337.265	+ 0.725	
		1850	3	4 19 53.808	339.014	1.063	338.289	0.725	
209	θ¹ Tauri	1755	5	4 14 36.522	+ 340.648	+ 1.137	+ 340.004	+ 0.644	
		1850	21	4 20 0.645	341.713	1.106	341.069	0.644	
210	θ² Tauri	1755	5	4 14 42.172	+ 340.528	+ 1.133	+ 339.806	+ 0.722	
		1850	29	4 20 6.180	341.591	1.104	340.867	0.724	
211	80 Tauri	1755	5	4 16 12.218	+ 340.031	+ 1.111	+ 339.405	+ 0.626	
		1850	9	4 21 35.743	341.071	1.080	340.449	0.622	
212	B. A. C. 1391	1755	- -	4 16 34.132	+ 341.239	+ 1.133	+ 340.696	+ 0.543	
		1850	7	4 21 58.815	342.299	1.100	341.749	0.550	
213	81 Tauri	1755	5	4 16 41.925	+ 340.426	+ 1.112	+ 339.568	+ 0.858	
		1850	15	4 22 5.824	341.467	1.080	340.608	0.859	
214	83 Tauri	1755	4	4 26 51.451	+ 335.888	+ 1.044	+ 335.179	+ 0.709	
		1850	3	4 22 11.011	336.866	1.015	336.160	0.706	
215	B. A. C. 1394	1755	- -	- - - - -	- - - -	+ 1.146	+ 340.610	- - -	
		1850	5	4 22 12.2	- - - -	1.114	341.663	- - -	
216	84 Tauri	1755	5	4 17 14.618	+ 338.537	+ 1.083	+ 338.293	+ 0.244	
		1850	6	4 22 36.711	339.550	1.050	339.307	0.243	
217	85 Tauri	1755	5	4 17 53.819	+ 340.735	+ 1.104	+ 340.029	+ 0.706	
		1850	17	4 23 18.010	341.768	1.071	341.063	0.705	
218	B. A. C. 1406	1755	4	4 19 38.677	+ 341.256	+ 1.108	+ 341.238	+ 0.018	
		1850	3	4 25 3.365	342.292	1.073	342.281	0.011	
219	B. A. C. 1408	1850	11	4 25 15.235	+ 374.120	+ 1.571	+ 374.053	+ 0.067	
220	ρ Tauri	1755	6	4 19 58.309	+ 338.529	+ 1.058	+ 337.912	+ 0.617	
		1850	5	4 25 20.383	339.518	1.025	338.904	0.614	
221	ε Tauri	1755	- -	4 21 53.409	+ 342.322	+ 1.082	+ 341.878	+ 0.444	−0.010
		1850	953	4 27 19.097	343.333	1.045	342.898	0.435	
		1900	- -	4 30 10.893	343.851	1.025	343.419	0.432	
222	W. B. 4ʰ 650	1850	- -	4 29 25.1	- - - -	+ 1.187	+ 353.016	- - -	
223	89 Tauri	1755	5	4 24 9.548	+ 341.570	+ 1.066	+ 340.939	+ 0.631	
		1850	3	4 29 34.516	342.566	1.030	341.939	0.627	
224	σ¹ Tauri	1755	5	4 25 11.494	+ 340.651	+ 1.044	+ 340.490	+ 0.161	
		1850	5	4 30 35.578	341.626	1.007	341.470	0.156	
225	σ² Tauri	1755	5	4 25 17.223	+ 341.320	+ 1.054	+ 340.760	+ 0.560	
		1850	7	4 30 41.947	342.304	1.018	341.748	0.556	

DECLINATIONS

No.	Star.	Mag.	Epoch.	Declination.	Centennial variation.	Secular variation.	Struve's precession.	Proper motion.	Sec. var. of proper motion.
				° ′ ″	″	″	″	″	″
206	ε Tauri	4.0	1755	+ 18 36 51.85	+ 888.25	— 46.00	+ 892.60	— 4.35	— 0.10
		3.7	1850	18 50 34.84	844.26	46.63	848.70	4.44	
			1900	18 57 31.12	820.86	46.95	825.36	4.50	
207	75 Tauri	6.0	1755	+ 15 47 23.66	+ 892.56	— 45.01	+ 891.66	+ 0.90	
		6.3	1850	16 1 11.19	849.50	45.65	848.65	0.85	
208	76 Tauri	7.0	1755	+ 14 10 26.56	+ 887.93	— 44.66	+ 891.07	— 3.14	
		6.3	1850	14 24 9.85	845.22	45.27	848.43	3.21	
209	θ¹ Tauri	5.0	1755	+ 15 23 45.21	+ 887.66	— 44.91	+ 890.47	— 2.81	
		4.0	1850	15 37 28.14	844.69	45.55	847.54	2.85	
210	θ² Tauri	5.5	1755	+ 15 18 17.73	+ 887.64	— 45.13	+ 889.80	— 2.16	
		4.0	1850	15 32 0.55	844.47	45.75	846.80	2.33	
211	80 Tauri	6.0	1755	+ 15 4 48.84	+ 874.46	— 45.11	+ 877.95	— 3.49	
		6.3	1850	15 18 19.13	831.30	45.75	834.95	3.65	
212	B. A. C. 1391 . .	5.5	1755	+ 15 38 19.53	+ 871.62	— 45.39	+ 875.18	— 3.56	
		5.0	1850	15 51 46.99	828.17	46.07	831.89	3.72	
213	81 Tauri	5.5	1755	+ 15 8 12.94	+ 871.03	— 45.31	+ 874.06	— 3.03	
		6.3	1850	15 21 39.89	827.68	45.96	830.96	3.28	
214	83 Tauri	6.0	1755	+ 13 10 10.25	+ 869.79	— 44.59	+ 872.82	— 3.03	
		6.0	1850	13 23 36.34	827.15	45.18	830.27	3.12	
215	B. A. C. 1394 . .	7.5	1755	+ 15 35 41.33	+ 870.91	— 45.32	+ 873.38	— 2.47	
		7.5	1850	15 49 8.16	827.55	45.97	830.08	2.53	
216	84 Tauri	7.0	1755	+ 14 33 16.34	+ 863.95	— 44.91	+ 869.78	— 5.83	
		7.3	1850	14 46 36.74	821.00	45.52	826.85	5.85	
217	85 Tauri	6.0	1755	+ 15 18 13.06	+ 860.67	— 45.32	+ 864.60	— 3.93	
		6.5	1850	15 31 30.15	817 31	45.95	821.36	4.05	
218	B. A. C. 1406 . .	7.5	1755	+ 15 47 3.39	+ 847.60	— 45.45	+ 850.79	— 3.19	
		7.5	1850	16 0 8.01	804.13	46.07	807.31	3.18	
219	B. A. C. 1408 . .	6.0	1850	+ 28 38 32.32	+ 801.49	— 50.34	+ 805.71	— 4.22	
220	ρ Tauri	5.0	1755	+ 14 18 25.75	+ 844.19	— 45.20	+ 848.19	— 4.00	
		5.3	1850	14 31 27.24	800.97	45 79	805.03	4.06	
221	α Tauri	1.0	1755	+ 15 59 38.38	+ 813.86	— 45.89	+ 832.95	—19.09	— 0.06
		1.0	1850	16 12 10.74	770.00	46.44	789.16	19.16	
			1900	16 18 29.92	746.70	46.72	765.90	19.20	
222	W 4ʰ 650 . . .	6.0	1850	+ 20 22 42.1	. . . .	— 47.86	+ 772.22	. . .	
223	89 Tauri	7.0	1755	+ 15 31 9.50	+ 812.17	— 45.97	+ 814.83	— 2.66	
		6.5	1850	15 43 40.22	768.22	46.55	770.93	2.71	
224	σ¹ Tauri	5.5	1755	+ 15 17 40.96	+ 798.48	— 45.87	+ 806.55	— 8.07	
		5.0	1850	15 29 58.72	754.63	46.44	762.71	8.08	
225	σ² Tauri	5.5	1755	+ 15 24 36.19	+ 802.90	— 46.02	+ 805.79	— 2.89	
		5.0	1850	15 36 58.09	758.91	46.60	761.86	2.95	

RIGHT ASCENSIONS.

No.	Star.	Epoch.	Number of observations.	Right ascension.			Centennial variation.	Secular variation.	Struve's precession.	Proper motion.	Sec. var. of proper motion.
				h.	*m.*	*s.*	*s.*	*s.*	*s.*	*s.*	*s.*
226	B. A. C. 1444 . .	1850	8	4	31	56.833	+ 374.134	+ 1.475	+ 373.977	+ 0.157	
227	τ Tauri	1755	5	4	27	34.297	+ 357.861	+ 1.284	+ 357.867	— 0.006	
		1850	89	4	33	14.837	359.056	1.232	359.057	0.001	
228	95 Tauri	1755	5	4	28	25.780	+ 360.911	+ 1.315	+ 360.834	+ 0.077	
		1850	4	4	34	9.230	362.134	1.259	362.061	0.073	
229	B. A. C. 1463 . .	1755	1	4	30	56.976	+ 359.895	+ 1.273	+ 359.927	— 0.032	
		1850	6	4	36	39.443	361.079	1.219	361.117	— 0.038	
230	B. A. C. 1468 . .	1850	8	4	37	31.624	+ 349.387	+ 1.036	+ 348.949	+ 0.429	
231	α Camelopardalis .	1755	. .	4	29	53.38	+ 582.73	+ 7.67	+ 583.04	— 0.31	
		1800	. .	4	34	16.38	586.13	7.40	586.44	0.31	
		1850	. .	4	39	10.35	589.75	7.09	590.06	0.31	
		1900	. .	4	44	6.10	593.22	6.75	593.53	0.31	
232	96 Tauri	1755	5	4	35	44.587	+ 341.523	+ 0.974	+ 341.508	+ 0.015	
		1850	15	4	41	9.467	342.428	0.933	342.411	0.017	
233	i Tauri	1755	5	4	37	4.026	+ 349.177	+ 1.041	+ 348.611	+ 0.566	
		1850	26	4	42	36.208	350.145	1.000	349.583	0.562	
234	ι Aurigæ . . .	1755	3	4	41	4.555	+ 388.059	+ 1.556	+ 387.990	+ 0.069	
		1850	212	4	47	13.901	389.495	1.467	389.425	0.070	
		1900	. .	4	50	28.830	390.216	1.417	390.148	0.068	
235	B. A. C. 1526 . .	1755	1	4	43	14.665	+ 344.891	+ 0.943	+ 345.008	— 0.117	
		1850	14	4	48	42.730	345.764	0.896	345.886	0.122	
236	99 Tauri	1755	1	4	42	58.513	+ 361.960	+ 1.153	+ 362.014	— 0.054	
		1850	9	4	48	42.885	363.026	1.092	363.078	0.052	
237	k Tauri	1755	5	4	43	11.415	+ 365.234	+ 1.177	+ 364.961	+ 0.273	
		1850	21	4	48	58.910	366.322	1.114	366.052	0.270	
238	101 Tauri	1755	5	4	45	41.878	+ 342.872	+ 0.885	+ 342.215	+ 0.657	
		1850	3	4	51	7.999	343.693	0.844	343.042	0.651	
239	ι Tauri	1755	5	4	48	28.574	+ 356.856	+ 1.014	+ 356.413	+ 0.443	
		1850	70	4	54	8.038	357.794	0.960	357.353	0.441	
240	11 Orionis	1755	5	4	50	35.410	+ 341.353	+ 0.836	+ 341.304	+ 0.049	
		1850	83	4	56	0.066	342.125	0.790	342.073	0.052	
		1900	. .	4	58	51.226	342.515	0.769	342.463	0.052	
241	m Tauri	1755	5	4	52	59.479	+ 353.077	+ 0.907	+ 349.336	+ 3.741	
		1850	21	4	58	35.303	353.913	0.853	350.165	3.748	
242	l Tauri	1755	5	4	53	19.806	+ 353.473	+ 0.930	+ 353.817	— 0.344	
		1850	16	4	58	56.016	354.328	0.871	354.675	0.347	
243	105 Tauri	1755	5	4	53	17.863	+ 357.043	+ 0.977	+ 357.098	— 0.055	
		1850	18	4	58	57.487	357.944	0.920	357.993	0.049	
244	103 Tauri	1755	5	4	53	12.435	+ 363.756	+ 1.044	+ 363.815	— 0.059	
		1870	16	4	58	58.464	364.716	0.976	364.779	0.063	

DECLINATIONS.

No.	Star.	Mag.	Epoch.	Declination.	Centennial variation.	Secular variation.	Struve's precession.	Proper motion.	Sec. var. of proper motion.
				° ′ ″	″	″	″	″	″
226	B. A. C. 1444 . .	5.7	1850	+ 28 19 9.16	+ 748.64	− 50.94	+ 751.77	− 3.13	
227	τ Tauri	5.0	1755	+ 22 27 47.45	+ 784.90	− 48.44	+ 787.42	− 2.52	
		4.3	1850	22 39 51.15	738.59	49.07	741.20	2.61	
228	95 Tauri	7.0	1755	+ 23 36 1.45	+ 777.95	− 48.85	+ 780.52	− 2.57	
		6.3	1850	23 47 58.36	731.24	49.49	733.81	2.57	
229	B. A. C. 1463 . .	7.5	1755	. . . . . .	. . . . .	− 48.91	+ 760.15	. . .	
		6.3	1850	+ 23 20 49.7	. . . . .	49.53	713.39	. . .	
230	B. A. C. 1468 . .	6.3	1850	+ 18 27 31.12	+ 697.30	− 48.06	+ 706.30	− 9.00	
231	α Camelopardalis .	4.7	1755	+ 65 53 11.88	+ 768.54	− 78.81	+ 768.73	− 0.19	
			1800	65 58 49.73	732.84	79.86	733.01	0.17	
			1850	66 4 46.11	692.62	80.99	692.77	0.15	
			1900	66 10 42.25	651.86	82.04	651.99	0.13	
232	96 Tauri	6.0	1755	+ 15 27 12.45	+ 720.65	− 46.80	+ 721.14	− 0.49	
		6.5	1850	15 38 15.87	675.94	47.32	676.44	0.50	
233	*i* Tauri	5.5	1755	+ 18 23 57.87	+ 705.86	− 48 06	+ 710.31	− 4.45	
		5.3	1850	18 34 46.66	659.95	48.60	664.53	4.58	
234	ι Aurigæ	4.0	1755	+ 32 45 5.80	+ 675.21	− 53.59	+ 677.38	− 2.17	
		3.0	1850	32 55 22.98	624.01	54.21	626.16	2.15	
			1900	33 0 28.19	596.83	54.53	598.98	2.15	
235	B. A. C. 1526 . .	6.5	1755	+ 16 44 45.52	+ 656.65	− 47.78	+ 659.51	− 2.86	
		5.3	1850	16 54 47.71	611.03	48.27	613.83	2.80	
236	99 Tauri	6.5	1755	+ 23 32 29.36	+ 659.22	− 50.26	+ 661.73	− 2.51	
		6.0	1850	23 42 32.87	611.20	50.84	613.82	2.62	
237	*k* Tauri	6.0	1755	+ 24 38 50.61	+ 653.80	− 50.67	+ 659.92	− 6.12	
		6.0	1850	24 48 48.75	605.37	51.31	611.60	6.23	
238	101 Tauri	7.0	1755	+ 15 31 26.45	+ 635.86	− 47.78	+ 639.17	− 3.31	
		7.0	1850	15 41 8.88	590.25	48.24	593.64	3.39	
239	ι Tauri	4.5	1755	+ 21 12 56.39	+ 610.98	− 50.00	+ 616.07	− 5.09	
		5.0	1850	21.22 14.09	563.26	50.47	568.51	5.25	
240	11 Orionis	5.0	1755	+ 15 2 21.71	+ 594.65	− 47.89	+ 598.46	− 3.81	− 0.03
		5.0	1850	15 11 24.96	548.98	48.27	552.82	3.84	
			1900	15 15 53.41	524.80	48.47	528.66	3.86	
241	*m* Tauri	5.0	1755	+ 18 17 29.97	+ 580.72	− 50.30	+ 578.36	+ 2.36	
		5.3	1850	18 26 18.88	532.74	50.70	531.03	1.71	
242	*l* Tauri	5.5	1755	+ 20 4 14.93	+ 572.09	− 49.72	+ 575 50	− 3.41	
		5.7	1850	20 12 55.90	524.66	50.12	528.11	3.45	
243	105 Tauri	6.0	1755	+ 21 21 21.61	+ 573.86	− 50.24	+ 575.76	− 1.90	
		6.0	1850	21 30 4.07	525.98	50.57	527.90	1.92	
244	103 Tauri	6.0	1755	+ 23 54 58.41	+ 575.39	− 51.07	+ 576.52	− 1.13	
		6.0	1850	24 3 41.92	526.65	51.55	527.76	1.11	

RIGHT ASCENSIONS.

No.	Star.	Epoch.	Number of observations.	Right ascension.			Centennial variation.	Secular variation.	Struve's precession.	Proper motion.	Sec. var. of proper motion.
				h.	m.	s.	s.	s.	s.	s.	s.
245	107 Tauri	1755	5	4	54	24.276	+ 352.450	+ 0.908	+ 352.479	− 0.029	
		1850	3	4	59	59.505	353.287	0.854	353.317	0.030	
246	W 4^h 1421 . . .	1850	. .	5	0	19.1	. . . .	+ 1.071	+ 375.398	. . .	
247	15 Orionis	1755	5	4	55	41.741	+ 342.013	+ 0.797	+ 342.062	− 0.049	
		1850	26	5	1	7.005	342.748	0.751	342.800	0.052	
248	α Aurigæ . . .	1755	. .	4	58	37.982	+ 440.137	+ 1.879	+ 439.270	+ 0.867	−0.052
		1850	. .	5	5	36.928	441.821	1.666	441.005	0.816	
		1900	. .	5	9	18.043	442.631	1.573	441.848	0.783	
249	108 Tauri	1755	5	5	0	45.440	+ 359.046	+ 0.901	+ 359.191	− 0.145	
		1850	2	5	6	26.931	359.872	0.837	360.019	0.147	
250	β Orionis	1755	. .	5	2	46.463	+ 287.588	+ 0.413	+ 287.613	− 0.025	
		1850	. .	5	7	19.856	287.974	0.400	287.999	0.025	
		1900	. .	5	9	43.892	288.172	0.391	288.199	0.027	
251	n Tauri	1755	5	5	4	34.475	+ 359.140	+ 0.861	+ 358.981	+ 0.159	
		1850	32	5	10	16.037	359.927	0.796	359.762	0.165	
252	22 Aurigæ . . .	1755	5	5	7	53.265	+ 378.373	+ 0.995	+ 378.273	+ 0.100	
		1850	3	5	13	53.155	379.276	0.907	379.166	0.110	
253	110 Tauri	1755	5	5	9	29.917	+ 345.197	+ 0.687	+ 345.521	− 0.324	
		1850	3	5	14	58.156	345.825	0.635	346.155	0.330	
254	111 Tauri	1755	5	5	10	8.643	+ 348.919	+ 0.706	+ 347.255	+ 1.664	
		1850	15	5	15	40.428	349.564	0.653	347.893	1.671	
255	β Tauri	1755	10	5	10	49.487	+ 377.795	+ 0.904	+ 377.597	+ 0.198	−0.010
		1850	. .	5	16	48.787	378.611	0.814	378.410	0.201	
		1900	. .	5	19	58.192	379.006	0.767	378.821	0.185	
256	113 Tauri	1755	5	5	11	57.240	+ 345.514	+ 0.665	+ 345.646	− 0.132	
		1850	5	5	17	25.770	346.121	0.613	346.258	0.137	
257	115 Tauri	1755	2	5	12	53.442	+ 348.914	+ 0.680	+ 348.855	+ 0.059	
		1850	19	5	18	25.209	349.533	0.624	349.473	0.060	
258	o Tauri	1755	5	5	12	56.163	+ 359.173	+ 0.756	+ 359.116	+ 0.057	
		1850	31	5	18	37.709	359.861	0.692	359.797	0.064	
259	B. A. C. 1699 . .	1755	1	5	13	25.553	+ 343.900	+ 0.640	+ 344.085	− 0.185	
		1850	3	5	18	52.538	344.483	0.588	344.672	0.189	
260	116 Tauri	1755	5	5	13	41.825	+ 343.713	+ 0.633	+ 343.690	+ 0.023	
		1850	9	5	19	8.630	344.290	0.581	344.270	0.020	
261	117 Tauri	1755	. .	5	13	49.317	+ 347.088	+ 0.652	+ 347.096	− 0.008	
		1850	6	5	19	19.338	347.682	0.598	347.690	0.008	
262	B. A. C. 1703 . .	1755	2	5	14	2.774	+ 344.681	+ 0.643	+ 345.109	− 0.428	
		1850	3	5	19	30.502	345.266	0.589	345.706	0.440	
263	Groombridge 966 .	1755	. .	5	7	9.10	+ 787.92	+10.61	+ 787.14	+ 0.78	
		1800	. .	5	13	4 72	792.49	9.63	791.70	0.79	
		1850	. .	5	19	42.12	797.00	8.38	796.20	0.80	
		1900	. .	5	26	21.60	800.83	7.02	800.04	0.79	

DECLINATIONS.

No.	Star.	Mag.	Epoch.	Declination.	Centennial variation.	Secular variation.	Struve's precession.	Proper motion.	Sec. var. of proper motion.
				° ′ ″	″	″	″	″	″
245	107 Tauri	7.0	1755	+ 19 31 0.31	+ 565.71	− 49.56	+ 566.47	− 0.76	
		6.5	1850	19 39 35.30	518.42	50.00	519.15	0.73	
246	W 4^h 1421 . . .	6.0	1850	+ 27 50 5.2	. . . .	− 53.14	+ 516.41	. . .	
247	15 Orionis	5.0	1755	+ 15 15 36.86	+ 555.07	− 48.17	+ 555.62	− 0.55	
		5.3	1850	15 24 2.38	509.12	48.57	509.65	0.53	
248	α Aurigæ	1.0	1755	+ 45 43 5.52	+ 487.59	− 62.35	+ 530.92	−43.33	− 0.11
		1.0	1850	45 50 20.51	428.07	62.96	471.48	43.41	
			1900	45 53 46.66	+ 396.51	63.28	439.98	43.47	
249	108 Tauri	7.0	1755	+ 21 58 45.88	+ 512.05	− 50.84	+ 512.90	− 0.85	
		6.3	1850	22 6 29.32	463.56	51.24	464.39	0.83	
250	β Orionis	1.0	1755	− 8 30 16.69	+ 495.19	− 40.88	+ 495.80	− 0.61	
		1.0	1850	8 22 44.74	456.26	41.09	456.87	0.61	
			1900	8 19 1.75	435.68	41.19	436.29	0.61	
251	π Tauri	5.5	1755	+ 21 49 4.43	+ 472.34	− 51.22	+ 480.54	− 8.20	
		5.7	1850	21 56 9.97	423.50	51.60	431.82	8.32	
252	22 Aurigæ	7.0	1755	+ 28 40 33.80	+ 459.22	− 53.99	+ 452.31	+ 6.91	
		7.0	1850	28 47 25.64	407.74	54.40	400.86	6.88	
253	110 Tauri	7.0	1755	+ 16 26 36.80	+ 435.86	− 49.28	+ 438.56	− 2.70	
		6.8	1850	16 33 8.57	388.89	49.60	391.55	2.66	
254	111 Tauri	6.0	1755	+ 17 7 51.40	+ 433.10	− 50.22	+ 433.05	+ 0.05	
		5.7	1850	17 14 20.15	385.25	50.52	385.50	− 0.25	
255	β Tauri	2.0	1755	+ 28 22 26.51	+ 409.20	− 54.12	+ 427.23	−18.03	− 0.03
		2.0	1850	28 28 30.78	357.64	54.44	375.70	18.06	
			1900	28 31 22.79	330.38	54.60	348.46	18.08	
256	113 Tauri	6.0	1755	+ 16 27 28.43	+ 416.43	− 49.46	+ 417.56	− 1.13	
		7.0	1850	16 33 41.67	369.30	49.76	370.38	1.08	
257	115 Tauri	5.5	1755	+ 17 43 34.90	+ 409.21	− 50.02	+ 409.50	− 0.29	
		6.0	1850	17 49 41.01	361.51	50.41	361.88	0.37	
258	ο Tauri	5.0	1755	+ 21 42 8.40	+ 408.01	− 51.60	+ 409.16	− 1.15	
		6.0	1850	21 48 12.71	358.84	51.93	360.10	1.26	
259	B. A. C. 1699 . .	8.0	1755	+ 15 48 24.03	+ 403.02	− 49.28	+ 404.92	− 1.90	
		8.0	1850	15 54 24.61	356.06	49.57	357.94	1.88	
260	116 Tauri	6.0	1755	+ 15 38 35.80	+ 398.93	− 49.30	+ 402.62	− 3.69	
		6.0	1850	15 44 32.49	351.96	49.58	355.65	3.69	
261	117 Tauri	. .	1755	+ 17 0 42.26	+ 393.62	− 49.98	+ 401.65	− 8.03	
		6.3	1850	17 6 33.59	346.01	50.27	354.12	8.11	
262	B. A. C. 1703 . .	7.0	1755	. . . . . .	. . . .	− 49.38	+ 399.62	. . .	
		6.9	1850	+ 16 18 40.4	. . . .	49.67	352.49	. . .	
263	Groombridge 966 .	. .	1755	+ 74 49 31.24	+ 460.64	−112.21	+ 458.59	+ 2.05	
			1800	74 52 47.12	409.84	113.48	407.85	1.99	
		6.5	1850	74 55 57.80	352.78	114.74	350.84	1.94	
			1900	74 58 39.81	+ 295.15	−115.84	+ 293.27	+ 1.88	

RIGHT ASCENSIONS.

No.	Star.	Epoch.	Number of observations.	Right ascension.			Centennial variation.	Secular variation.	Struve's precession.	Proper motion.	Sec. var. of proper motion.
				h.	*m.*	*s.*	*s.*	*s.*	*s.*	*s.*	*s.*
264	118 Tauri	1755	5	5	14	12.796	+ 368.030	+ 0.797	+ 367.923	+ 0.107	
		1850	8	5	20	2.773	368.750	0.720	368.644	0.106	
265	119 Tauri	1755	5	5	17	51.757	+ 350.782	+ 0.641	+ 350.711	+ 0.071	
		1850	32	5	23	25.279	351.362	0.581	351.289	0.073	
266	B. A. C. 1728 . .	1850	. .	5	23	32.9	. . . .	+ 0.558	+ 347.391	. . .	
267	δ Orionis	1755	10	5	19	30.004	+ 305.809	+ 0.410	+ 305.844	— 0.035	
		1850	607	5	24	20.704	306.187	0.385	306.220	0.033	
		1900	. .	5	26	53.845	306.376	0.371	306.410	0.034	
268	120 Tauri	1755	5	5	19	10.871	+ 350.685	+ 0.632	+ 350.638	+ 0.047	
		1850	18	5	24	44.298	351.258	0.574	351.203	0.055	
269	α Leporis . . .	1755	5	5	21	55.947	+ 264.069	+ 0.311	+ 264.090	— 0.021	+0.002
		1850	209	5	26	6.951	264.358	0.298	264.377	0.019	
		1900	. .	5	28	19.167	264.506	0.293	264.528	0.022	
270	121 Tauri	1755	5	5	20	30.319	+ 365.279	+ 0.696	+ 365.262	+ 0.017	
		1850	15	5	26	17.637	365.904	0.620	365.884	0.020	
271	122 Tauri	1755	5	5	22	51.337	+ 347.313	+ 0.557	+ 347.014	+ 0.299	
		1850	5	5	28	21.527	347.816	0.501	347.524	0.292	
272	ε Orionis	1755	10	5	23	47.501	+ 303.743	+ 0.382	+ 303.815	— 0.072	
		1850	420	5	28	36.227	304.095	0.358	304.167	0.072	
		1900	. .	5	31	8.319	304.271	0.346	304.343	0.072	
273	ζ Tauri	1755	5	5	23	0.961	+ 357.612	+ 0.617	+ 357.593	+ 0.019	
		1850	119	5	28	40.961	358.167	0.552	358.147	0.020	
274	26 Aurigæ	1755	3	5	22	55.275	+ 383.979	+ 0.785	+ 384.202	— 0.223	
		1850	6	5	29	0.394	384.676	0.684	384.901	0.225	
275	B. A C. 1772 . .	1850	. .	5	29	46.2	. . . .	+ 0.651	+ 381.024	. . .	
276	125 Tauri	1755	5	5	24	34.046	+ 370.742	+ 0.675	+ 370.701	+ 0.041	
		1850	23	5	30	26.541	371.340	0.586	371.300	0.040	
277	126 Tauri	1755	5	5	27	8.772	+ 346.020	+ 0.510	+ 345.916	+ 0.104	
		1850	10	5	32	37.713	346.479	0.456	346.381	0.098	
278	B. A. C. 1796 . .	1755	1	5	28	4.368	+ 352.261	+ 0.521	+ 352.210	+ 0.051	
		1850	3	5	33	39.241	352.726	0.458	352.675	0.051	
279	127 Tauri	1755	5	5	28	29.680	+ 352.002	+ 0.522	+ 352.199	— 0.197	
		1850	3	5	34	4.308	352.468	0.459	352.670	0.202	
280	B. A. C. 1801 . .	1850	. .	5	34	13.1	. . . .	+ 0.507	+ 363.926	. . .	
281	α Columbæ . . .	1755	. .	5	30	46.941	+ 216.902	+ 0.281	+ 216.757	+ 0.145	—0.002
		1850	179	5	34	13.124	217.166	0.275	217.023	0.143	
		1900	. .	5	36	1.741	217.303	0.272	217.163	0.140	
282	128 Tauri	1755	5	5	30	47.008	+ 344.886	+ 0.472	+ 344.928	— 0.042	
		1850	6	5	36	14.854	345.308	0.416	345.356	0.048	
283	129 Tauri	1755	5	5	32	40.685	+ 344.326	+ 0.449	+ 344.333	— 0.007	
		1850	21	5	38	7.989	344.726	0.394	344.738	0.012	

DECLINATIONS.

No.	Star.	Mag.	Epoch.	Declination.	Centennial variation.	Secular variation.	Struve's precession.	Proper motion.	Sec. var. of proper motion.
				° ' "	"	"	"	"	"
264	118 Tauri	7.0	1755	+ 24 55 33.01	+ 395.19	− 52.81	+ 398.19	− 3.00	
		5.7	1850	25 1 24.57	344.87	53.14	347.90	3.03	
265	119 Tauri	5.5	1755	+ 18 23 13.74	+ 366.33	− 50.57	+ 366.80	− 0.47	
		5.0	1850	18 28 38.91	318.18	50.84	318.75	0.57	
266	B. A. C. 1728 . .	6.0	1850	+ 16 56 26.5	. . . .	− 50.18	+ 317.66	. . .	
267	δ Orionis	2.0	1755	− 0 30 7.62	+ 352.28	− 44.13	+ 352.76	− 0.48	− 0.02
		2.3	1850	0 24 52.89	310.27	44.29	310.76	0.49	
			1900	0 22 23.30	288.11	44.37	288.62	0.51	
268	120 Tauri	6.0	1755	+ 18 20 25.60	+ 356.22	− 50.69	+ 355.57	+ 0.65	
		6.0	1850	18 25 41.09	307.94	50.95	307.37	0.57	
269	ι Leporis . . .	3.5	1755	− 18 0 58.59	+ 331.72	− 38.19	+ 331.77	− 0.05	− 0.01
		2.7	1850	17 56 0.71	295.39	38.29	295.45	0.06	
			1900	17 53 37.80	276.23	38.35	276.29	0.06	
270	121 Tauri	6.0	1755	+ 23 51 3.66	+ 341.07	− 52.79	+ 344.07	− 3.00	
		6.0	1850	23 56 3.82	290.78	53.11	293.92	3.14	
271	122 Tauri	6.0	1755	+ 16 51 52.78	+ 318.50	− 50.18	+ 323.77	− 5.27	
		6.0	1850	16 56 32.68	270.72	50.42	276.00	5.28	
272	ε Orionis	2.3	1755	− 1 22 48.15	+ 315.73	− 43.92	+ 315.70	+ 0.03	
		1.8	1850	1 18 8.05	273.94	44.05	273.90	0.04	
			1900	1 15 56.59	251.90	44.13	251.86	0.04	
273	ζ Tauri	3.4	1755	+ 20 58 5.69	+ 318.62	− 51.77	+ 322.40	− 3.78	
		3.3	1850	21 2 44.97	269.32	52.00	273.21	3.89	
274	26 Aurigæ	5.0	1755	30 19 10.33	+ 322.39	− 55.39	+ 323.20	− 0.81	
		6.0	1850	30 23 51.54	269.63	55.69	270.41	0.78	
275	B. A. C. 1772 . .	6.3	1850	+ 29 7 24.6	. . . .	− 55.22	+ 263.80	. . .	
276	125 Tauri	6.0	1755	+ 25 44 0.74	+ 305.99	− 53.74	+ 309.00	− 3.01	
		6.0	1850	25 48 27.13	254.84	53.95	257.96	3.12	
277	126 Tauri	5.5	1755	+ 16 22 56.42	+ 284.27	− 50.10	+ 286.64	− 2.37	
		5.7	1850	16 27 3.84	236.58	50.30	238.96	2.38	
278	B. A. C. 1796 . .	8.0	1755	+ 18 50 38.81	+ 270.10	− 51.02	+ 278.61	− 8.51	
		7.5	1850	18 54 32.36	221.54	51.22	230.02	8.48	
279	127 Tauri	8.0	1755	+ 18 50 14.02	+ 270.64	− 50.95	+ 274.96	− 4.32	
		6.3	1850	18 54 8.09	222.14	51.16	226.36	4.22	
280	B. A. C. 1801 . .	6.0	1850	+ 23 7 41.5	. . . .	− 52.91	+ 225.15	. . .	
281	α Columbæ . . .	. .	1755	− 34 13 9.25	+ 250.64	− 31.55	+ 255.10	− 4.46	− 0.03
		2.5	1850	34 9 25.39	220.64	31.61	225.14	4.50	
			1900	34 7 39.02	204.82	31.65	209.34	4.52	
282	128 Tauri	6.0	1755	+ 15 57 18.39	+ 253.73	− 50.01	+ 255.09	− 1.36	
		6.9	1850	16 0 56.85	206.19	50.07	207.50	1.31	
283	129 Tauri	6.0	1755	+ 15 42 8.98	+ 236.57	− 49.97	+ 238.63	− 2.06	
		6.3	1850	15 45 31.13	189.02	50.14	191.07	2.05	

RIGHT ASCENSIONS.

No.	Star.	Epoch.	Number of observations.	Right ascension.	Centennial variation.	Secular variation.	Struve's precession.	Proper motion.	Sec. var. of proper motion.
				h. m. s.	s.	s.	s.	s.	s.
284	130 Tauri	1755	5	5 33 9.684	+ 349.043	+ 0.464	+ 349.169	— 0.126	
		1850	11	5 38 41.475	349.455	0.404	349.584	0.129	
285	132 Tauri	1755	5	5 33 59.491	+ 367.381	+ 0.527	+ 367.447	— 0.066	
		1850	26	5 39 48.728	367.843	0.446	367.908	0.065	
286	136 Tauri	1755	5	5 37 56.206	+ 376.470	+ 0.492	+ 376.390	+ 0.080	
		1850	43	5 43 54.057	376.895	0.402	376.820	0.075	
287	B. A. C. 1867 . .	1755	1	5 38 45.734	+ 356.144	+ 0.409	+ 356.045	+ 0.099	
		1850	12	5 44 24.195	356.500	0.342	356.412	0.088	
288	χ^1 Orionis	1755	5	5 39 52.959	+ 354.689	+ 0.394	+ 356.037	— 1.348	
		1850	44	5 45 30.081	355.031	0.327	356.388	1.357	
289	χ^2 Orionis	1755	5	5 40 26.900	+ 354.586	+ 0.402	+ 354.656	— 0.070	
		1850	11	5 46 3.928	354.936	0.335	355.003	0.067	
290	α Orionis	1755	. .	5 41 54.894	+ 324.308	+ 0.321	+ 324.170	+ 0.138	+0.001
		1850	. .	5 47 3.125	324.593	0.279	324.452	0.141	
		1900	. .	5 49 45.456	324.728	0.261	324.589	0.139	
291	139 Tauri	1755	3	5 42 47.976	+ 371.718	+ 0.411	+ 371.757	— 0.039	
		1850	17	5 48 41.281	372.069	0.328	372.112	0.043	
292	140 Tauri	1755	. .	5 45 37.480	+ 363.172	+ 0.354	+ 363.277	— 0.105	
		1850	3	5 51 22.647	363.472	0.279	363.577	0.105	
293	141 Tauri	1755	5	5 46 54.385	+ 361.750	+ 0.336	+ 361.952	— 0.202	
		1850	12	5 52 38.187	362.034	0.263	362.237	0.203	
294	χ^3 Orionis	1755	3	5 48 58.106	+ 354.221	+ 0.295	+ 354.755	— 0.534	
		1850	7	5 54 34.739	354.470	0.230	355.007	0.537	
295	1 Geminorum . .	1755	5	5 49 13.828	+ 364.418	+ 0.292	+ 364.413	+ 0.005	
		1850	92	5 55 0.144	364.655	0.216	364.653	0.002	
296	χ^4 Orionis	1755	. .	5 49 22.636	+ 355.766	+ 0.292	+ 355.946	— 0.180	
		1850	18	5 55 0.734	356.011	0.224	356.194	0.183	
297	2 Geminorum . .	1755	5	5 51 52.639	+ 365.535	+ 0.273	+ 365.498	+ 0.037	
		1850	7	5 57 40.009	365.757	0.195	365.721	0.036	
298	ν Orionis	1755	5	5 53 35.016	+ 342.472	+ 0.235	+ 342.257	+ 0.215	
		1850	132	5 59 0.462	342.670	0.181	342.458	0.212	
		1900	. .	6 1 51.819	342.754	0.153	342.541	0.213	
299	3 Geminorum . .	1755	5	5 54 51.421	+ 364.107	+ 0.219	+ 364.116	— 0.009	
		1850	18	6 0 37.410	364.282	0.150	364.298	0.006	
300	4 Geminorum . .	1755	5	5 55 38.384	+ 363.776	+ 0.214	+ 363.811	— 0.035	
		1850	3	6 1 24.056	363.942	0.137	363.988	0.046	
301	22 (H) Camelopardalis	1755	. .	5 51 49.60	+ 661.72	+ 0.90	+ 661.87	— 0.15	
		1800	. .	5 56 47.44	661.99	+ 0.30	662.17	0.18	
		1850	. .	6 2 18.45	661.98	— 0.36	662.19	0.21	
		1900	. .	6 7 49.37	+ 661.64	— 1.02	+ 661.87	— 0.23	

DECLINATIONS.

No.	Star.	Mag.	Epoch.	Declination.	Centennial variation.	Secular variation.	Struve's precession.	Proper motion.	Sec. var. of proper motion.
				° ′ ″	″	″	″	″	″
284	130 Tauri	6.0	1755	+ 17 36 43.61	+ 234.99	− 50.72	+ 234.42	+ 0.57	
		6.0	1850	17 40 3.96	186.89	50.55	186.20	0.69	
285	132 Tauri	5.0	1755	+ 24 27 32.26	+ 225.29	− 53.50	227.28	− 1.99	
		5.3	1850	24 30 42.11	174.36	53.72	176.45	2.09	
286	136 Tauri	4.5	1755	+ 27 31 38.86	+ 191.88	− 54.85	+ 192.84	− 0.96	
		5.3	1850	27 34 16.39	139.70	55.01	140.78	1.08	
287	B. A. C. 1867	7.5	1755	+ 20 13 7.85	+ 176.27	− 51.82	+ 185.69	− 9.42	
		7.2	1850	20 15 31.90	126.97	51.96	136.37	9.40	
288	χ^1 Orionis	5.0	1755	+ 20 12 20.95	+ 165.48	− 51.42	+ 175.88	−10.40	
		4.7	1850	20 14 34.93	116.57	51.54	126.81	10.24	
289	χ^2 Orionis	6.0	1755	+ 19 40 37.02	+ 169.65	− 51.77	+ 171.00	− 1.35	
		6.0	1850	19 42 54.81	120.44	51.81	121.87	1.43	
290	α Orionis	1.0	1755	+ 7 20 17.74	+ 158.88	− 47.29	+ 158.18	+ 0.70	− 0.02
		1.3	1850	7 22 27.34	113.94	47.34	113.25	0.69	
			1900	7 23 18.39	90.26	47.36	89.59	0.67	
291	139 Tauri	5.5	1755	+ 25 53 48.55	+ 150.44	− 54.29	+ 150.44	0.00	
		5.3	1850	25 55 46.97	98.77	54.50	98.95	− 0.18	
292	140 Tauri	8.0	1755	+ 22 51 33.28	+ 125.16	− 52.91	+ 125.77	− 0.61	
		7.0	1850	22 53 8.28	74.85	53.00	75.41	0.56	
293	141 Tauri	6.0	1755	+ 22 22 5.76	+ 111.99	− 52.82	+ 114.56	− 2.57	
		6.7	1850	22 23 28.30	61.78	52.88	64.42	2.64	
294	χ^3 Orionis	5.0	1755	+ 19 40 10.15	+ 93.95	− 51.54	+ 96.54	− 2.59	
		6.0	1850	19 41 16.11	44.94	51.64	47.44	2.50	
295	1 Geminorum	5.0	1755	+ 23 15 1.82	+ 84.13	− 53.23	+ 94.23	−10.10	
		5.0	1850	23 15 57.72	33.53	53.35	43.74	10.21	
296	χ^4 Orionis	5.5	1755	+ 20 7 7.77	+ 92.34	− 52.03	+ 93.00	− 0.66	
		5.0	1850	20 8 11.99	42.86	52.14	43.64	0.78	
297	2 Geminorum	6.5	1755	+ 23 38 6.78	+ 69.63	− 53.33	+ 71.10	− 1.47	
		7.2	1850	23 38 48.86	18.95	53.37	20.43	1.48	
298	ν Orionis	4.5	1755	+ 14 46 24.93	+ 53.24	− 50.11	+ 56.15	− 2.81	
		4.7	1850	14 46 52.89	+ 5.65	50.08	+ 8.67	3.02	
			1900	14 46 49.46	− 19.38	50.04	− 16.31	− 3.07	
299	3 Geminorum	6.0	1755	+ 23 7 39.13	+ 43.80	− 53.30	+ 45.04	− 1.24	
		6.3	1850	23 7 56.69	− 6.84	53.30	− 5.45	1.39	
300	4 Geminorum	7.0	1755	+ 23 0 59.89	+ 31.75	− 53.06	+ 38.17	− 6.42	
		7.4	1850	23 1 6.10	− 18.67	53.08	− 12.25	6.42	
301	22 (H) Camelopardalis	. .	1755	+ 69 21 32.94	+ 59.66	− 96.48	+ 71.53	−11.87	
			1800	69 21 50.05	+ 16.24	96.55	+ 28.09	11.85	
		4.7	1850	69 21 46.10	− 32.03	96.52	− 20.19	11.84	
			1900	+ 69 21 18.03	− 80.26	− 96.40	− 68.43	−11.83	

RIGHT ASCENSIONS.

No.	Star.	Epoch.	Number of observations.	Right ascension.	Centennial variation.	Secular variation.	Struve's precession.	Proper motion.	Sec. var. of proper motion.
				h. m. s.	s.	s.	s.	s.	s.
302	5 Geminorum . .	1755	5	5 56 30.891	+ 367.767	+ 0.203	+ 367.803	— 0.036	
		1850	13	6 2 20.349	367.921	0.121	367.960	0.039	
303	68 Orionis	1755	2	5 57 30.465	+ 355.463	+ 0.191	+ 355.206	+ 0.257	
		1850	10	6 3 8.231	355.613	0.124	355.360	0.253	
304	6 Geminorum . .	1755	4	5 57 28.040	+ 363.561	+ 0.196	+ 363.601	— 0.040	
		1850	9	6 3 13.499	363.711	0.120	363.754	0.043	
305	f^1 Orionis	1755	4	5 57 55.908	+ 345.634	+ 0.193	+ 345.757	— 0.123	
		1850	4	6 3 24.338	345.790	0.135	545.918	0.128	
306	κ Aurigæ	1755	4	5 59 45.882	+ 382.364	+ 0.113	+ 382.864	— 0.500	
		1850	35	6 5 49.164	382.424	0.013	382.948	0.524	
307	η Geminorum . .	1755	5	6 0 5.307	+ 362.130	+ 0.163	+ 362.547	— 0.417	
		1850	214	6 5 49.393	362.250	0.090	362.667	0.417	
308	71 Orionis	1755	4	6 0 26.121	+ 352.826	+ 0.128	+ 353.620	— 0.794	
		1850	17	6 6 1.354	352.917	0.064	353.729	0.812	
309	f^2 Orionis	1755	3	6 1 17.515	+ 345.919	+ 0.158	+ 345.840	+ 0.079	
		1850	3	6 6 46.200	346.042	0.101	345.969	0.073	
310	8 Geminorum . .	1755	4	6 1 21.079	+ 366.411	+ 0.140	+ 366.617	— 0.206	
		1850	3	6 7 9.220	366.506	0.060	366.717	0.211	
311	9 Geminorum . .	1755	5	6 2 1.981	+ 365.916	+ 0.135	+ 365.984	— 0.068	
		1850	3	6 7 49.650	366.006	0.056	366.079	0.073	
312	10 Geminorum . .	1755	4	6 3 58.751	+ 365.454	+ 0.101	+ 365.627	— 0.173	
		1850	3	6 9 45.976	365.513	0.023	365.697	0.184	
313	11 Geminorum . .	1755	3	6 4 24.316	+ 365.352	+ 0.106	+ 365.259	+ 0.093	
		1850	3	6 10 11.437	365.416	0.028	365.324	0.092	
314	12 Geminorum . .	1755	. .	. . . . . .	. . . .	+ 0.111	+ 364.719	. . .	
		1850	. .	6 10 15.7	. . . .	0.033	364.778	. . .	
315	μ Geminorum . .	1755	5	6 8 8.100	+ 363.162	+ 0.040	+ 362.679	+ 0.483	—0.008
		1850	498	6 13 53.111	363.166	— 0 032	362.691	0.475	
		1900	. .	6 16 54.688	363.141	0.070	362.674	0.467	
316	Lal. 12148 . . .	1755	. .	. . . . . .	. . . .	. . .	+ 349.586	. . .	
		1850	. .	6 14 5.1	. . . .	+ 0.017	349.626	. . .	
317	14 Geminorum . .	1755	4	6 11 0.447	+ 360.126	+ 0.024	+ 360.314	— 0.188	
		1850	3	6 16 42.567	360.115	— 0.049	360.308	0.193	
318	15 Geminorum (2d star)	1755	4	6 13 10.304	+ 357.732	0.000	+ 358.030	— 0.298	
		1850	15	6 18 50.139	357.699	— 0.070	358.001	0.302	
319	48 Aurigæ	1755	. .	6 12 48.974	+ 385.893	— 0.071	+ 386.035	— 0.142	
		1850	13	6 18 55.524	385.776	0.176	385.920	0.144	
320	16 Geminorum . .	1755	5	6 13 22.233	+ 356.987	+ 0.005	+ 357.240	— 0.253	
		1850	5	6 19 1.362	356.959	— 0.063	357.210	0.251	
321	ν Geminorum . .	1755	5	6 14 24.816	+ 356.380	— 0.013	+ 356.489	— 0.109	
		1850	55	6 20 3.361	356.338	0.075	356.448	0.110	

DECLINATIONS.

No.	Star.	Mag.	Epoch.	Declination.	Centennial variation.	Secular variation.	Struve's precession.	Proper motion.	Sec. var. of proper motion.
				° ′ ″	″	″	″	″	″
302	5 Geminorum . .	7.0	1755	+ 24 26 52.83	+ 24.06	— 53.65	+ 30.51	— 6.45	
		6.7	1850	24 26 51.48	— 26.91	53.66	— 20.47	6.44	
303	68 Orionis	6.0	1755	+ 19 49 12.13	+ 19.43	— 52.06	+ 21.82	— 2.39	
		6.0	1850	19 49 7.07	— 30.05	52.11	— 27.45	2.60	
304	6 Geminorum . .	6.5	1755	+ 22 56 16.39	+ 22.23	— 53.11	+ 22.18	+ 0.05	
		6.7	1850	22 56 13.58	— 28.21	53.08	— 28.22	0.01	
305	f^1 Orionis	6.0	1755	+ 16 9 40.26	+ 16.71	— 50.41	+ 18.11	— 1.40	
		5.7	1850	16 9 33.39	— 31.18	50.41	— 29.80	1.38	
306	κ Aurigæ	4.0	1755	+ 29 33 40.92	— 25.94	— 55.70	+ 2.06	—28.00	
		4.7	1850	29 32 51.15	78.84	55.67	— 50.91	27.93	
307	η Geminorum . .	4.5	1755	+ 22 33 8.01	— 2.31	— 52.83	— 0.78	— 1.53	
		3.3	1850	22 32 41.99	52.54	52.92	50.95	1.59	
308	71 Orionis	5.5	1755	+ 19 12 55.49	— 25.30	— 51.52	— 3.78	—21.52	
		6.0	1850	19 12 8.21	74.30	51.66	52.70	21.60	
309	f^2 Orionis	6.0	1755	+ 16 11 37.31	— 12.42	— 50.47	— 11.31	— 1.11	
		5.7	1850	16 11 2.75	60.35	50.44	59.25	1.10	
310	8 Geminorum . .	7.0	1755	+ 24 1 26.02	— 15.77	— 53.42	— 11.83	— 3.94	
		6.5	1850	24 0 45.95	66.50	53.38	62.56	3.94	
311	9 Geminorum . .	7.0	1755	+ 23 47 51.59	— 18.92	— 53.36	— 17.80	— 1.12	
		6.3	1850	23 47 9.55	69.59	53.32	68.49	1.10	
312	10 Geminorum . .	7.5	1755	+ 23 40 19.99	— 41.26	— 53.26	— 34.84	— 6.42	
		7.0	1850	23 39 16.79	91.78	53.11	85.45	6.33	
313	11 Geminorum . .	7.0	1755	+ 23 32 23.40	— 37.91	— 53.28	— 38.56	+ 0.65	
		7.3	1850	23 31 23.41	88.51	53.23	89.15	0.64	
314	12 Geminorum . .	8.0	1755	. . . . .	. . . .	— 53.41	— 39.18	. . .	
		7.5	1850	+ 23 19 47.2	. . . .	53.36	89.78	. . .	
315	μ Geminorum . .	3.0	1755	+ 22 36 49.96	— 83.22	— 53.07	— 71.16	—12.06	— 0.08
		3.0	1850	22 35 6.97	133.58	52.93	121.44	12.14	
			1900	22 33 53.58	160.02	52.85	147.84	12.18	
316	Lal. 12148 . . .	. .	1755	+ 17 40 6.47	— 74.85	— 50.93	— 74.85	0.00	
		7.0	1850	17 38 32.37	123.22	50.85	123.22	0.00	
317	14 Geminorum . .	7.5	1755	+ 21 45 19.68	— 99.12	— 52.40	— 96.33	— 2.79	
		7.2	1850	21 43 21.88	148.86	52.30	146.11	2.75	
318	15 Geminorum(2d star)	6.0	1755	+ 20 54 52.06	— 119.96	— 52.01	— 115.25	— 4.71	
		7.0	1850	20 52 34.65	169.31	51.89	164.65	4.66	
319	48 Aurigæ	6.0	1755	+ 30 37 2.45	— 114.99	— 56.14	— 112.12	— 2.87	
		5.7	1850	30 34 47.90	168.25	55.99	165.42	2.83	
320	16 Geminorum . .	6.0	1755	+ 20 37 7.89	— 117.58	— 51.90	— 117.00	— 0.58	
		6.8	1850	20 34 52.79	166.83	51.78	166.29	0.54	
321	ν Geminorum . .	5.0	1755	+ 20 20 31.93	— 128.14	— 52.00	— 126.10	— 2.04	
		4.7	1850	20 18 6.76	177.45	51.80	175.30	2.15	

RIGHT ASCENSIONS.

No.	Star.	Epoch.	Number of observations.	Right ascension.	Centennial variation.	Secular variation.	Struve's precession.	Proper motion.	Sec. var. of proper motion.
				h. m. s.	s.	s.	s.	s.	s.
322	α rgus	1850	. .	6 20 37.48	+ 132.99	+ 0.10	+ 132.90	+ 0.09	
		1875	. .	6 21 10.73	133.02	0.10	132.93	0.09	
		1900	. .	6 21 43.99	133.05	0.10	132.96	0.09	
323	19 Geminorum . .	1755	5	6 17 31.743	+ 345.321	− 0.010	+ 345.368	− 0.047	
		1850	4	6 22 59.785	345.285	0.066	345.335	0.048	
324	20 Geminorum . .	1755	5	6 17 59.571	+ 350.468	− 0.027	+ 350.139	+ 0.329	
		1850	3	6 23 32.494	350.415	0.085	350.085	0.330	
325	21 Geminorum . .	1755	2	6 18 0.471	+ 350.358	− 0.024	+ 350.150	+ 0.208	
		1850	3	6 23 33.290	350.307	0.084	350.097	0.211	
326	49 Aurigæ	1755	5	6 19 45.769	+ 378.397	− 0.160	+ 378.413	− 0.016	
		1850	15	6 25 45.160	378.201	0.252	378.222	0.021	
327	22 Geminorum . .	1755	3	6 20 11.579	+ 354.256	− 0.067	+ 354.422	− 0.166	
		1850	3	6 25 48.083	354.162	0.131	354.327	0.165	
328	23 Geminorum . .	1755	3	6 21 50.665	+ 347.674	− 0.064	+ 347.631	+ 0.043	
		1850	6	6 27 20.918	347.587	0.121	347.548	0.039	
329	51 (H) Cephei . .	1755	. .	5 39 27.33	+3091.51	+103.66	+3097.40	− 5.89	
		1775	. .	5 49 47.43	3106.84	+ 49.94	3113.03	6.19	
		1800	. .	6 2 45.07	3110.80	− 18.22	3117.35	6.55	
		1825	. .	6 15 41.55	3097.79	85.76	3104.64	6.85	
		1850	. .	6 28 32.65	3068.16	150.40	3075.27	7.11	
		1875	. .	6 41 14.35	3022.99	210.09	3030.33	7.34	
		1900	. .	6 53 42.95	+2963.67	−263.18	+2971.20	− 7.53	
330	53 Aurigæ	1755	4	6 22 50.332	+ 381.037	− 0.221	+ 381.284	− 0.247	
		1850	3	6 28 52.203	380.781	0.317	381.030	0.249	
331	γ Geminorum . .	1755	10	6 23 33.234	+ 346.891	− 0.078	+ 346.621	+ 0.270	−0.001
		1850	426	6 29 2.737	346.790	0.133	346.525	0.265	
		1900	. .	6 31 56.114	346.715	0.164	346.453	0.262	
332	54 Aurigæ	1755	5	6 24 5.718	+ 378.845	− 0.232	+ 379.082	− 0.237	
		1850	13	6 30 5.501	378.580	0.326	378.822	0.242	
333	25 Geminorum . .	1755	5	6 25 53.849	+ 378.772	− 0.254	+ 378.850	− 0.078	
		1850	7	6 31 53.554	378.486	0.348	378.565	0.079	
334	26 Geminorum . .	1755	5	6 28 7.951	+ 349.790	− 0.147	+ 349.774	+ 0.016	
		1850	12	6 33 40.176	349.623	0.206	349.611	0.014	
335	ε Geminorum . .	1755	5	6 28 50.942	+ 369.755	− 0.254	+ 369.864	− 0.109	
		1850	93	6 34 42.084	369.476	0.335	369.586	0.112	
336	28 Geminorum . .	1755	5	6 29 13.181	+ 381.061	− 0.325	+ 381.124	− 0.063	
		1850	9	6 35 15.028	380.707	0.421	380.772	0.065	
337	α Canis Majoris . .	1755	⊙	6 34 21.105	+ 264.453	− 0.059	+ 268.073	− 3.620	−0.086
		1850	. .	6 38 32.307	264.392	0.069	268.094	3.702	
		1900	. .	6 40 44.494	264.356	0.073	268.102	3.746	
338	33 Geminorum . .	1755	1	6 35 43.319	+ 345.822	− 0.196	+ 346.028	− 0.206	
		1850	8	6 41 11.753	345.610	0.250	345.820	0.210	

DECLINATIONS.

No.	Star.	Mag.	Epoch.	Declination.	Centennial variation.	Secular variation.	Struve's precession.	Proper motion.	Sec. var. of proper motion.
				° ′ ″	″	″	″	″	″
322	α Argus	0.4	1850	− 52 36 55.39	− 179.35	− 19.25	− 180.24	+ 0.89	
			1875	52 37 40.83	184.16	19.25	185.06	0.90	
			1900	52 38 27.48	188.97	19.25	189.87	0.90	
323	19 Geminorum . .	6.5	1755	+ 16 3 2.36	− 155.31	− 50.16	− 153.32	− 1.99	
		6.6	1850	16 0 12.20	202.90	50.02	200.90	2.00	
324	20 Geminorum . .	8.0	1755	+ 17 55 39.61	− 156.11	− 50.95	− 157.36	+ 1.25	
		6.3	1850	17 52 48.33	204.45	50.81	205.64	1.19	
325	21 Geminorum . .	7.0	1755	+ 17 55 55.53	− 154.60	− 50.92	− 157.49	+ 2.89	
		6.5	1850	17 53 5.70	202.90	50.78	205.74	2.84	
326	49 Aurigæ	6.0	1755	+ 28 11 13.18	− 175.82	− 54.92	− 172.81	− 3.01	
		5.7	1850	28 8 1.40	227.90	54.72	224.90	3.00	
327	22 Geminorum . .	7.5	1755	+ 19 35 32.63	− 176.79	− 51.38	− 176.56	− 0.23	
		7.2	1850	19 32 21.53	225.52	51.22	225.34	0.18	
328	23 Geminorum . .	8.0	1755	+ 16 58 14.57	− 193.95	− 50.41	− 190.96	− 2.99	
		7.1	1850	16 54 47.60	241.76	50.25	238.77	2.99	
329	51 (H) Cephei . . .	. .	1755	87 15 59.39	+ 174.55	−448.40	+ 179.62	− 5.07	
			1775	87 16 25.15	+ 84.45	451.92	+ 89.35	4.90	
			1800	87 16 32.16	− 28.75	452.81	− 24.08	4.67	
			1825	87 16 10.85	141.66	449.79	137.24	4.42	
		5.3	1850	87 15 21.41	253.32	442.99	249.15	4.17	
			1875	87 14 4.35	362.82	432.67	358.91	3.91	
			1900	87 12 20.24	− 469.33	−419.26	− 465.69	− 3.64	
330	53 Aurigæ	7.5	1755	+ 29 10 2.85	− 201.94	− 55.19	− 199.62	− 2.32	
		6.0	1850	29 6 26.16	254.21	54.86	251.96	2.25	
331	γ Geminorum . .	3.0	1755	+ 16 35 2.90	− 210.50	− 50.37	− 205.80	− 4.70	− 0.05
		2.3	1850	16 31 20.22	258.25	50.15	253.50	4.75	
			1900	16 29 4.84	283.29	50.03	278.52	4.77	
332	54 Aurigæ	6.0	1755	+ 28 27 12.77	− 214.60	− 54.84	− 210.56	− 4.04	
		6.0	1850	28 23 24.19	266.58	54.60	262.57	4.01	
333	25 Geminorum . .	7.0	1755	+ 28 23 47.03	− 226.97	− 54.79	− 226.24	− 0.73	
		6.5	1850	28 19 46.72	278.91	54.55	278.20	0.71	
334	26 Geminorum .	5.5	1755	+ 17 51 38.56	− 255.75	− 50.65	− 245.67	−10.08	
		5.7	1850	17 47 12.75	303.78	50.46	293.56	10.22	
335	ε Geminorum . .	3.0	1755	+ 25 16 27.38	− 253.08	− 53.57	− 251.87	+ 1.21	
		3.3	1850	25 20 51.96	303.86	53.33	302.52	1.34	
336	28 Geminorum . .	6.0	1755	+ 29 11 31.67	− 258.24	− 55.02	− 255.12	− 3.12	
		6.0	1850	29 7 1.56	310.38	54.74	307.28	3.10	
337	α Canis Majoris . .	1.0	1755	− 16 23 53.31	− 420.92	− 37.56	− 299.62	−121.30	
		1.0	1850	16 30 51.11	456.54	37.44	335.67	120.87	
			1900	16 34 44.06	475.24	37.37	354.58	120.66	
338	33 Geminorum . .	6.0	1755	+ 16 27 23.22	− 310.14	− 49.67	− 311.47	+ 1.33	
		6.0	1850	16 22 6.21	357.20	49.42	358.58	1.38	

RIGHT ASCENSIONS.

No.	Star.	Epoch.	Number of observations.	Right ascension.			Centennial variation.	Secular variation.	Struve's precession.	Proper motion.	Sec. var. of proper motion.
				h.	m.	s.	s.	s.	s.	s.	s.
339	d Geminorum . .	1755	5	6	36	51.365	+ 360.348	— 0.307	+ 360.409	— 0.061	
		1850	7	6	42	33.547	360.025	0.374	360.092	0.067	
340	37 Geminorum . .	1755	5	6	40	13.831	+ 369.966	— 0.414	+ 370.259	— 0.293	
		1850	7	6	46	5.100	369.536	0.493	369.831	0.295	
341	39 Geminorum . .	1755	5	6	43	40.456	+ 370.871	— 0.466	+ 372.129	— 1.258	
		1850	7	6	49	32.560	370.390	0.546	371.646	1.256	
342	40 Geminorum . .	1755	5	6	44	19.393	+ 371.522	— 0.489	+ 371.649	— 0.127	
		1850	7	6	50	12.106	371.020	0.568	371.150	0.130	
343	41 Geminorum . .	1755	4	6	46	10.536	+ 345.418	— 0.302	+ 345.551	— 0.133	
		1850	3	6	51	38.539	345.106	0.354	345.244	0.138	
344	ε Canis Majoris . .	1755	5	6	49	0.020	+ 235.590	+ 0.138	+ 235.564	+ 0.026	
		1850	349	6	52	43.891	235.718	0.132	235.691	0.027	
		1900	. .	6	54	41.767	235.784	0.131	235.759	0.025	
345	ω Geminorum . .	1755	5	6	47	28.157	+ 366.658	— 0.490	+ 366.766	— 0.108	
		1850	16	6	53	16.249	366.158	0.562	366.266	0.108	
346	W 6h 1656 . . .	1850	. .	6	54	20.7	. . . .	— 0.651	+ 373.534	. . .	
347	ζ Geminorum .	1755	5	6	49	33.754	+ 356.880	— 0.431	+ 356.903	— 0.023	
		1850	195	6	55	12.586	356.442	0.492	356.464	0.022	
348	44 Geminorum . .	1755	5	6	50	32.452	+ 362.302	— 0.491	+ 362.345	— 0.043	
		1850	8	6	56	16.407	361.804	0.558	361.849	0.045	
349	45 Geminorum . .	1755	5	6	54	18.260	+ 344.922	— 0.394	+ 345.014	— 0.092	
		1850	11	6	59	45.751	344.525	0.442	344.628	0.103	
350	τ Geminorum . .	1755	5	6	55	31.206	+ 383.562	— 0.790	+ 383.858	— 0.296	
		1850	23	7	1	35.220	382.769	0.881	383.067	0.298	
351	47 Geminorum . .	1755	5	6	56	9.953	+ 373.700	— 0.684	+ 373.787	— 0.087	
		1850	18	7	2	4.647	373.013	0.762	373.100	0.087	
352	δ Canis Majoris . .	1755	5	6	58	26.028	+ 243.690	+ 0.116	+ 243.805	— 0.115	+0.003
		1850	129	7	2	17.586	243.801	0.118	243.915	0.114	
		1900	. .	7	4	19.501	243.860	0.117	243.973	0.113	
353	B. A. C. 2347 . .	1755	1	6	57	16.864	+ 343.280	— 0.400	+ 343.476	— 0.196	
		1850	3	7	2	42.792	342.877	0.447	343.057	0.180	
354	48 Geminorum . .	1755	5	6	57	31.970	+ 365.960	— 0.620	+ 366.081	— 0.121	
		1850	15	7	3	19.342	365.338	0.690	365.463	0.125	
355	49 Geminorum . .	1755	3	6	57	44.032	+ 370.437	— 0.674	+ 370.550	— 0.113	
		1850	6	7	3	35.632	369.762	0.748	369.883	0.121	
356	50 Geminorum . .	1755	. .	6	58	49.137	+ 343.015	— 0.404	+ 343.034	— 0.019	
		1850	3	7	4	14.812	342.609	0.451	342.626	0.017	
357	51 Geminorum . .	1755	4	6	59	17.445	+ 345.350	— 0.438	+ 345.425	— 0.075	
		1850	71	7	4	45.323	344.911	0.486	344.990	0.079	
358	B. A. C. 2363 . .	1755	1	6	59	28.828	+ 367.061	— 0.658	+ 367.583	— 0.522	
		1850	9	7	5	17.228	366.404	0.727	366.911	0.507	

DECLINATIONS.

No.	Star.	Mag.	Epoch.	Declination.	Centennial variation.	Secular variation.	Struve's precession.	Proper motion.	Sec. var. of proper motion.
				° ′ ″	″	″	″	″	″
339	*d* Geminorum . .	6.5	1755	+ 22 1 31.53	− 326.25	− 51.74	− 321.28	− 4.97	
		6.0	1850	21 55 58.30	375.26	51.45	370.31	4.95	
340	37 Geminorum . .	6.0	1755	+ 25 39 25.48	− 350.55	− 52.95	− 350.41	− 0.14	
		6.3	1850	25 33 28.61	400.69	52.61	400.59	0.10	
341	39 Geminorum . .	6.5	1755	+ 26 22 40.56	− 373.15	− 52.78	− 380.06	+ 6.91	
		6.3	1850	26 16 22.31	423.11	52.41	430.19	7.08	
342	40 Geminorum . .	6.5	1755	+ 26 13 17.91	− 388.46	− 53.00	− 385.63	− 2.83	
		6.3	1850	26 6 45.02	438.63	52.63	435.81	2.82	
343	41 Geminorum . .	7.0	1755	+ 16 23 36.31	− 401.73	− 49.18	− 401.53	− 0.20	
		6.0	1850	16 16 52.53	448.30	48.86	448.12	0.18	
344	ε Canis Majoris . .	2.5	1755	− 28 39 15.70	− 427.01	− 33.41	− 425.73	− 1.28	0.00
		1.8	1850	28 46 16.42	458.69	33.27	457.41	1.28	
			1900	28 50 9.92	475.31	33.21	474.03	1.28	
345	ω Geminorum . .	6.0	1755	+ 24 32 24.26	− 414.66	− 52.27	− 412.60	− 2.06	
		5.7	1850	24 25 26.82	464.14	51.92	462.00	2.14	
346	W 6h 1656 . . .	8.2	1850	+ 27 3 3.9	. . . . .	− 52.76	− 472.42	. . . .	
347	ζ Geminorum . .	4.0	1755	+ 20 54 20.90	− 432.33	− 50.78	− 430.54	− 1.79	
		4.0	1850	20 47 7.36	480.34	50.33	478.50	1.84	
348	44 Geminorum . .	6.5	1755	+ 22 58 46.56	− 440.49	− 51.38	− 438.90	− 1.59	
		6.0	1850	22 51 24.95	489.16	50.98	487.53	1.63	
349	45 Geminorum . .	6.0	1755	+ 16 17 55.79	− 482.32	− 48.69	− 470.99	−11.33	
		5.7	1850	16 9 55.67	528.40	48.32	517.09	11.31	
350	τ Geminorum . .	5.0	1755	+ 30 37 15.08	− 486.78	− 54.07	− 481.33	− 5.45	
		4.7	1850	30 29 8.32	537.91	53.56	532.50	5.41	
351	47 Geminorum . .	6.0	1755	+ 27 14 3.02	− 491.63	− 52.75	− 486.82	− 4.81	
		6.0	1850	27 5 52.24	541.52	52.25	536.63	4.89	
352	δ Canis Majoris . .	. .	1755	− 26 1 15.07	− 505.17	− 34.15	− 506.10	+ 0.93	0.00
		2.1	1850	26 9 30.36	537.53	33.97	538.46	0.93	
			1900	26 14 3.37	554.49	33.89	555.42	0.93	
353	B. A. C. 2347 . .	. .	1755	+ 15 42 46.01	− 500.28	− 48.29	− 496.21	− 4.07	
		7.3	1850	15 34 29.02	545.97	47.90	542.00	3.97	
354	48 Geminorum . .	6.0	1755	+ 24 30 48.75	− 503.15	− 51.63	− 498.42	− 4.73	
		6.0	1850	24 22 27.55	551.97	51.16	547.12	4.85	
355	49 Geminorum . .	8.0	1755	+ 26 8 0.01	− 504.68	− 52.10	− 500.14	− 4.54	
		7.2	1850	25 59 37.13	553.95	51.62	549.39	4.56	
356	50 Geminorum . .	7.5	1755	+ 15 33 55.03	− 508.79	− 48.13	− 509.28	+ 0.49	
		7.5	1850	15 25 30.08	554.37	47.79	554.95	0.58	
357	51 Geminorum . .	5.0	1755	+ 16 33 4.80	− 517.93	− 48.47	− 513.28	− 4.65	
		5.7	1850	16 24 30.95	563.79	48.07	559.19	4.60	
358	B. A. C. 2363 . .	. .	1755	. . . . . .	. . . . .	− 51.46	− 514.91	. . .	
		7.3	1850	+ 24 57 40.8	. . . . .	50.99	563.40	. . .	

RIGHT ASCENSIONS.

No.	Star.	Epoch.	Number of observations.	Right ascension.			Centennial variation.	Secular variation.	Struve's precession.	Proper motion.	Sec. var. of proper motion.
				h.	m.	s.	s.	s.	s.	s.	s.
359	52 Geminorum . .	1755	4	6	59	41.611	+ 368.476	— 0.701	+ 368.056	+ 0.420	
		1850	15	7	5	31.338	367.781	0.763	367.378	0.403	
360	53 Geminorum . .	1755	5	7	0	37.704	+ 376.334	— 0.782	+ 376.510	— 0.176	
		1850	10	7	6	34.857	375.553	0.861	375.733	0.180	
361	λ Geminorum . .	1755	5	7	3	59.809	+ 345.908	— 0.490	+ 346.207	— 0.299	
		1850	86	7	9	28.194	345.423	0.533	345.722	0.299	
362	δ Geminorum . .	1755	5	7	5	28.210	+ 359.725	— 0.653	+ 359.922	— 0.197	
		1850	619	7	11	9.646	359.079	0.710	359.271	0.192	
		1900	. .	7	14	9.096	358.717	0.739	358.910	0.193	
363	56 Geminorum . .	1755	5	7	7	28.291	+ 355.352	— 0.629	+ 355.831	— 0.479	
		1850	11	7	13	5.584	354.729	0.684	355.208	0.479	
364	A Geminorum . .	1755	5	7	8	30.944	+ 367.405	— 0.792	+ 367.960	— 0.555	
		1850	6	7	14	19.610	366.620	0.859	367.178	0.558	
365	58 Geminorum . .	1755	2	7	8	43.577	+ 361.984	— 0.725	+ 362.279	— 0.295	
		1850	3	7	14	27.126	361.266	0.787	361.564	0.298	
366	Piazzi VII 67 . .	1755	. .	7	5	8.40	+ 640.44	— 7.23	+ 640.16	+ 0.28	
		1800	. .	7	9	55.84	637.09	7.66	636.82	0.27	
		1850	. .	7	15	13.39	633.14	8.12	632.88	0.26	
		1900	. .	7	20	28.93	628.96	8.58	628.73	0.23	
367	59 Geminorum . .	1755	5	7	9	17.146	+ 375.181	— 0.890	+ 375.140	+ 0.041	
		1850	14	7	15	13.155	374.299	0.967	374.257	0.042	
368	ι Geminorum . .	1755	5	7	10	38.828	+ 374.627	— 0.928	+ 375.526	— 0.899	
		1850	48	7	16	24.294	373.710	1.003	374.615	0.905	
369	61 Geminorum . .	1755	5	7	12	28.753	+ 354.969	— 0.680	+ 355.094	— 0.125	
		1850	3	7	18	5.658	354.298	0.732	354.426	0.128	
370	63 Geminorum . .	1755	5	7	13	10.394	+ 357.728	— 0.734	+ 358.108	— 0.380	
		1850	29	7	18	49.896	357.004	0.791	357.389	0.385	
371	b^1 Geminorum . .	1755	5	7	14	2.881	+ 375.840	— 0.990	+ 376.149	— 0.309	
		1850	6	7	19	59.471	374.865	1.063	375.179	0.314	
372	b^2 Geminorum . .	1755	7	7	14	32.521	+ 375.341	— 0.982	+ 375.520	— 0.179	
		1850	9	7	20	28.641	374.373	1.056	374.555	0.182	
373	B. A. C. 2472 . .	1850	7	7	21	19.638	+ 374.243	— 1.070	+ 374.466	— 0.223	
374	W 7^h 685 . . .	1850	. .	7	23	9.4	. . . .	— 0.676	+ 346.354	. . .	
375	67 Geminorum . .	1755	. .	7	19	25.779	+ 342.973	— 0.605	+ 343.408	— 0.435	
		1850	4	7	24	51.324	342.379	0.645	342.818	0.439	
376	a^2 Geminorum . .	1755		7	18	55.563	+ 385.577	— 1.236	+ 386.899	— 1.322	—0.003
		1850	623	7	25	1.292	384.367	1.321	385.692	1.325	
		1900	. .	7	28	13.309	383.696	1.363	385.023	1.327	
377	68 Geminorum . .	1755	1	7	19	36.343	+ 343.768	— 0.606	+ 343.848	— 0.080	
		1850	43	7	25	2.643	343.173	0.646	343.248	0.075	

DECLINATIONS.

No.	Star.	Mag.	Epoch.	Declination.	Centennial variation.	Secular variation.	Struve's precession.	Proper motion.	Sec. var. of proper motion.
				° ′ ″	″	″	″	″	″
359	52 Geminorum . .	7.0	1755	+ 25 17 9.51	− 527.42	− 51.73	− 516.71	−10.71	
		6.3	1850	25 8 25.17	576.40	51.41	565.62	10.78	
360	53 Geminorum . .	6.0	1755	+ 28 17 55.07	− 526.83	− 52.74	− 524.62	− 2.21	
		6.3	1850	28 9 10.87	576.68	52.22	574.50	2.18	
361	λ Geminorum . .	4.5	1755	+ 16 57 34.47	− 558.18	− 48.37	− 553.00	− 5.18	
		4.0	1850	16 48 22.44	603.91	47.93	598.70	5.21	
362	δ Geminorum . .	3.5	1755	+ 22 24 33.97	− 567.01	− 50.16	− 565.37	− 1.64	
		3.3	1850	22 15 12.75	614.41	49.62	612.78	1.63	
			1900	22 9 59.35	639.15	49.34	637.52	− 1.63	
363	56 Geminorum . .	5.5	1755	+ 20 52 55.78	− 584.19	− 49.48	− 582.15	− 2.04	
		5.7	1850	20 43 18.53	630.98	49.05	628.86	2.12	
364	A Geminorum . .	6.0	1755	+ 25 29 48.75	− 594.58	− 50.89	− 590.95	− 3.63	
		5.7	1850	25 20 1.02	642.66	50.34	639.10	3.56	
365	58 Geminorum . .	7.0	1755	+ 23 23 34.86	− 597.21	− 50.15	− 592.71	− 4.50	
		6.3	1850	23 13 44.96	644.61	49.63	640.14	4.47	
366	Piazzi VII 67 . .	. .	1755	+ 68 55 26.43	− 566.35	− 89.48	− 562.64	− 3.71	
			1800	68 51 2.58	606.38	88.42	602.66	3.72	
		5.5	1850	68 45 48.39	650.28	87.19	646.54	3.74	
			1900	68 40 12.41	693.54	85.88	689.78	3.76	
367	59 Geminorum . .	6.5	1755	+ 28 5 10.75	− 595.94	− 52.00	− 597.41	+ 1.47	
		6.9	1850	27 55 21.23	645.06	51.42	646.50	1.44	
368	ι Geminorum . .	4.0	1755	+ 28 15 36.46	− 616.35	− 51.70	− 607.36	− 8.99	
		4.0	1850	28 5 27.70	665.18	51.11	656.31	8.87	
369	61 Geminorum . .	7.5	1755	+ 20 43 25.67	− 626.54	− 48.93	− 624.02	− 2.52	
		6.0	1850	20 33 8.45	672.78	48.41	670.30	2.48	
370	63 Geminorum . .	6.0	1755	+ 21 55 21.19	− 641.77	− 49.30	− 629.80	−11.97	
		5.7	1850	21 44 49.33	688.37	48.80	676.36	12.01	
371	b^1 Geminorum . .	5.5	1755	+ 28 35 54.59	− 643.50	− 51.67	− 637.06	− 6.44	
		5.3	1850	28 25 20.04	692.29	51.05	685.87	6.42	
372	b^2 Geminorum . .	5.5	1755	+ 28 23 48.60	− 644.05	− 51.58	− 641.16	− 2.89	
		6.3	1850	28 13 13.57	692.76	50.96	689.90	2.86	
373	B. A. C. 2472 . .	8.0	1850	+ 28 13 0.90	− 701.88	− 50.86	− 696.88	− 5.00	
374	W 7^h 685 . . .	6.2	1850	+ 17 24 4.4	. . . .	− 46.93	− 711.85	. . .	
375	67 Geminorum . .	7.0	1755	+ 16 8 32.36	− 682.92	− 46.70	− 681.47	− 1.45	
		7.5	1850	15 57 22.59	727.05	46.19	725.75	1.30	
376	α^2 Geminorum . .	. .	1755	+ 32 23 57.98	− 685.44	− 52.48	− 677.37	− 8.07	+ 0.20
		1.7	1850	32 12 43.25	734.94	51.72	727.06	7.88	
			1900	32 6 29.33	760.70	51.32	752.92	7.78	
377	68 Geminorum . .	5.0	1755	+ 16 19 53.01	− 685.56	− 46.93	− 682.98	− 2.58	
		5.7	1850	16 8 40.65	729.87	46.36	727.30	2.57	

RIGHT ASCENSIONS.

No.	Star.	Epoch.	Number of observations.	Right ascension.			Centennial variation.	Secular variation.	Struve's precession.	Proper motion.	Sec. var. of proper motion.
				h.	m.	s.	s.	s.	s.	s.	s
397	6 Cancri	1755	5	7	48	25.691	+ 371.395	− 1.421	+ 371.536	− 0.141	
		1850	156	7	54	17.868	370.021	1.471	370.166	0.145	
398	7 Cancri	1755	5	7	49	20.798	+ 356.371	− 1.122	+ 356.788	− 0.417	
		1850	3	7	54	58.839	355.287	1.161	355.707	0.420	
399	8 Cancri	1755	5	7	51	24.176	+ 335.887	− 0.776	+ 336.036	− 0.149	
		1850	16	7	56	42.915	335.139	0.799	335.294	0.155	
400	μ^1 Cancri	1755	5	7	51	45.421	+ 357.719	− 1.176	+ 357.922	− 0.203	
		1850	9	7	57	24.717	356.584	1.215	356.790	0.206	
401	B. A. C. 2703 . .	1755	1	7	52	4.808	+ 356.307	− 1.167	+ 357.460	− 1.153	
		1850	8	7	57	42.767	355.180	1.205	356.331	1.151	
402	3 (H) Ursæ Majoris.	1755	. . .	7	48	4.57	+ 620.59	−11.31	+ 620.02	+ 0.57	
		1800	. . .	7	52	42.68	615.43	11.56	614.87	0.56	
		1850	. . .	7	57	48.94	609.59	11.80	609.03	0.56	
		1900	. . .	8	2	52.26	603.63	12.02	603.08	0.55	
403	μ^2 Cancri	1755	5	7	53	18.806	+ 355.364	− 1.142	+ 355.220	+ 0.144	
		1850	20	7	58	55.880	354.263	1.176	354.115	0.148	
404	11 Cancri	1755	4	7	53	48.010	+ 369.874	− 1.461	+ 369.981	− 0.107	
		1850	3	7	59	38.725	368.464	1.508	368.574	0.110	
405	12 Cancri	1755	5	7	54	59.410	+ 337.024	− 0.812	+ 336.963	+ 0.061	
		1850	24	8	0	19.212	336.242	0.835	336.182	0.060	
406	ψ^1 Cancri	1755	5	7	55	22.390	+ 365.204	− 1.385	+ 365.525	− 0.321	
		1850	3	8	1	8.702	363.868	1.427	364.195	0.327	
407	15 Argus	1755	5	7	57	6.810	+ 255.314	+ 0.091	+ 255.989	− 0.675	+0.004
		1850	207	8	1	9.400	255.404	0.098	256.074	0.670	
		1900	. . .	8	3	17.114	255.454	0.102	256.123	0.669	
408	ψ^2 Cancri	1755	5	7	55	39.435	+ 364.090	− 1.413	+ 364.686	− 0.596	
		1850	32	8	1	24.678	362.730	1.452	363.350	0.620	
409	ζ^1 Cancri	1755	5	7	58	7.990	+ 346.033	− 1.015	+ 345.635	+ 0.398	
		1850	64	8	3	36.259	345.055	1.038	344.674	0.381	
410	χ Cancri	1755	5	8	5	8.161	+ 367.627	− 1.621	+ 367.753	− 0.126	
		1850	11	8	10	56.670	366.070	1.657	366.224	0.154	
411	B. A. C. 2788 . .	1850	13	8	11	35.620	+ 351.121	− 1.239	+ 350.763	+ 0.358	
412	λ Cancri	1755	5	8	5	55.702	+ 359.486	− 1.380	+ 359.642	− 0.156	
		1850	45	8	11	36.587	358.160	1.413	358.313	0.153	
413	d^1 Cancri	1755	5	8	9	18.295	+ 345.652	− 1.112	+ 346.135	− 0.483	
		1850	82	8	14	46.161	344.588	1.130	345.070	0.482	
414	21 Cancri	1755	5	8	10	29.932	+ 329.668	− 0.786	+ 329.718	− 0.050	
		1850	8	8	15	42.760	328.916	0.798	328.969	0.053	
415	B. A. C. 2810 . .	1755	1	8	10	47.088	+ 343.437	− 1.079	+ 343.492	− 0.055	
		1850	3	8	16	12.863	342.403	1.098	342.462	0.059	

DECLINATIONS.

No.	Star.	Mag.	Epoch.	Declination.	Centennial variation.	Secular variation.	Struve's precession.	Proper motion.	Sec. var. of proper motion.
				° ′ ″	″	″	″	″	″
397	6 Cancri	5.5	1755	+ 28 27 31.98	— 919.66	— 47.82	— 914.26	— 5.40	
		5.4	1850	28 12 36.86	964.68	46.97	959.28	5.40	
398	7 Cancri	7.5	1755	+ 22 44 9.29	— 922.18	— 45.74	— 921.40	— 0.78	
		6.3	1850	22 29 12.70	965.27	44.97	964.54	0.73	
399	8 Cancri	6.0	1755	+ 13 47 47.45	— 945.40	— 42.90	— 937.36	— 8.04	
		6.0	1850	13 32 30.06	985.84	42.25	977.82	8.02	
400	μ^1 Cancri	6.0	1755	+ 23 18 49.53	— 942.66	— 45.67	— 940.11	— 2.55	
		6.3	1850	23 3 33.51	985.68	44.89	983.12	2.56	
401	B. A. C. 2703 . .	7.8	1755	+ 23 8 19.01	— 945.93	— 45.33	— 942.58	— 3.35	
		7.5	1850	22 53 0.03	988.63	44.56	985.36	3.27	
402	3 (H) Ursæ Majoris.	. .	1755	+ 69 9 30.51	— 911.22	— 80.33	— 911.51	+ 0.29	
			1800	69 2 32.39	947.03	78.80	947.29	0.26	
		5.3	1850	68 54 29.10	986.00	77.08	986.22	0.22	
			1900	68 46 6.54	1024.10	75.32	1024.28	0.18	
403	μ^2 Cancri	6.5	1755	+ 22 16 18.61	— 959.82	— 45.33	— 952.12	— 7.70	
		5.7	1850	22 0 46.46	1002.51	44.55	994.71	7.80	
404	11 Cancri	7.0	1755	+ 28 10 17.85	— 960.19	— 47.02	— 955.86	— 4.33	
		7.0	1850	27 54 44.60	1004.44	46.15	1000.13	4.31	
405	12 Cancri	6.0	1755	+ 14 20 1.83	— 966.76	— 42.70	— 965.02	— 1.74	
		6.3	1850	14 4 24.24	1007.00	42.02	1005.24	1.76	
406	ψ^1 Cancri	7.5	1755	+ 26 32 37.92	— 973.91	— 46.21	— 967.96	— 5.95	
		6.8	1850	26 16 51.98	1017.41	45.36	1011.48	5.93	
407	15 Argus	3.5	1755	— 23 36 47.59	— 976.82	— 31.99	— 981.26	+ 4.44	+ 0.09
		3.2	1850	23 52 29.96	1007.05	31.62	1011.57	4.52	
			1900	24 0 57.43	1022.83	31.48	1027.39	4.56	
408	ψ^2 Cancri	7.5	1755	+ 26 13 46.22	— 1006.85	— 45.99	— 970.13	—36.72	
		5.7	1850	25 57 29.09	1050.14	45.15	1013.50	36.64	
409	ζ^1 Cancri	6.0	1755	+ 18 21 54.76	— 1002.36	— 43.61	— 988.90	—13.46	
		4.7	1850	18 5 42.94	1043.45	42.91	1030.02	13.43	
410	χ Cancri	6.0	1755	+ 27 59 23.16	— 1080.13	— 45.53	— 1041.88	—38.25	
		5.3	1850	27 41 56.63	1122.94	44.61	1084.60	38.34	
411	B. A. C. 2788 . .	6.0	1850	+ 21 13 0.17	— 1094.62	— 42.58	— 1089.39	— 5.23	
412	λ Cancri	6.0	1755	+ 24 46 25.00	— 1052.23	— 44.29	— 1047.87	— 4.36	
		5.7	1850	24 29 25.52	1093.89	43.42	1089.45	4.44	
413	d^1 Cancri	6.0	1755	+ 19 5 55.41	— 1075.42	— 42.21	— 1072.90	— 2.52	
		6.0	1850	18 48 34.86	1115.13	41.40	1112.60	2.53	
414	21 Cancri	7.0	1755	+ 11 24 10.12	— 1084.68	— 40.03	— 1081.75	— 2.93	
		6.3	1850	11 6 41.72	1122.38	39.32	1119.45	2.93	
415	B. A. C. 2810 . .	7.5	1755	+ 17 57 43.97	— 1096.07	— 41.68	— 1083.84	—12.23	
		7.0	1850	17 40 4.02	1135.29	40.89	1123.13	12.16	

RIGHT ASCENSIONS.

No.	Star.	Epoch.	Number of observations.	Right ascension.	Centennial variation.	Secular variation.	Struve's precession.	Proper motion.	Sec. var. of proper motion.
				h. m. s.	s.	s.	s.	s.	s.
416	d^2 Cancri	1755	5	8 11 55.965	+ 341.701	− 1.080	+ 343.094	− 1.393	
		1850	18	8 17 20.090	340.664	1.104	342.068	1.404	
417	ϕ^2 Cancri (mean) . .	1755	2	8 11 55.676	+ 365.805	− 1.612	+ 365.943	− 0.138	
		1850	35	8 17 42.457	364.258	1.644	364.398	0.140	
418	υ^1 Cancri	1755	3	8 12 3.061	+ 359.611	− 1.473	+ 360.065	− 0.454	
		1850	12	8 17 44.022	358.198	1.501	358.652	0.454	
419	27 Cancri	1755	5	8 13 9.476	+ 333.534	− 0.892	+ 333.727	− 0.193	
		1850	6	8 18 25.929	332.680	0.906	332.880	0.200	
420	υ^2 Cancri	1755	5	8 14 2.781	+ 358.499	− 1.465	+ 358.856	− 0.357	
		1850	9	8 19 42.690	357.095	1.492	357.456	0.361	
421	29 Cancri	1755	5	8 14 55.480	+ 336.606	− 0.963	+ 336.810	− 0.204	
		1850	36	8 20 14.819	335.685	0.976	335.897	0.212	
422	υ^3 Cancri	1755	5	8 16 58.805	+ 357.641	− 1.483	+ 358.294	− 0.653	
		1850	16	8 22 37.891	356.221	1.507	356.875	0.654	
423	θ Cancri	1755	5	8 17 35.687	+ 344.291	− 1.158	+ 344.793	− 0.502	
		1850	43	8 23 2.239	343.182	1.176	343.685	0.503	
424	η Cancri	1755	3	8 18 30.151	+ 349.573	+ 1.286	+ 349.809	− 0.236	
		1850	221	8 24 1.661	348.344	1.303	348.583	0.239	
		1900	. .	8 26 55.670	347.690	1.313	347.931	0.241	
425	υ^4 Cancri	1755	5	8 18 28.728	+ 357.457	− 1.493	+ 358.099	− 0.642	
		1850	12	8 24 7.635	356.028	1.516	356.661	0.633	
426	35 Cancri	1755	5	8 21 12.435	+ 347.183	− 1.252	+ 347.624	− 0.441	
		1850	7	8 26 41.691	345.986	1.269	346.422	0.436	
427	B. A. C. 2899 . .	1850	3	8 29 10.498	+ 344.820	− 1.259	+ 345.443	− 0.623	
428	B. A. C. 2907 . .	1850	4	8 30 31.989	+ 346.198	− 1.297	+ 345.958	+ 0.240	
429	38 Cancri	1755	. .	8 25 35.962	+ 347.205	− 1.288	+ 347.514	− 0.309	
		1850	5	8 31 5.224	345.975	1.303	346.279	0.304	
430	B. A. C. 2914 . .	1850	5	8 31 13.898	+ 345.733	− 1.298	+ 345.777	− 0.044	
431	39 Cancri	1755	1	8 25 58.875	+ 347.445	− 1.305	+ 347.958	− 0.513	
		1850	25	8 31 28.357	346.199	1.320	346.712	0.513	
432	40 Cancri	1755	1	8 26 3.979	+ 347.481	− 1.303	+ 347.868	− 0.387	
		1850	23	8 31 33.495	346.236	1.317	346.624	0.388	
433	B. A. C. 2919 . .	1755	. .	8 26 16.031	+ 346.943	− 1.293	+ 347.202	− 0.259	
		1850	3	8 31 45.041	345.708	1.307	345.966	0.258	
434	ε Cancri	1755	. .	8 26 21.943	+ 346.366	− 1.282	+ 346.921	− 0.555	
		1850	7	8 31 50.409	345.141	1.296	345.694	0.553	
435	c Cancri	1755	. .	8 26 36.651	+ 347.361	− 1.296	+ 347.267	+ 0.094	
		1850	12	8 32 6.057	346.123	1.310	346.025	0.098	
436	B. A. C. 2925 .	1755	. .	8 26 51.241	+ 346.321	− 1.288	+ 346.938	− 0.617	
		1850	7	8 32 19.663	345 091	1.302	345.708	0.615	

DECLINATIONS.

No.	Star.	Mag.	Epoch.	Declination.	Centennial variation.	Secular variation.	Struve's precession.	Proper motion.	Sec. var. of proper motion.
				° ′ ″	″	″	″	″	″
416	d^2 Cancri	6.0	1755	+ 17 50 1.39	− 1107.09	− 41.25	− 1092.29	−14.80	
		6.0	1850	17 32 11.18	1145.88	40.42	1131.21	14.67	
417	ϕ^2 Cancri (mean) . .	6.0	1755	+ 27 42 54.08	− 1093.62	− 44.28	− 1092.25	− 1.37	
		5.7	1850	27 25 15.31	1135.23	43.33	1133.90	1.33	
418	υ^1 Cancri (1st star) .	7.5	1755	+ 25 19 10.41	− 1102.87	− 43.46	− 1093.17	− 9.70	
		6.0	1850	25 1 23.20	1143.73	42.56	1134.09	9.64	
419	27 Cancri	6.5	1755	+ 13 26 38.74	− 1111.76	− 40.27	− 1101.26	−10.50	
		5.7	1850	13 8 44.51	1149.65	39.50	1139.12	10.53	
420	υ^2 Cancri	6.5	1755	+ 24 56 17.85	− 1115.72	− 43.08	− 1107.78	− 7.94	
		5.8	1850	24 38 18.61	1156.22	42.17	1148.32	7.90	
421	29 Cancri	6.0	1755	+ 15 0 11.73	− 1116.61	− 40.43	− 1114.15	− 2.46	
		6.0	1850	14 42 12.81	1154.67	39.67	1152.16	2.51	
422	υ^3 Cancri	6.5	1755	+ 24 53 17.53	− 1136.73	− 42.57	− 1129.10	− 7.63	
		6.0	1850	24 34 58.56	1176.77	41.74	1169.18	7.59	
423	θ Cancri	5.5	1755	+ 18 54 12.46	− 1140.32	− 40.99	− 1133.49	− 6.83	
		5.7	1850	18 35 50.79	1178.85	40.13	1172.05	6.80	
424	η Cancri	6.0	1755	+ 21 15 15.14	− 1145.33	− 41.52	− 1140.10	− 5.23	
		5.7	1850	20 56 48.47	1184.38	40.70	1179.08	5.30	
			1900	20 46 51.21	1204.63	40.28	1199.26	5.37	
425	υ^4 Cancri	7.5	1755	+ 24 53 55.22	− 1145.55	− 42.35	− 1139.92	− 5.63	
		5.7	1850	24 35 27.98	1185.34	41.42	1179.78	5.56	
426	35 Cancri	8.0	1755	+ 20 24 48.18	− 1160.93	− 40.78	− 1159.50	− 1.43	
		6.3	1850	20 6 7.02	1199.26	39.92	1197.90	1.36	
427	B. A. C. 2899 . .	7.2	1850	+ 19 47 9.67	− 1211.62	− 39.42	− 1215.17	+ 3.55	
428	B. A. C. 2907 . .	8.8	1850	+ 20 6 55.48	− 1227.92	− 39.50	− 1224.69	− 3.23	
429	38 Cancri	7.0	1755	+ 20 37 18.77	− 1189.42	− 40.22	− 1190.65	+ 1.23	
		7.0	1850	20 18 10.81	1227.21	39.33	1228.49	1.28	
430	B. A. C. 2914 . .	7.2	1850	+ 20 3 55.67	− 1233.15	− 39.31	− 1229.49	− 3.66	
431	39 Cancri	6.0	1755	+ 20 51 12.89	− 1194.44	− 40.17	− 1193.38	− 1.06	
		7.0	1850	20 32 0.18	1232.18	39.28	1231.14	1.04	
432	40 Cancri	6.0	1755	+ 20 49 1.40	− 1193.88	− 40.18	− 1193.95	+ 0.07	
		7.3	1850	20 29 49.22	1231.63	39.29	1231.74	0.11	
433	B. A. C. 2919 . .	7.0	1755	+ 20 31 2.33	− 1199.17	− 40.11	− 1195.31	− 3.86	
		7.3	1850	20 11 45.15	1236.85	39.22	1233.12	3.73	
434	ε Cancri	6.5	1755	+ 20 23 32.34	− 1197.43	− 39.99	− 1196.03	− 1.40	
		7.2	1850	20 4 16.87	1235.00	39.10	1233.69	1.31	
435	c Cancri	7.5	1755	+ 20 34 4.74	− 1198.70	− 40.15	− 1197.68	− 1.02	
		7.1	1850	20 14 47.99	1236.42	39.26	1235.50	0.92	
436	B. A. C. 2925 . .	7.5	1755	+ 20 25 49.17	− 1202.63	− 39.91	− 1199.46	− 3.17	
		7.7	1850	20 6 28.79	1240.13	39.02	1237.07	3.06	

RIGHT ASCENSIONS.

No.	Star.	Epoch.	Number of observations.	Right ascension.	Centennial variation.	Secular variation.	Struve's precession.	Proper motion.	Sec. var. of proper motion.
				h. m. s.	s.	s.	s.	s.	s.
437	B. A. C. 2931 . .	1755	1	8 27 43.694	+ 347.028	— 1.308	+ 347.463	— 0.435	
		1850	15	8 33 12.778	345.779	1.322	346.208	0.429	
438	γ Cancri	1755	5	8 29 4.085	+ 349.949	— 1.406	+ 350.720	— 0.771	
		1850	146	8 34 35.901	348.609	1.417	349.376	0.767	
439	44 Cancri	1755	1	8 29 10.529	+ 343.404	— 1.224	+ 343.659	— 0.255	
		1850	7	8 34 36.209	342.237	1.233	342.490	0.253	
440	A¹ Cancri	1755	1	8 29 40.566	+ 332.552	— 0.957	+ 332.584	— 0.032	
		1850	12	8 34 56.057	331.640	0.962	331.669	0.029	
441	δ Cancri	1755	5	8 30 43.603	+ 343.389	— 1.262	+ 343.524	— 0.135	
		1850	105	8 36 9.251	342.185	1.272	342.329	0.144	
442	b Cancri	1755	1	8 31 25.628	+ 327.394	— 0.854	+ 327.432	— 0.038	
		1850	3	8 36 36.267	326.583	0.854	326.621	0.038	
443	A² Cancri	1755	5	8 33 28.742	+ 330.642	— 0.954	+ 331.193	— 0.551	
		1850	17	8 38 42.422	329.736	0.955	330.288	0.552	
444	ε Hydræ	1755	5	8 33 46.884	+ 319.133	— 0.708	+ 320.418	— 1.285	0.000
		1850	580	8 38 49.741	318.461	0.707	319.747	1.286	
		1900	. .	8 41 28.883	318.107	0.707	319.392	1.285	
445	54 Cancri	1755	3	8 37 20.952	+ 336.226	— 1.110	+ 337.134	— 0.908	
		1850	5	8 42 39.865	335.170	1.113	336.074	0.904	
446	52 Cancri	1755	3	8 37 25.961	+ 338.093	— 1.150	+ 338.407	— 0.314	
		1850	8	8 42 46.630	337.000	1.153	337.315	0.315	
447	60 Cancri	1755	5	8 42 31.242	+ 329.504	— 0.960	+ 329.587	— 0.083	
		1850	18	8 47 43.840	328.594	0.957	328.674	0.080	
448	o¹ Cancri	1755	5	8 43 33.108	+ 336.884	— 1.143	+ 336.464	+ 0.420	
		1850	10	8 48 52.632	335.797	1.145	335.374	0.423	
449	ι Ursæ Majoris . .	1755	3	8 42 18.455	+ 419.299	— 4.430	+ 423.804	— 4.505	+0.004
		1850	322	8 48 54.788	415.085	4.442	419.585	4.500	
		1900	. .	8 52 21.775	412.863	4.446	417.365	4.502	
450	o² Cancri	1755	5	8 43 52.369	+ 337.240	— 1.155	+ 336.923	+ 0.317	
		1850	18	8 49 12.228	336.144	1.154	335.822	0.322	
451	ι² Cancri	1755	5	8 45 3.649	+ 330.012	— 0.984	+ 329.822	+ 0.190	
		1850	170	8 50 16.716	329.078	0.982	328 890	0.188	
452	68 Cancri	1755	4	8 47 56.526	+ 339.141	— 1.255	+ 339.349	— 0.208	
		1850	7	8 53 18.144	337.949	1.255	338.155	0.206	
453	ν Cancri	1755	3	8 48 21.985	+ 354.055	— 1.721	+ 354.128	— 0.073	
		1850	20	8 53 57.560	352.419	1.724	352.490	0.071	
454	σ² Ursæ Majoris . .	1755	4	8 48 27.47	+ 553.61	—13.58	+ 553.84	— 0.24	
		1800	. .	8 52 35.21	547.52	13.52	547.76	0.24	
		1850	. .	8 57 7.28	540.77	13.44	541.02	0.25	
		1900	. .	9 1 35.99	+ 534.07	—13.36	+ 534.34	— 0.27	

DECLINATIONS.

No.	Star.	Mag.	Epoch.	Declination.	Centennial variation.	Secular variation.	Struve's precession.	Proper motion.	Sec. var. of proper motion.
				° ′ ″	″	″	″	″	″
437	B. A. C. 2931 . .	7.0	1755	+ 20 43 41.02	− 1206.56	− 39.91	− 1205.60	− 0.96	
		7.5	1850	20 24 16.91	1244.04	39.00	1243.13	0.91	
438	γ Cancri	5.0	1755	+ 22 19 50.99	− 1219.39	− 40.08	− 1214.94	− 4.45	
		4.3	1850	22 0 14.65	1257.00	39.10	1252.63	4.37	
439	44 Cancri	8.0	1755	+ 19 0 35.71	− 1215.54	− 39.30	− 1215.72	+ 0.18	
		8.3	1850	18 41 3.34	1252.46	38.42	1252.68	0.22	
440	A¹ Cancri	6.5	1755	+ 13 32 30.18	− 1219.80	− 38.03	− 1219.09	− 0.71	
		6.0	1850	13 12 54.34	1255.54	37.22	1254.94	0.60	
441	δ Cancri	4.5	1755	+ 19 2 11.72	− 1249.76	− 39.20	− 1226.49	−23.27	
		4.0	1850	18 42 6.88	1286.57	38.28	1263.24	23.33	
442	b Cancri	6.5	1755	+ 10 57 4.14	− 1232.37	− 37.18	− 1231.36	− 1.01	
		5.7	1850	10 37 16.73	1267.32	36.40	1266.30	1.02	
443	A² Cancri	6.0	1755	+ 12 59 29.78	− 1250.80	− 37.25	− 1245.48	− 5.32	
		6.0	1850	12 39 24.84	1285.80	36.43	1280.50	5.30	
444	ε Hydræ	4.0	1755	+ 7 18 2.84	− 1253.07	− 35.82	− 1247.54	− 5.53	+ 0.13
		3.3	1850	6 57 56.37	1286.75	35.06	1281.35	5.40	
			1900	6 47 8.64	1304.17	34.66	1298.83	5.34	
445	54 Cancri	6.5	1755	+ 16 14 30.89	− 1265.85	− 37.29	− 1271.87	+ 6.02	
		6.3	1850	15 54 11.64	1300.87	36.43	1306.98	6.11	
446	52 Cancri	7.5	1755	+ 16 53 39.58	− 1269.15	− 37.55	− 1272.43	+ 3.28	
		8.0	1850	16 33 17.07	1304.41	36.68	1307.74	3.33	
447	60 Cancri	6.0	1755	+ 12 32 43.81	− 1308.32	− 35.97	− 1306.53	− 1.79	
		6.0	1850	12 11 44.80	1342.05	35.07	1340.28	1.77	
448	o¹ Cancri	6.0	1755	+ 16 14 42.59	− 1311.46	− 36.67	− 1313.39	+ 1.93	
		5.7	1850	15 53 40.27	1345.88	35.82	1347.73	1.85	
449	ι Ursæ Majoris . .	3.5	1755	+ 48 59 0.28	− 1331.00	− 45.41	− 1305.14	−25.86	+ 0.48
		3.0	1850	48 37 35.58	1373.37	43.76	1347.97	25.40	
			1900	48 26 3.46	1395.08	42.90	1369.92	25.16	
450	o² Cancri	6.0	1755	+ 16 30 17.25	− 1313.23	− 36.66	− 1315.52	+ 2.29	
		6.0	1850	16 9 13.27	1347.62	35.73	1349.85	2.23	
451	ι² Cancri . . .	5.0	1755	+ 12 47 23.11	− 1327.39	− 35.71	− 1323.36	− 4.03	
		4.0	1850	12 26 6.08	1360.88	34.82	1356.80	4.08	
452	68 Cancri	7.5	1755	+ 18 1 27.11	− 1341 45	− 36.18	− 1342.21	+ 0.76	
		7.5	1850	17 39 56.55	1375.38	35.25	1376.16	0.78	
453	ν Cancri	6.0	1755	+ 25 23 58.33	− 1346.72	− 37.74	− 1344.96	− 1.76	
		5.3	1850	25 2 22.07	1382.08	36.70	1380.34	1.74	
454	σ² Ursæ Majoris . .	. .	1755	+ 68 6 6.39	− 1351.17	− 59.30	− 1345.56	− 5.61	
		5.5	1800	67 55 52.41	1377.49	57.66	1371.90	5.59	
		5.0	1850	67 44 16.53	1405.86	55.85	1400.28	5.58	
			1900	+ 67 32 26.70	− 1433.34	− 54.08	− 1427.74	− 5.60	

RIGHT ASCENSIONS.

No.	Star.	Epoch.	Number of observations.	Right ascension.	Centennial variation.	Secular variation.	Struve's precession.	Proper motion.	Sec. var. of proper motion.
				h. m. s.	s.	s.	s.	s.	s.
455	71 Cancri	1755	1	8 51 58.401	+ 339.196	− 1.289	+ 339.451	− 0.255	
		1850	3	8 57 20.056	337.973	1.286	338.224	0.251	
456	B. A. C. 3103	1755	1	8 52 29.273	+ 338.883	− 1.277	+ 338.865	+ 0.018	
		1850	3	8 57 50.637	337.672	1.273	337.654	0.018	
457	73 Cancri	1755	. .	8 52 43.781	+ 335.280	− 1.175	+ 335.401	− 0.121	
		1850	7	8 58 1.768	334.166	1.171	334.290	0.124	
458	κ Cancri	1755	5	8 54 26.998	+ 326.851	− 0.946	+ 326.944	− 0.093	
		1850	246	8 59 37.082	325.958	0.936	326.047	0.089	
		1900	. .	9 2 19.944	325.490	0.932	325.579	0.089	
459	78 Cancri	1755	3	8 55 16.090	+ 338.791	− 1.309	+ 339.188	− 0.397	
		1850	5	9 0 37.352	337.551	1.302	337.949	0.398	
460	ξ Cancri	1755	4	8 55 13.796	+ 347.899	− 1.589	+ 348.011	− 0.112	
		1850	16	9 0 43.583	346.390	1.586	346.503	0.113	
461	79 Cancri	1755	5	8 56 13.715	+ 347.741	− 1.591	+ 347.751	− 0.010	
		1850	15	9 1 43.352	346.231	1.587	346.239	0.008	
462	80 Cancri	1755	3	8 58 9.013	+ 339.597	− 1.352	+ 339.884	− 0.287	
		1850	13	9 3 31.021	338.315	1.346	338.604	0.289	
463	π¹ Cancri	1755	4	8 58 51.316	+ 330.426	− 1.143	+ 334.193	− 3.767	
		1850	5	9 4 4.707	329.344	1.136	333.097	3.753	
464	B. A. C. 3138	1755	2	8 59 35.064	+ 345.668	− 1.562	+ 345.815	− 0.147	
		1850	22	9 5 2.745	344.187	1.556	344.337	0.150	
465	π² Cancri	1755	5	9 1 40.256	+ 333.581	− 1.184	+ 333.815	− 0.234	
		1850	20	9 6 56.626	332.460	1.175	332.696	0.236	
466	83 Cancri	1755	5	9 5 16.088	+ 337.533	− 1.311	+ 338.321	− 0.788	
		1850	196	9 10 36.145	336.268	1.354	337.033	0.765	
467	ι Argus	1850	. .	9 12 24.47	+ 160.20	− 0.23	+ 161.10	− 0.90	
		1875	. .	9 13 4.51	160.14	0.23	161.04	0.90	
		1900	. .	9 14 24.58	160.09	0.23	160.99	0.90	
468	1 (H) Draconis	1755	. .	8 59 52.86	+1012.49	−88.92	+1013.25	− 0.76	
		1775	. .	9 3 13.57	994.85	87.36	995.62	0.77	
		1800	. .	9 7 19.55	973.23	85.45	974.00	0.77	
		1825	. .	9 11 20.21	952.08	83.50	952.85	0.77	
		1850	. .	9 15 15.63	931.44	81.51	932.21	0.77	
		1875	. .	9 19 5.96	911.29	79.49	912.05	0.77	
		1900	. .	9 22 51.32	+ 891.68	−77.46	+ 892.44	− 0.77	
469	B. A. C. 3206	1755	2	9 10 55.549	+ 340.520	− 1.508	+ 341.230	− 0.710	
		1850	10	9 16 18.365	339.094	1.495	339.804	0.710	
470	c Hydræ	1755	10	9 15 32.662	+ 295.116	− 0.168	+ 295.246	− 0.130	+0.002
		1850	. .	9 20 12.950	294.967	0.144	295.096	0.129	
		1900	. .	9 22 40.416	294.898	0.131	295.025	0.127	

DECLINATIONS.

No.	Star.	Mag.	Epoch.	Declination.	Centennial variation.	Secular variation.	Struve's precession.	Proper motion.	Sec. var. of proper motion.
				° ′ ″	″	″	″	″	″
455	71 Cancri	7.5	1755	+ 18 21 1.69	− 1367.99	− 35.58	− 1368.26	+ 0.27	
		8.0	1850	17 59 6.19	1401.34	34.64	1401.60	0.26	
456	B. A. C. 3103 . .	7.5	1755	. . . . .	. . . .	− 35.50	− 1371.52	. . .	
		7.5	1850	+ 17 42 40.6	. . . .	34.56	1404.82	. . .	
457	73 Cancri	8.0	1755	+ 16 14 15.93	− 1372.84	− 35.06	− 1373.02	+ 0.18	
		8.1	1850	15 52 16.04	1405.72	34.15	1405.97	0.25	
458	κ Cancri	5.5	1755	+ 11 38 17.38	− 1384.44	− 33.88	− 1384.02	− 0.42	+ 0.04
		5.0	1850	11 16 7.00	1416.22	33.01	1415.84	0.38	
			1900	11 4 14.79	1432.60	32.55	1432.24	0.36	
459	78 Cancri	7.0	1755	+ 18 26 43.33	− 1391.67	− 35.02	− 1389.18	− 2.49	
		7.8	1850	18 4 25.58	1424 49	34.07	1422.05	2.44	
460	ξ Cancri . . .	5.5	1755	+ 23 1 10.28	− 1387.91	− 36.02	− 1388.94	+ 1.03	
		5.0	1850	22 38 55.67	1421.64	34.99	1422.69	1.05	
461	79 Cancri	6.0	1755	+ 22 58 28.14	− 1394.85	− 35.86	− 1395.24	+ 0.39	
		6.3	1850	22 36 7.00	1428.43	34.83	1428.83	0.40	
462	80 Cancri	7.5	1755	+ 19 1 53.58	− 1409.52	− 34.68	− 1407.30	− 2.22	
		6.8	1850	18 39 19.01	1442.08	33.87	1439.78	2.30	
463	π^1 Cancri	6.5	1755	+ 15 58 6.44	− 1390.43	− 33.26	− 1411.68	+21.25	
		6.3	1850	15 35 50.65	1421.60	32.36	1443.23	21.63	
464	B. A. C. 3138 . .	6.0	1775	+ 22 16 36.92	− 1420.00	− 35.10	− 1416.20	− 3.80	
		6.3	1850	21 53 52.23	1452.86	34.08	1449.08	3.78	
465	π^2 Cancri	6.0	1755	+ 15 56 30.17	− 1428.90	− 33.52	− 1429.08	+ 0.18	
		6.0	1850	15 33 37.73	1460.30	32.59	1460.51	0.21	
466	83 Cancri	6.0	1755	+ 18 43 44.46	− 1465.53	− 33.30	− 1451.03	−14.50	
		5.7	1850	18 20 17.30	1496.70	32.32	1482.27	14.43	
467	ι Argus	2.5	1850	− 58 38 49.83	− 1493.96	− 14.82	− 1496.77	+ 2.81	
			1875	58 45 3.78	1497.66	14.77	1500.48	2.82	
			1900	58 51 18.65	1501.35	14.71	1504.18	2.83	
468	1 (H) Draconis . .	. .	1755	+ 82 22 6.86	− 1419.97	−103.96	− 1418.04	− 1.93	
			1775	82 17 20.80	1440.41	100.47	1438.50	1.91	
			1800	82 11 17.61	1465.01	96.40	1463.12	1.89	
			1825	82 5 8.38	1488.63	92.51	1486.76	1.87	
		4.3	1850	81 58 53.38	1511.28	88.74	1509.42	1.86	
			1875	81 52 32.83	1533.00	85.10	1531.16	1.84	
			1900	+ 81 46 6.96	− 1553.83	− 81.52	− 1552.03	− 1.82	
469	B. A. C. 3206 . .	7.0	1755	+ 20 49 54.42	− 1497.25	− 32.70	− 1484.78	−12.47	
		6.3	1850	20 25 57.43	1527.83	31.68	1515.44	12.39	
470	α Hydræ	2.0	1755	− 7 36 34.21	− 1508.67	− 27.66	− 1511.68	+ 3.01	+ 0.01
		2.1	1850	8 0 39.81	1534.61	26.95	1537.63	3.02	
			1900	8 13 30.47	1547.99	26.57	1551.01	3.02	

RIGHT ASCENSIONS.

No.	Star.	Epoch.	Number of observations.	Right ascension.			Centennial variation.	Secular variation.	Struve's precession.	Proper motion.	Sec. var. of proper motion.
				h.	m.	s.	s.	s.	s.	s.	s.
471	ω Leonis	1755	2	9	15	18.712	+ 323.067	— 0.953	+ 322.699	+ 0.368	
		1850	19	9	20	25.205	322.192	0.890	321.846	0.346	
472	3 Leonis . . .	1755	2	9	15	25.175	+ 320.975	— 0.861	+ 321.318	— 0.343	
		1850	4	9	20	29.715	320.166	0.844	320.508	0.342	
473	d Ursæ Majoris . .	1755	. .	9	12	20.11	+ 563.30	—17.63	+ 564.82	— 1.52	
		1800	. .	9	16	31.82	555.41	17.39	556.91	1.50	
		1850	. .	9	21	7.37	546.79	17.10	548.26	1.47	
		1900	. .	9	25	38.64	538.31	16.81	539.76	1.45	
474	θ Ursæ Majoris . .	1755	. .	9	16	19.237	+ 411.594	— 5.627	+ 422.332	—10.738	+0.067
		1850	175	9	22	47.724	406.289	5.541	416.979	10.690	
		1900	. .	9	26	10.177	403.528	5.503	414.200	10.672	
475	ξ Leonis	1755	5	9	18	42.829	+ 325.248	— 1.024	+ 325.982	— 0.734	
		1850	87	9	23	51.354	324.283	1.008	325.017	0.734	
476	h Leonis	1755	2	9	18	47.913	+ 323.541	— 0.946	+ 323.475	+ 0.066	
		1850	29	9	23	54.854	322.654	0.923	322.589	0.065	
477	7 Leonis	1755	5	9	22	27.556	+ 330.181	— 1.194	+ 330.470	— 0.289	
		1850	5	9	27	40.691	329.056	1.174	329.344	0.288	
478	8 Leonis	1755	5	9	23	29.433	+ 333.479	— 1.317	+ 333.646	— 0.167	
		1850	9	9	28	45.646	332.237	1.297	332.409	0.172	
479	10 Leonis	1755	2	9	24	15.414	+ 318.158	— 0.797	+ 318.692	— 0.534	
		1850	26	9	29	17.308	317.410	0.776	317.945	0.535	
480	11 Leonis	1755	5	9	24	37.257	+ 329.615	— 1.198	+ 330.168	— 0.553	
		1850	3	9	29	49.853	328.487	1.177	329.042	0.555	
481	o Leonis	1755	5	9	28	2.999	+ 321.962	— 0.960	+ 322.990	— 1.028	
		1850	223	9	33	8.433	321.061	0.936	322.096	1.035	
482	ψ Leonis	1755	5	9	30	21.514	+ 328.906	— 1.181	+ 328.968	— 0.062	
		1850	14	9	35	33.445	327.795	1.158	327.857	0.062	
483	ε Leonis	1755	5	9	31	53.773	+ 343.933	— 1.825	+ 344.367	— 0.434	+0.001
		1850	653	9	37	19.690	342.213	1.797	342.645	0.432	
		1900	. .	9	40	10.573	341.318	1.784	341.750	0.432	
484	18 Leonis	1755	5	9	33	9.687	+ 325.239	— 1.051	+ 325.346	— 0.107	
		1850	23	9	38	18.193	324.252	1.028	324.357	0.105	
485	19 Leonis	1755	4	9	34	14.174	+ 324.311	— 1.040	+ 324.885	— 0.574	
		1850	5	9	39	21.804	323.335	1.014	323.909	0.574	
486	B. A. C. 3345 . .	1755	1	9	34	21.270	+ 324.592	— 1.051	+ 324.680	— 0.088	
		1850	33	9	39	29..161	323.605	1.026	323.701	0.096	
487	20 Leonis	1755	5	9	36	4.805	+ 338.821	— 1.642	+ 339.241	— 0.420	
		1850	6	9	41	25.948	337.274	1.614	337.697	0.423	
488	21 Leonis	1755	3	9	37	36.715	+ 324.757	— 1.057	+ 324.935	— 0.178	
		1850	5	9	42	44.761	323.766	1.030	323.944	0.178	

DECLINATIONS.

No.	Star.	Mag.	Epoch.	Declination.	Centennial variation.	Secular variation.	Struve's precession.	Proper motion.	Sec. var. of proper motion.
				° ′ ″	″	″	″	″	″
471	ω Leonis	6.5	1755	+ 10 6 34 15	− 1510.89	− 30.46	− 1510.35	− 0.54	
		5.9	1850	9 42 25.70	1539.39	29.54	1538.77	0.62	
472	3 Leonis . . .	6.5	1755	+ 9 14 34.01	− 1512.62	− 30.13	− 1510.96	− 1.66	
		6.3	1850	8 50 23.56	1540.82	29.25	1539.20	1.62	
473	d Ursæ Majoris .	5.0	1755	+ 70 53 2.70	− 1487.23	− 54.11	− 1493.05	+ 5.82	
			1800	70 41 48.02	1511.17	52.22	1517.05	5.88	
		4.7	1850	70 29 5.99	1536.74	50.17	1542.70	5.96	
			1900	70 16 11.43	1561.32	48.16	1567.36	6.04	
474	θ Ursæ Majoris . .	3.0	1755	+ 52 46 38.36	− 1572.91	− 37.70	− 1516.17	−56.74	+ 0.91
		3.0	1850	52 21 27.34	1607.89	35.94	1552.01	55.88	
			1900	52 7 58.95	1625.63	35.02	1570.21	55.42	
475	ξ Leonis	5.0	1755	+ 12 22 14.87	− 1538.01	− 30.04	− 1529.82	− 8.19	
		5.3	1850	11 57 40.35	1566.10	29.13	1557.88	8.22	
476	h Leonis	6.0	1755	+ 10 46 56.05	− 1531.48	− 29.92	− 1530.26	− 1.22	
		5.7	1850	10 22 27.79	1559.46	28.99	1558.21	1.25	
477	7 Leonis	6.5	1755	+ 15 27 34.01	− 1551.99	− 29.87	− 1550.79	− 1.20	
		6.3	1850	15 2 46.29	1579.91	28.90	1578.73	1.18	
478	8 Leonis	6.5	1755	+ 17 31 19.89	− 1558.32	− 30.01	− 1556.50	− 1.82	
		5.7	1850	17 6 26.09	1586.36	29.02	1584.56	1.80	
479	10 Leonis . . .	5.5	1755	+ 7 55 15.98	− 1560.63	− 28.45	− 1560.72	+ 0.09	
		5.4	1850	7 30 20.67	1587.24	27.58	1587.39	0.15	
480	11 Leonis	7.0	1755	+ 15 26 24.36	− 1570.13	− 29.44	− 1562.73	− 7.40	
		6.8	1850	15 1 19.60	1597.63	28.47	1590.27	7.36	
481	o Leonis	4.0	1755	+ 10 59 37.65	− 1584.82	− 28.47	− 1581.39	− 3.43	
		3.7	1850	10 34 19.36	1611.43	27.56	1607.80	3.63	
482	ψ Leonis	6.0	1755	+ 15 7 47.11	− 1595.47	− 28.46	− 1593.74	− 1.73	
		6.0	1850	14 42 18.73	1622.04	27.48	1620.32	1.72	
483	ε Leonis	3.0	1755	+ 24 53 21.19	− 1604.05	− 29.53	− 1601.87	− 2.18	+ 0.02
		3.0	1850	24 27 44.19	1631.57	28.40	1629.40	2.17	
			1900	24 14 4.88	1645.62	27.80	1643.46	2.16	
484	18 Leonis	6.0	1755	+ 12 55 34.35	− 1608.19	− 27.70	− 1608.52	+ 0.33	
		6.0	1850	12 29 54.21	1634.03	26.71	1634.37	0.34	
485	19 Leonis	7.0	1755	+ 12 41 18.48	− 1612.70	− 27.36	− 1614.14	+ 1.44	
		7.0	1850	12 15 34.21	1638.24	26.41	1639.72	1.48	
486	B. A. C. 3345 . .	8.0	1755	+ 12 33 10.00	− 1619.61	− 27.45	− 1614.79	− 4.82	
			1850	12 7 19.14	1645.23	26.52	1640.34	4.89	
487	20 Leonis	7.0	1755	+ 22 18 31.32	− 1627.27	− 28.30	− 1623.67	− 3.60	
		6.0	1850	21 52 32.80	1653.65	27.22	1650.07	3.58	
488	21 Leonis	7.5	1755	+ 12 58 27.03	− 1631.39	− 26.86	− 1631.52	+ 0.13	
		6.8	1850	12 32 25.24	1656.45	25.90	1656.60	0.15	

RIGHT ASCENSIONS.

No.	Star.	Epoch.	Number of observations.	Right ascension.			Centennial variation.	Secular variation.	Struve's precession.	Proper motion.	Sec. var. of proper motion.
				h.	m.	s.	s.	s.	s.	s.	s.
489	23 Leonis	1755	2	9	37	44.540	+ 326.950	— 1.130	+ 326.682	+ 0.268	
		1850	7	9	42	54.637	325.890	1.103	325.620	0.270	
490	μ Leonis	1755	5	9	38	46.679	+ 344.868	— 2.001	+ 346.640	— 1.772	+0.005
		1850	220	9	44	13.405	342.981	1.970	344.747	1.766	
		1900	. .	9	47	4.650	342.001	1.952	343.764	1.763	
491	9 Sextantis . . .	1755	5	9	41	17.443	+ 314.722	— 0.697	+ 315.113	— 0.391	
		1850	3	9	46	16.119	314.074	0.668	314.465	0.391	
492	10 Sextantis . . .	1755	5	9	43	25.372	+ 319.641	— 0.895	+ 320.345	— 0.704	
		1850	3	9	48	28.632	318.805	0.865	319.508	0.703	
493	26 Leonis	1755	4	9	44	50.498	+ 328.533	— 1.254	+ 328.860	— 0.327	
		1850	12	9	50	2.043	327.356	1.225	327.684	0.328	
494	ν Leonis	1755	5	9	45	0.943	+ 324.731	— 1.093	+ 324.976	— 0.245	
		1850	47	9	50	8.948	323.704	1.070	323.955	0.251	
495	11 Sextantis . . .	1755	5	9	45	7.540	+ 319.412	— 0.866	+ 319.385	+ 0.027	
		1850	5	9	50	10.596	318.604	0.836	318.577	0.027	
496	π Leonis	1755	5	9	47	14.757	+ 318.543	— 0.850	+ 318.867	— 0.324	
		1850	214	9	52	16.994	317.750	0.819	318.075	0.325	
497	14 Sextantis . . .	1755	2	9	53	57.717	+ 314.964	— 0.713	+ 315.326	— 0.362	
		1850	14	9	58	56.615	314.303	0.679	314.665	0.362	
498	η Leonis	1755	5	9	53	56.437	+ 329.581	— 1.362	+ 329.643	— 0.062	
		1850	48	9	59	8.932	328.312	1.310	328.381	0.069	
499	A Leonis	1755	1	9	54	52.725	+ 320.093	— 0.945	+ 320.732	— 0.639	
		1850	17	9	59	56.391	319.212	0.910	319.853	0.641	
500	α Leonis	1755	. .	9	55	17.835	+ 321.432	— 1.045	+ 323.179	— 1.747	+0.004
		1850	. .	10	0	22.728	320.454	1.011	322.200	1.746	
		1900	. .	10	3	2.829	319.953	0.992	321.695	1.742	
501	B. A. C. 3460 . .	1755	1	9	55	39.228	+ 332.028	— 1.467	+ 331.758	+ 0.270	
		1850	7	10	0	53.998	330.652	1.429	330.391	0.261	
502	16 Sextantis . . .	1755	3	9	56	23.214	+ 315.928	— 0.736	+ 315.873	+ 0.055	
		1850	13	10	1	23.022	315.248	0.697	315.188	0.060	
503	34 Leonis	1755	3	9	58	25.645	+ 324.922	— 1.134	+ 324.568	+ 0.354	
		1850	20	10	3	33.814	323.863	1.097	323.507	0.356	
504	19 Sextantis . . .	1755	5	10	0	2.535	+ 313.299	— 0.650	+ 313.805	— 0.506	
		1850	3	10	4	59.881	312.699	0.615	313.204	0.505	
505	32 Ursæ Majoris . .	1755	5	9	59	55.09	+ 457.91	—12.14	+ 459.58	— 1.67	
		1800	. .	10	3	19.94	452.50	11.89	454.15	1.65	
		1850	. .	10	7	4.72	446.62	11.62	448.26	1.64	
		1900	. .	10	10	46.59	+ 440.87	—11.35	+ 442.50	— 1.63	
506	B. A. C. 3506 . .	1850	14	10	8	5.434	+ 328.499	— 1.361	+ 328.147	+ 0.352	
507	37 Leonis . . .	1755	5	10	3	29.931	+ 324.103	— 1.146	+ 324.368	— 0.265	
		1850	23	10	8	37.317	323.032	1.108	323.298	0.266	

DECLINATIONS.

No.	Star.	Mag.	Epoch.	Declination.	Centennial variation.	Secular variation.	Struve's precession.	Proper motion.	Sec. var. of proper motion.
				° ′ ″	″	″	″	″	″
489	23 Leonis	7.5	1755	+ 14 11 59.60	− 1635.06	− 27.06	− 1632.18	− 2.88	
		6.3	1850	13 45 54.24	1660.30	26.08	1657.50	2.80	
490	μ Leonis	3.0	1755	+ 27 8 52.04	− 1642.89	− 28.21	− 1637.41	− 5.48	+ 0.14
		4.0	1850	26 42 38.74	1669.14	27.06	1663 84	5.30	
			1900	26 28 40.81	1682.52	26.46	1677.30	5.22	
491	9 Sextantis . . .	7.0	1755	+ 6 5 16.49	− 1649.17	− 25.38	− 1650.05	+ 0.88	
		6.9	1850	5 38 58.46	1672.86	24.49	1673.77	0.91	
492	10 Sextantis . . .	6.0	1755	+ 10 4 57.44	− 1659.67	− 25.40	− 1660.60	+ 0.93	
		6.0	1850	9 38 29.44	1683.35	24.47	1684.34	0.99	
493	26 Leonis	7.5	1755	+ 16 22 42.99	− 1671.41	− 25.91	− 1667.54	− 3.87	
		7.7	1850	15 56 3.61	1695.55	24.90	1691.70	3.85	
494	ν Leonis . . .	5.5	1755	+ 13 36 8.00	− 1671.26	− 25.65	− 1668.38	− 2.88	
		5.3	1850	13 9 28.88	1695.16	24.67	1692.23	2.93	
495	11 Sextantis . . .	6.0	1755	+ 9 28 19.37	− 1672.15	− 25.15	− 1668.92	− 3.23	
		6.0	1850	9 1 39.62	1695.60	24.22	1692.36	3.24	
496	π Leonis	4.5	1755	+ 9 12 30.00	− 1681.35	− 24.77	− 1679.14	− 2.21	
		5.2	1850	8 45 41.68	1704.42	23.82	1702.20	2.22	
497	14 Sextantis . . .	6.0	1755	+ 6 47 42.30	− 1711.11	− 23.25	− 1710.61	− 0.50	
		6.6	1850	6 20 26.40	1732.76	22.34	1732.29	0.47	
498	η Leonis	3.5	1755	+ 17 56 46.75	− 1710.46	− 24.43	− 1710.52	+ 0.06	
		3.3	1850	17 29 30.97	1733.17	23.39	1733.16	− 0.01	
499	A Leonis	5.0	1755	+ 11 11 15.35	− 1720.69	− 23.46	− 1714.80	− 5.89	
		4.7	1850	10 43 50.26	1742.52	22.50	1736.66	5.86	
500	α Leonis	1.0	1755	+ 13 9 15.16	− 1716.96	− 23.41	− 1716.70	− 0.26	+ 0.13
		1.3	1850	12 41 53.64	1738.73	22.43	1738.60	0.13	
			1900	12 27 21.50	1749.81	21.91	1749.74	0.07	
501	B. A. C. 3460 . .	7.8	1755	+ 19 43 25.23	− 1724.62	− 24.29	− 1718.31	− 6.31	
		6.3	1850	19 15 56.05	1747.19	23.22	1740.82	6.37	
502	16 Sextantis . . .	6.0	1755	+ 7 21 41.41	− 1722.91	− 22.95	− 1721.64	− 1.27	
		6.9	1850	6 54 14.46	1744.26	22.00	1742.96	1.30	
503	34 Leonis	6.0	1755	+ 14 33 13.57	− 1733.56	− 23.26	− 1730.73	− 2.83	
		6.3	1850	14 5 36.34	1755.18	22.26	1752.34	2.84	
504	19 Sextantis . . .	7.0	1755	+ 5 48 54.71	− 1737.98	− 22.06	− 1737.80	− 0.18	
		6.2	1850	5 21 13.81	1758.50	21.15	1758.41	0.09	
505	32 Ursæ Majoris . .	5.5	1755	+ 66 19 1.22	− 1739.97	− 32.55	− 1737.28	− 2.69	
			1800	66 5 54.97	1754.36	31.30	1751.72	2.64	
		6.0	1850	65 51 13.94	1769.66	29.95	1767.08	2.58	
			1900	65 36 25.43	1784.28	28.64	1781.76	2.52	
506	B. A. C. 3506 . .	6.0	1850	+ 18 29 5.14	− 1773.63	− 21.74	− 1771.25	− 2.38	
507	37 Leonis	6.0	1755	+ 14 56 26.13	− 1756.47	− 22.24	− 1752.76	− 3.71	
		5.7	1850	14 28 27.59	1777.12	21.23	1773.43	3.69	

RIGHT ASCENSIONS.

No.	Star.	Epoch.	Number of observations.	Right ascension.	Centennial variation.	Secular variation.	Struve's precession.	Proper motion.	Sec. var. of proper motion.
				h. m. s.	s.	s.	s.	s	s.
508	γ^1 Leonis	1755	10	10 6 25.590	+ 333.543	− 1.560	+ 331.503	+ 2.040	−0.048
		1850	380	10 11 41.759	332.081	1.518	330.050	2.031	
		1900	. .	10 14 27.611	331.328	1.493	329.305	2.023	
509	23 Sextantis . . .	1755	1	10 8 22.312	+ 310.522	− 0.516	+ 310.838	− 0.316	
		1850	9	10 13 17.081	310.051	0.477	310.368	0.317	
510	42 Leonis	1755	5	10 8 37.925	+ 324.887	− 1.207	+ 325.149	− 0.262	
		1850	26	10 13 46.028	323.756	1.174	324.015	0.259	
511	43 Leonis	1755	5	10 10 10.223	+ 315.218	− 0.734	+ 315.420	− 0.202	
		1850	16	10 15 9.355	314.540	0.694	314.745	0.205	
512	44 Leonis	1850	14	10 17 20.675	+ 316.995	− 0.805	+ 316.913	+ 0.082	
513	45 Leonis	1755	3	10 14 41.133	+ 318.553	− 0.893	+ 318.531	+ 0.022	
		1850	26	10 19 43.363	317.724	0.854	317.703	0.021	
514	B. A. C. 3579 . .	1755	1	10 15 40.987	+ 322.827	− 1.155	+ 323.401	− 0.574	
		1850	24	10 20 47.157	321.748	1.118	322.321	0.573	
515	9 (H) Draconis . .	1755	. .	10 13 27.71	+ 565.93	−31.44	+ 565.94	− 0.01	
		1775	. .	10 15 20.27	559.70	30.84	559.71	0.01	
		1800	. .	10 17 39.24	552.08	30.11	552.10	0.02	
		1825	. .	10 19 56.33	544.64	29.40	544.66	0.02	
		1850	. .	10 22 11.58	537.38	28.69	537.40	0.02	
		1875	. .	10 24 25.04	530.30	28.00	530.32	0.02	
		1900	. .	10 26 36.74	+ 523.38	−27.33	+ 523.41	− 0.02	
516	31 Sextantis . . .	1755	1	10 17 50.835	+ 310.881	− 0.483	+ 310.422	+ 0.459	
		1850	6	10 22 45.960	310.442	0.442	309.983	0.459	
517	*i* Leonis	1755	5	10 19 5.330	+ 322.320	− 1.142	+ 322.688	− 0.368	
		1850	17	10 24 11.027	321.256	1.097	321.626	0.370	
518	32 Sextantis . . .	1755	3	10 19 34.108	+ 312.583	− 0.600	+ 312.856	− 0.273	
		1850	6	10 24 30.798	312.033	0.559	312.305	0.272	
519	ρ Leonis	1755	5	10 19 53.278	+ 317.527	− 0.852	+ 317.546	− 0.019	
		1850	330	10 24 54.554	316.737	0.810	316.756	0.019	
		1900	. .	10 27 32.822	316.337	0.787	316.357	0.020	
520	48 Leonis	1755	5	10 22 0.128	+ 314.218	− 0.711	+ 314.996	− 0.778	
		1850	13	10 26 58.321	313.563	0.670	314.338	0.775	
521	49 Leonis	1755	4	10 22 9.633	+ 316.290	− 0.812	+ 316.671	− 0.381	
		1850	8	10 27 9.748	315.537	0.774	315.922	0.385	
522	50 Leonis	1755	4	10 25 44.253	+ 323.993	− 1.247	+ 323.750	+ 0.243	
		1850	7	10 30 51.489	322.830	1.201	322.590	0.240	
523	34 Sextantis . . .	1755	4	10 29 57.713	+ 310.659	− 0.511	+ 311.347	− 0.688	
		1850	148	10 34 52.615	310.194	0.467	310.881	0.687	
524	35 Sextantis (1st star)	1755	4	10 30 37.017	+ 311.916	− 0.574	+ 312.365	− 0.449	
		1850	6	10 35 33.085	311.393	0.529	311.842	0.449	

DECLINATIONS.

No.	Star.	Mag.	Epoch.	Declination.	Centennial variation.	Secular variation.	Struve's precession.	Proper motion.	Sec. var. of proper motion.
				° ′ ″	″	″	″	″	″
508	γ^1 Leonis	2.0	1755	+ 21 4 14.08	− 1779.47	− 22.52	− 1765.09	−14.38	− 0.14
		2.0	1850	20 35 53.60	1800.33	21.41	1785.82	14.51	
			1900	20 20 50.78	1810.89	20.83	1796.30	14.59	
509	23 Sextantis . . .	6.0	1755	+ 3 30 46.71	− 1773.57	− 20.42	− 1773.11	− 0.46	
		6.6	1850	3 2 32.74	1792.54	19.52	1792.11	0.43	
510	42 Leonis	6.0	1755	+ 16 12 6.22	− 1777.73	− 21.36	− 1774.20	− 3.53	
		6.0	1850	15 43 47.87	1797.52	20.31	1794.00	3.52	
511	43 Leonis	6.0	1755	+ 7 46 39.27	− 1791.56	− 20.42	− 1780.44	−11.12	
		6.5	1850	7 18 8.22	1810.51	19.48	1799.41	11.10	
512	44 Leonis	6.0	1850	+ 9 32 43.68	− 1812.16	− 19.26	− 1807.78	− 4.38	
513	45 Leonis	6.0	1755	+ 11 0 7.55	− 1798.33	− 19.87	− 1798.33	0.00	
		6.0	1850	10 31 30.33	1816.74	18.88	1816.70	− 0.04	
514	B. A. C. 3579 . .	. .	1755	+ 15 35 12.35	− 1804.09	− 19.95	− 1802.06	− 2.03	
		7.2	1850	15 6 29.63	1822.54	18.92	1820.63	1.91	
515	9 (H) Draconis . .	5.5	1755	+ 76 57 39.57	− 1795.15	− 36.24	− 1793.54	− 1.61	
			1775	76 51 39.82	1802.30	35.22	1800.69	1.61	
			1800	76 44 8.16	1810.95	34.01	1809.34	1.61	
			1825	76 36 34.38	1819.30	32.82	1817.69	1.61	
		4.7	1850	76 28 58.55	1827.35	31.66	1825.74	1.61	
			1875	76 21 20.73	1835.14	30.54	1833.53	1.61	
			1900	+ 76 13 41.00	− 1842.64	− 29.44	− 1841.03	− 1.61	
516	31 Sextantis . . .	7.0	1755	+ 3 24 0.01	− 1815.05	− 18.80	− 1810.44	4.61	
		7.0	1850	2 55 7.37	1832.47	17.88	1827.83	4.64	
517	*i* Leonis	6.0	1755	+ 15 23 11.18	− 1813.81	− 19.31	− 1815.08	+ 1.27	
		5.7	1850	14 54 19.52	1831.65	18.26	1832.90	1.25	
518	32 Sextantis . . .	7.0	1755	+ 5 53 41.01	− 1814.67	− 18.55	− 1816.88	+ 2.21	
		8.0	1850	5 24 48.84	1831.85	17.62	1834.07	2.22	
519	ρ Leonis	4.0	1755	+ 10 33 32.85	− 1818.94	− 18.79	− 1818.06	− 0.88	
		3.9	1850	10 4 36.51	1836.34	17.83	1835.46	0.88	
			1900	9 49 16.13	1845.13	17.33	1844.25	0.88	
520	48 Leonis	5.5	1755	+ 8 12 25.93	− 1821.29	− 18.18	− 1825.81	+ 4.52	
		5.5	1850	7 43 27.64	1838.11	17.22	1842.66	4.55	
521	49 Leonis	6.0	1755	+ 9 54 30.45	− 1827.91	− 18.34	− 1826.41	− 1.50	
		6.0	1850	9 25 25.80	1844.89	17.40	1843.32	1.57	
522	50 Leonis	6.5	1755	+ 17 23 41.45	− 1841.97	− 18.12	− 1839.12	− 2.85	
		6.3	1850	16 54 23.56	1858.69	17.08	1855.83	2.86	
523	34 Sextantis . . .	6.0	1755	+ 4 51 21.99	− 1852.07	− 16.51	− 1853.61	+ 1.54	
		6.7	1850	4 21 55.21	1867.32	15.58	1868.88	1.56	
524	35 Sextantis (1st star)	7.0	1755	+ 6 1 32.80	− 1862.52	− 16.48	− 1855.79	− 6.73	
		6.2	1850	5 31 56.12	1877.72	15.53	1871.04	6.68	

RIGHT ASCENSIONS.

No.	Star.	Epoch.	Number of observations.	Right ascension.	Centennial variation.	Secular variation.	Struve's precession.	Proper motion.	Sec. var. of proper motion.
				h. m. s.	s.	s.	s.	s.	s.
525	36 Sextantis	1755	5	10 32 31.454	+ 309.862	− 0.437	+ 310.278	− 0.416	
		1850	14	10 37 25.630	309.463	0.403	309.875	0.412	
526	37 Sextantis	1755	4	10 33 19.214	+ 313.675	− 0.652	+ 313.638	+ 0.037	
		1850	28	10 38 16.917	313.077	0.606	313.042	0.035	
527	*k* Leonis	1755	5	10 33 25.120	+ 319.682	− 1.097	+ 320.655	− 0.973	
		1850	18	10 38 28.329	318.663	1.049	319.642	0.979	
528	η Argus	1850	. .	10 39 15.22	+ 230.50	+ 2.13	+ 230.70	− 0.20	
		1875	. .	10 40 12.91	231.04	2.16	231.24	0.20	
		1900	. .	10 41 10.74	231.58	2.20	231.78	0.20	
529	38 Sextantis	1755	5	10 34 33.946	+ 312.901	− 0.641	+ 313.524	− 0.623	
		1850	13	10 39 30.920	312.315	0.594	312.936	0.621	
530	*l* Leonis	1755	5	10 36 21.402	+ 316.974	− 0.870	+ 317.001	− 0.027	
		1850	265	10 41 22.142	316.170	0.823	316.196	0.026	
		1900	. .	10 44 0.125	315.765	0.797	315.791	0.026	
531	55 Leonis	1755	5	10 43 5.677	+ 309.215	− 0.317	+ 308.564	+ 0.651	
		1850	11	10 47 59.296	308.937	0.270	308.285	0.652	
532	56 Leonis	1755	1	10 43 17.243	+ 312.690	− 0.600	+ 312.770	− 0.080	
		1850	6	10 48 14.038	312.143	0.553	312.223	0.080	
533	57 Leonis	1755	2	10 43 35.930	+ 308.431	− 0.300	+ 308.321	+ 0.110	
		1850	6	10 48 28.811	308.169	0.252	308.061	0.108	
534	*d* Leonis	1755	5	10 47 53.956	+ 310.509	− 0.448	+ 310.566	− 0.057	
		1850	130	10 52 48.745	310.107	0.399	310.165	0.058	
535	*c* Leonis	1755	5	10 48 2.101	+ 311.904	− 0.576	+ 312.371	− 0.467	
		1850	30	10 52 58.157	311.380	0.528	311.846	0.466	
536	α Ursæ Majoris	1755	10	10 48 22.731	+ 386.039	− 8.704	+ 387.947	− 1.908	−0.008
		1850	576	10 54 25.606	377.977	8.267	379.857	1.880	
		1900	. .	10 57 33.571	373.900	8.040	375.761	1.861	
537	*p*² Leonis	1755	5	10 51 4.022	+ 307.321	− 0.246	+ 307.906	− 0.585	
		1850	9	10 55 55.873	307.110	0.198	307.697	0.587	
538	χ Leonis	1755	5	10 52 21.908	+ 310.525	− 0.624	+ 312.920	− 2.395	
		1850	190	10 57 16.634	309.959	0.570	312.350	2.391	
539	*p*³ Leonis	1755	5	10 54 24.137	+ 306.373	− 0.330	+ 309.161	− 2.788	
		1850	18	10 59 15.050	306.082	0.283	308.871	2.789	
540	*p*⁴ Leonis	1755	. .	10 56 42.629	+ 306.859	− 0.150	+ 306.989	− 0.130	
		1850	6	11 1 34.084	306.740	0.101	306.870	0.130	
541	*p*⁵ Leonis	1755	5	11 1 12.806	+ 307.485	− 0.194	+ 307.742	− 0.257	
		1850	27	11 6 4.837	307.325	0.143	307.583	0.258	
542	δ Leonis	1755	5	11 1 2.517	+ 321.660	− 1.410	+ 320.652	+ 1.008	−0.007
		1850	746	11 6 7.468	320.350	1.348	319.347	1.003	
		1900	. .	11 8 47.476	319.684	1.315	318.685	0.999	

DECLINATIONS.

No.	Star.	Mag.	Epoch.	Declination.	Centennial variation.	Secular variation.	Struve's precession.	Proper motion.	Sec. var. of proper motion.
				° ′ ″	″	″	″	″	″
525	36 Sextantis . . .	6.0	1755	+ 3 46 8.96	— 1863.70	— 16.01	— 1862.07	— 1.63	
		6.6	1850	3 16 31.36	1878.47	15.09	1876.86	1.61	
526	37 Sextantis . . .	6.0	1755	+ 7 39 26.16	— 1868.43	— 16.13	— 1864.61	— 3.82	
		6.3	1850	7 9 44.03	1883.29	15.16	1879.50	3.79	
527	*k* Leonis	6.0	1755	+ 15 28 53.20	— 1872.92	— 16.35	— 1864.97	— 7.95	
		5.7	1850	14 59 6.70	1887.97	15.33	1880.07	7.90	
528	η Argus	. .	1850	— 58 53 48.72	— 1882.83	— 10.78	— 1882.45	— 0.38	
			1875	59 1 39.76	1885.51	10.67	1885.12	0.39	
			1900	59 9 31.47	1888.17	10.56	1887.77	0.40	
529	38 Sextantis . . .	7.0	1755	+ 7 37 53.23	— 1867.90	— 15.79	— 1868.66	+ 0.76	
		7.8	1850	7 8 11.75	1882.44	14.83	1883.23	0.79	
530	*l* Leonis	6.0	1755	+ 11 50 5.51	— 1877.19	— 15.69	— 1874.31	— 2.88	0.00
		5.3	1850	11 20 15.24	1891.63	14.69	1888.75	2.88	
			1900	11 4 27.61	1898.85	14.17	1895.97	2.88	
531	55 Leonis	6.0	1755	+ 2 2 15.18	— 1895.93	— 14.11	— 1894.56	— 1.37	
		6.2	1850	1 32 7.83	1908.89	13.18	1907.45	1.44	
532	56 Leonis	7.0	1755	+ 7 29 11.51	— 1895.01	— 14.17	— 1895.10	+ 0.09	
		6.6	1850	6 59 5.01	1908.01	13.20	1908.11	0.10	
533	57 Leonis	7.0	1755	+ 1 44 4.95	— 1898.16	— 13.91	— 1896.00	— 2.16	
		6.9	1850	1 13 55.56	1910.94	12.99	1908.77	2.17	
534	*d* Leonis	5.0	1755	+ 4 55 39.21	— 1910.79	— 13.23	— 1907.95	— 2.84	
		5.3	1850	4 25 18.21	1922.89	12.26	1920.07	2.82	
535	*c* Leonis	5.5	1755	+ 7 24 43.01	— 1910.59	— 13.21	— 1908.34	— 2.25	
		5.3	1850	6 54 22.14	1922.68	12.25	1920.46	2.22	
536	α Ursæ Majoris . .	1.5	1755	+ 63 4 1.85	— 1916.08	— 16.32	— 1909.32	— 6.76	+ 0.08
		2.0	1850	62 33 34.46	1930.79	14.64	1924.11	6.68	
			1900	62 17 27.27	1937.89	13.76	1931.25	6.64	
537	p^2 Leonis . . .	6.0	1755	+ 1 18 48.08	— 1917.66	— 12.48	— 1916.32	— 1.34	
		5.4	1850	0 48 20.81	1929.08	11.56	1927.78	1.30	
538	χ Leonis	4.5	1755	+ 8 39 18.09	— 1924.06	— 12.29	— 1919.73	— 4.33	
		4.8	1850	8 8 44.84	1935.28	11.33	1931 00	4.28	
539	p^3 Leonis	5.5	1755	+ 3 16 49.58	— 1933.56	— 11.68	— 1924.83	— 8.73	
		5.9	1850	2 46 7.56	1944.24	10.81	1935.59	8.65	
540	p^4 Leonis	7.0	1755	— 0 0 37.46	— 1930.75	— 11.37	— 1930.42	— 0.33	
		6.9	1850	0 31 16.66	1941.11	10.44	1940.82	0.29	
541	p^5 Leonis	5.5	1755	+ 1 15 34.41	— 1942.00	— 10.54	— 1940.81	— 1.19	
		5.7	1850	0 44 44.90	1951.57	9.60	1950.39	1.18	
542	δ Leonis	3.0	1755	+ 21 51 42.84	— 1954.61	— 11.15	— 1940.42	—14.19	— 0.04
		2.3	1850	21 20 41.09	1964.71	10.09	1950.48	14.23	
			1900	21 4 17.50	1969.61	9.53	1955.36	14.25	

RIGHT ASCENSIONS.

No.	Star.	Epoch.	Number of observations.	Right ascension.			Centennial variation.	Secular variation.	Struve's precession.	Proper motion.	Sec. var. of proper motion.
				h.	m.	s.	s.	s.	s.	s.	s.
543	B. A. C. 3837 . .	1850	17	11	6	14.025	+ 312.460	− 0.578	+ 312.034	+ 0.426	
544	φ Leonis	1755	5	11	4	12.473	+ 304.961	+ 0.001	+ 305.692	− 0.731	
		1850	33	11	9	2.193	304.986	0.052	305.718	0.732	
545	75 Leonis	1755	5	11	4	40.768	+ 309.085	− 0.290	+ 308.876	+ 0.209	
		1850	11	11	9	34.275	308.832	0.240	308.622	0.210	
546	76 Leonis	1755	5	11	6	20.352	+ 308.168	− 0.259	+ 308.617	− 0.449	
		1850	4	11	11	13.002	307.947	0.208	308.397	0.450	
547	δ Crateris . . .	1755	5	11	7	6.552	+ 298.815	+ 0.568	+ 299.677	− 0.862	+0.001
		1850	454	11	11	50.691	299.381	0.623	300.240	0.859	
		1900	. .	11	14	20.461	299.700	0.653	300.559	0.859	
548	σ Leonis	1755	5	11	8	29.521	+ 310.209	− 0.480	+ 310.852	− 0.643	
		1850	146	11	13	24.010	309.778	0.428	310.421	0.643	
549	ι Leonis	1755	5	11	11	8.252	+ 313.919	− 0.725	+ 312.948	+ 0.971	
		1850	31	11	16	6.157	313.257	0.669	312.289	0.968	
550	79 Leonis	1755	4	11	11	27.873	+ 308.096	− 0.220	+ 308.346	− 0.250	
		1850	15	11	16	20.475	307.913	0.166	308.163	0.250	
551	82 Leonis	1755	1	11	13	3.225	+ 309.090	− 0.312	+ 309.208	− 0.118	
		1850	9	11	17	56.727	308.818	0.260	308.937	0.119	
552	80 Leonis	1755	5	11	13	14.203	+ 308.871	− 0.340	+ 309.466	− 0.595	
		1850	5	11	18	7.485	308.574	0.287	309.168	0.594	
553	83 Leonis	1755	5	11	14	21.020	+ 303.966	− 0.277	+ 309.007	− 5.041	
		1850	25	11	19	9.671	303.728	0.226	308.764	5.036	
554	τ Leonis	1755	5	11	15	19.986	+ 308.932	− 0.278	+ 308.901	+ 0.031	
		1850	201	11	20	13.354	308.693	0.226	308.663	0.030	
		1900	. .	11	22	47.674	308.589	0.195	308.558	0.031	
555	λ Draconis . . .	1755	5	11	16	32.76	+ 377.80	−12.30	+ 378.91	− 1.11	
		1800	. .	11	19	21.55	372.37	11.85	373.46	1.09	
		1850	. .	11	22	26.28	366.56	11.37	367.64	1.08	
		1900	. .	11	25	28.15	360.98	10.92	362.05	1.07	
556	e Leonis	1755	5	11	17	47.993	+ 306.376	+ 0.039	+ 306.290	+ 0.086	
		1850	39	11	22	39.076	306.438	0.091	306.352	0.086	
557	89 Leonis	1755	5	11	21	49.286	+ 307.481	− 0.254	+ 308.710	− 1.229	
		1850	11	11	26	41.286	307.265	0.200	308.495	1.230	
558	υ Leonis	1755	5	11	24	24.393	+ 307.126	− 0.040	+ 307.187	− 0.061	+0.001
		1850	271	11	29	16.154	307.116	+ 0.019	307.178	0.062	
		1900	. .	11	31	49.716	307.132	0.047	307.194	0.062	
559	ω Virginis . . .	1755	5	11	25	48.970	+ 310.203	− 0.506	+ 310.318	− 0.115	
		1850	6	11	30	43.443	309.749	0.451	309.866	0.117	
560	ξ Virginis . . .	1755	5	11	32	38.615	+ 310.131	− 0.478	+ 309.696	+ 0.435	
		1850	17	11	37	33.032	309.704	0.421	309.270	0.434	

DECLINATIONS.

No.	Star.	Mag.	Epoch.	Declination.	Centennial variation.	Secular variation.	Struve's precession.	Proper motion.	Sec. var. of proper motion.
				° ′ ″	″	″	″	″	″
543	B. A. C. 3837 . .	6.3	1850	+ 8 52 49.98	− 1960.50	− 9.77	− 1950.70	− 9.80	
544	φ Leonis	5.0	1755	− 2 18 58.05	− 1952.19	− 9.86	− 1947.32	− 4.87	
		4.2	1850	2 49 56.94	1961.12	8.95	1956.26	4.86	
545	75 Leonis	5.5	1755	+ 3 21 14.87	− 1964.60	− 9.99	− 1948.26	−16.34	
		5.7	1850	2 50 4.14	1973.63	9.04	1957.28	16.35	
546	76 Leonis	6.0	1755	+ 2 59 23.82	− 1958.31	− 9.58	− 1951.71	− 6.60	
		6.3	1850	2 28 19.25	1966.96	8.63	1960.38	6.58	
547	δ Crateris . . .	3.5	1755	− 13 27 20.51	− 1934.94	− 9.11	− 1953.25	+18.31	+ 0.03
		3.8	1850	13 58 2.68	1943.18	8.24	1961.52	18.34	
			1900	14 14 15.28	1947.18	7.78	1965.54	18.36	
548	σ Leonis	4.0	1755	+ 7 22 5.65	− 1957.19	− 9.24	− 1956.00	− 1.19	
		4.1	1850	6 51 2.29	1965.52	8.28	1964.33	1.19	
549	ι Leonis	4.0	1755	+ 11 52 32.23	− 1969.37	− 8.88	− 1961.03	− 8.34	
		4.0	1850	11 21 17.48	1977.33	7.88	1968.95	8.38	
550	79 Leonis	5.5	1755	+ 2 44 57.45	− 1962.87	− 8.60	− 1961.63	− 1.24	
		6.0	1850	2 13 49.00	1970.58	7.64	1969.36	1.22	
551	82 Leonis	7.0	1755	+ 4 38 51.66	− 1969.94	− 8.37	− 1964.50	− 5.46	
		6.9	1850	4 7 36.59	1977.44	7.41	1971.96	5.48	
552	80 Leonis	7.0	1755	+ 5 12 23.59	− 1970.95	− 8.27	− 1964.84	− 6.11	
		6.5	1850	4 41 7.60	1978.35	7.32	1972.25	6.10	
553	83 Leonis	8.0	1755	+ 4 20 43.54	− 1949.18	− 7.78	− 1966.79	+17.61	
		6.5	1850	3 49 48.44	1956.14	6.88	1973.88	17.74	
554	τ Leonis	4.0	1755	+ 4 12 10.01	− 1970.57	− 7.89	− 1968.46	− 2.11	− 0.01
		5.3	1850	3 40 54.56	1977.61	6.93	1975.50	2.11	
			1900	3 24 24.92	1980.94	6.43	1978.84	2.10	
555	λ Draconis . . .	3.5	1755	+ 70 40 48.10	− 1972.59	− 9.51	− 1970.50	− 2.09	
			1800	70 25 59.48	1976.71	8.70	1974.63	2.08	
		3.3	1850	70 9 30.08	1980.83	7.85	1978.76	2.07	
			1900	69 52 58.72	1984.55	7.00	1982.49	2.06	
556	ε Leonis	4.5	1755	− 1 39 16.94	− 1973.88	− 7.35	− 1972.55	− 1.33	
		5.3	1850	2 10 35.30	1980.41	6.41	1979.07	1.34	
557	89 Leonis	6.0	1755	+ 4 25 7.41	− 1989.75	− 6.56	1978.68	−11.07	
		6.2	1850	3 53 34.33	1995.53	5.61	1984.49	11.04	
558	υ Leonis	4.5	1755	+ 0 31 36.93	− 1978.80	− 6.09	− 1982.30	+ 3.50	0.00
		4.4	1850	+ 0 0 14.46	1984.14	5.14	1987.64	3.50	
			1900	− 0 16 18.23	1986.58	4.64	1990.08	3.50	
559	ω Virginis . . .	6.5	1755	+ 9 29 20.94	− 1986.31	− 5.87	− 1984.18	− 2.13	
		5.9	1850	8 57 51.44	1991.43	4.90	1989.29	2.14	
560	ξ Virginis . . .	5.5	1755	+ 9 37 7.56	− 1995.18	− 4.54	− 1992.18	− 3.00	
		5.3	1850	9 5 30.25	1999.03	3.57	1996.02	3.01	

RIGHT ASCENSIONS.

No.	Star.	Epoch.	Number of observations.	Right ascension.			Centennial variation.	Secular variation.	Struve's precession.	Proper motion.	Sec. var. of proper motion.
				h.	m.	s.	s.	s.	s.	s.	s.
561	ν Virginis . . .	1755	5	11	33	15.555	+ 308.972	− 0.383	+ 309.159	− 0.187	
		1850	57	11	38	8.914	308.635	0.328	308.823	0.187	
562	A¹ Virginis . . .	1755	5	11	35	18.978	+ 309.051	− 0.459	+ 309.429	− 0.378	
		1850	11	11	40	12.378	308.642	0.403	309.019	0.377	
563	β Leonis	1755	20	11	36	32.622	+ 307.414	− 0.795	+ 310.900	− 3.486	+0.009
		1850	. .	11	41	24.315	306.686	0.736	310.164	3.478	
		1900	. .	11	43	57.567	306.326	0.703	309.800	3.474	
564	β Virginis . . .	1755	10	11	37	55.992	+ 312.587	− 0.107	+ 307.703	+ 4.884	
		1850	129	11	42	52.910	312.512	0.051	307.632	4.880	
		1900	. .	11	45	29.161	312.494	0.021	307.616	4.878	
565	B. A. C. 4006 . .	1755	2	11	38	31.019	+ 306.422	+ 0.266	+ 306.128	+ 0.294	
		1850	26	11	43	22.250	306.705	0.330	306.414	0.293	
566	γ Ursæ Majoris . .	1755	10	11	40	49.117	+ 324.210	− 4.645	+ 323.000	+ 1.210	−0.016
		1850	390	11	45	55.054	319.903	4.423	318.699	1.204	
		1900	. .	11	48	34.458	317.721	4.304	316.527	1.194	
567	A² Virginis . . .	1755	5	11	42	28.460	+ 308.511	− 0.430	+ 308.762	− 0.251	
		1850	5	11	47	21.360	308.130	0.373	308.382	0.252	
568	B. A. C. 4039 . .	1755	2	11	45	40.184	+ 307.822	− 0.145	+ 307.705	+ 0.117	
		1850	9	11	50	32.558	307.711	0.090	307.593	0.118	
569	b Virginis . . .	1755	5	11	47	23.863	+ 307.491	− 0.144	+ 307.642	− 0.151	
		1850	26	11	52	15.923	307.381	0.088	307.531	0.150	
570	π Virginis . . .	1755	5	11	48	18.908	+ 307.757	− 0.297	+ 307.952	− 0.195	
		1850	160	11	53	11.152	307.502	0.240	307.697	0.195	
571	o Virginis . . .	1755	5	11	52	43.270	+ 306.216	− 0.381	+ 307.753	− 1.537	
		1850	189	11	57	34.012	305.882	0.323	307.416	1.534	
		1900	. .	12	0	6.914	305.728	0.293	307.261	1.533	
572	α Corvi	1755	5	11	55	49.504	+ 306.341	+ 1.442	+ 305.950	+ 0.391	
		1850	5	12	0	41.190	307.749	1.522	307.354	0.395	
573	10 Virginis . . .	1755	5	11	57	8.168	+ 307.359	+ 0.002	+ 307.092	+ 0.267	
		1850	43	12	2	0.168	307.387	0.059	307.121	0.266	
574	11 Virginis . . .	1755	4	11	57	34.089	+ 305.999	− 0.197	+ 307.175	− 1.176	
		1850	11	12	2	24.708	305.838	0.141	307.013	1.175	
575	4 (H) Draconis . .	1755	. .	12	0	21.25	+ 306.99	−15.13	+ 305.94	+ 1.05	
		1775	. .	12	1	22.34	304.00	14.67	302.96	1.04	
		1800	. .	12	2	37.89	300.40	14.12	299.38	1.02	
		1825	. .	12	3	52.56	296.93	13.59	295.92	1.01	
		1850	. .	12	5	6.38	293.60	13.09	292.60	1.00	
		1875	. .	12	6	19.37	290.38	12.61	289.39	0.99	
		1900	. .	12	7	31.58	+ 287.29	−12.16	+ 286.31	+ 0.98	
576	γ Corvi	1755	5	12	3	14.246	+ 306.456	+ 1.074	+ 307.544	− 1.088	
		1850	25	12	8	5.873	307.505	1.135	308.595	1.090	
		1900	. .	12	10	39.769	308.081	1.168	309.172	1.091	

DECLINATIONS.

No.	Star.	Mag.	Epoch.	Declination.	Centennial variation.	Secular variation.	Struve's precession.	Proper motion.	Sec. var. of proper motion.
				° ′ ″	″	″	″	″	″
561	ν Virginis . . .	4.5	1755	+ 7 54 2.76	— 2010.58	— 4.42	— 1992.79	—17.79	
		4.0	1850	7 22 10.87	2014.31	3.44	1996.53	17.78	
562	A¹ Virginis . . .	5.5	1755	+ 9 36 20.16	— 1994.80	— 3.98	— 1994.83	+ 0.03	
		5.8	1850	9 4 43.45	1998.13	3.02	1998.16	0.03	
563	β Leonis	2.5	1755	+ 15 56 26 48	— 2007.92	— 3.67	— 1995.96	—11.96	+ 0.04
		2.0	1850	15 24 37.44	2010.96	2.73	1999.04	11.92	
			1900	15 7 51.64	2012.20	2.23	2000.30	11.90	
564	β Virginis . . .	3.5	1755	+ 3 8 40.51	— 2025.34	— 3.59	— 1997.15	—28.19	— 0.05
		3.7	1850	2 36 34.95	2028.29	2.59	2000.05	28.24	
			1900	2 19 40.51	2029.45	2.07	2001.19	28.26	
565	B. A. C. 4006 . .	. .	1755	— 3 58 16.19	— 1999.88	— 3.35	— 1997.64	— 2.24	
		6.1	1850	4 29 57.45	2002.61	2.40	2000.36	2.25	
566	γ Ursæ Majoris . .	2.0	1755	+ 55 3 24.25	— 1999.58	— 3.07	— 1999.44	— 0.14	— 0.01
		2.3	1850	54 31 43.41	2002.01	2.01	2001.86	0.15	
			1900	54 15 2.18	2002.87	1.45	2002.72	0.15	
567	A² Virginis . . .	6.0	1755	+ 9 48 23.08	— 2001.31	— 2.58	— 2000.59	— 0.72	
		6.1	1850	9 16 40.82	2003.31	1.62	2002.59	c. 72	
568	B. A. C. 4039 . .	7.0	1755	+ 4 50 47.24	— 2003.97	— 1.95	— 2002.54	— 1.43	
		7.5	1850	4 19 2.72	2005.37	0.99	2003.94	1.43	
569	b Virginis . . .	5.5	1755	+ 5 1 12.38	— 2005.50	— 1.60	— 2003.42	— 2.08	
		5.8	1850	4 29 26.58	2006.57	0.65	2004.50	2.07	
570	π Virginis . . .	5.0	1755	+ 7 58 50.40	— 2007.72	— 1.42	— 2003.85	— 3.87	
		4.9	1850	7 27 2.57	2008.62	0.47	2004.76	3.86	
571	o Virginis . . .	4.5	1755	+ 10 5 40.49	— 2001.68	— 0.55	— 2005.46	+ 3.78	0.00
		4.2	1850	9 33 58.79	2001.75	+ 0.39	2005.53	3.78	
			1900	9 17 17.99	— 2001.43	+ 0.89	2005.21	3.78	
572	α Corvi	4.5	1755	— 23 21 40.39	— 2010.82	+ 0.05	— 2006.14	— 4.68	
		4.2	1850	23 53 30.50	2010.33	1.00	2005.64	4.69	
573	10 Virginis . . .	6.0	1755	+ 3 16 29.76	— 2025.97	+ 0.30	— 2006.30	—19.67	
		6.4	1850	2 44 25.38	2025.23	1.25	2005.56	19.67	
574	11 Virginis . . .	7.0	1755	+ 7 10 12.25	— 2004.85	+ 0.39	— 2006.35	+ 1.50	
		6.1	1850	6 38 27.96	2004.03	1.33	2005.53	1.50	
575	4 (H) Draconis . .	5.0	1755	+ 78 58 43.42	— 2004.36	+ 0.93	— 2006.49	+ 2.13	
			1775	78 52 2.71	2004.14	1.13	2006.27	2.13	
			1800	78 43 41.74	2003.82	1.37	2005.95	2.13	
			1825	78 35 20.84	2003.45	1.60	2005.58	2.13	
		4.7	1850	78 27 0.02	2003.01	1.82	2005.14	2.13	
			1875	78 18 39.34	2002.53	2.03	2004.66	2.13	
			1900	+ 78 10 18.77	— 2002.01	+ 2.24	— 2004.14	+ 2.13	
576	γ Corvi	3.0	1755	— 16 10 47.69	— 2004.66	+ 1.49	— 2006.23	+ 1.57	
		2.5	1850	16 42 31.29	2002.79	2.44	2004.36	1.57	
			1900	16 59 12.36	2001.45	2.94	2003.04	1.59	

RIGHT ASCENSIONS.

No.	Star.	Epoch.	Number of observations.	Right ascension.			Centennial variation.	Secular variation.	Struve's precession.	Proper motion.	Sec. var. of proper motion.
				h.	m.	s.	s.	s.	s.	s.	s.
577	β Chamæleontis . .	1850	. .	12	9	39.51	+ 330.42	+17.08	+ 334.85	− 4.43	
		1875	. .	12	11	2.65	334.73	17.71	339.22	4.49	
		1900	. .	12	12	26.89	339.24	18.36	343.78	4.54	
578	B. A. C. 4134 . .	1850	. .	12	10	27.8	. . . .	+ 0.412	+ 307.510	. . .	
579	13 Virginis . . .	1755	5	12	6	7.233	+ 307.013	+ 0.195	+ 306.960	+ 0.053	
		1850	30	12	10	58.991	307.224	0.250	307.172	0.052	
580	14 Virginis . . .	1755	. .	12	6	44.531	+ 307.735	+ 0.626	+ 307.517	+ 0.218	
		1850	13	12	11	37.170	308.356	0.682	308.139	0.217	
581	η Virginis . . .	1755	5	12	7	22.660	+ 306.547	+ 0.196	+ 306.945	− 0.398	
		1850	385	12	12	13.976	306.759	0.250	307.156	0.397	
		1900	. .	12	14	47.388	306.891	0.279	307.289	0.398	
582	c Virginis . . .	1755	4	12	7	54.528	+ 304.602	− 0.006	+ 306.618	− 2.016	
		1850	26	12	12	43.905	304.623	+ 0.050	306.639	2.016	
583	17 Virginis . . .	1755	3	12	10	4.499	+ 305.241	− 0.098	+ 306.308	− 1.067	
		1850	14	12	14	54.443	305.175	0.041	306.242	1.067	
584	α^1 Crucis	1850	. .	12	18	17.54	+ 325.13	+ 6.61	+ 327.46	− 2.33	
		1875	. .	12	19	39.03	326.79	6.71	329.13	2.34	
		1900	. .	12	21	0.94	328.48	6.82	330.83	2.35	
585	q Virginis . . .	1755	4	12	21	9.501	+ 308.047	+ 0.737	+ 308.751	− 0.704	
		1850	54	12	26	2.486	308.773	0.791	309.477	0.704	
586	β Corvi	1755	2	12	21	33.885	+ 312.014	+ 1.552	+ 312.085	− 0.071	
		1850	521	12	26	31.009	313.521	1.620	313.592	0.071	
		1900	. .	12	29	7.973	314.340	1.656	314.410	0.070	
587	κ Draconis . . .	1755	. .	12	22	52.25	+ 266.86	− 6.10	+ 267.98	− 1.12	
		1800	. .	12	24	51.75	264.17	5.83	265.28	1.11	
		1850	. .	12	27	3.13	261.33	5.55	262.42	1.09	
		1900	. .	12	29	13.11	258.62	5.29	259.70	1.08	
588	f Virginis . . .	1755	4	12	24	11.310	+ 307.804	+ 0.561	+ 308.100	− 0.296	
		1850	22	12	29	3.985	308.363	0.615	308.658	0.295	
589	B. A. C. 4254 . .	1850	14	12	30	43.474	+ 305.642	+ 0.502	+ 306.340	− 0.698	
590	χ Virginis . . .	1755	5	12	26	37.342	+ 308.226	+ 0.687	+ 308.802	− 0.576	
		1850	57	12	31	30.475	308.904	0.742	309.482	0.578	
591	γ Virginis . . .	1755	5	12	29	15.411	+ 303.295	+ 0.361	+ 307.029	− 3.734	
		1850	95	12	34	3.712	303.664	0.414	307.393	3.729	
592	28 Virginis . . .	1755	1	12	29	18.760	+ 308.864	+ 0.678	+ 308.835	+ 0.029	
		1850	24	12	34	12.494	309.534	0.732	309.504	0.030	
593	38 Virginis . . .	1755	2	12	40	39.659	+ 305.903	+ 0.531	+ 307.911	− 2.008	
		1850	28	12	45	30.515	306.431	0.581	308.438	2.007	
594	ψ Virginis . . .	1755	4	12	41	38.322	+ 310.217	+ 0.852	+ 310.480	− 0.263	
		1850	56	12	46	33.420	311.052	0.906	311.316	0.264	

DECLINATIONS.

No.	Star.	Mag.	Epoch.	Declination.	Centennial variation.	Secular variation.	Struve's precession.	Proper motion.	Sec. var. of proper motion.
				° ′ ″	″	″	″	″	″
577	β Chamæleontis . .	4.6	1850	− 78 28 44.50	− 1999.55	+ 2.87	− 2003.86	+ 4.31	
			1875	78 37 4.29	1998.79	3.18	2003.09	4.30	
			1900	78 45 23.88	1997.95	3.51	2002.26	4.31	
578	B. A. C. 4134 . .	6.3	1850	− 3 7 12.5	. . . .	+ 2.92	− 2003.54	. . .	
579	13 Virginis . . .	6.0	1755	+ 0 34 37.64	− 2009.53	+ 2.06	− 2005.74	− 3.79	
		6.1	1850	0 2 49.67	2007.12	3.01	2003.34	3.78	
580	14 Virginis . . .	6.5	1755	− 7 33 1.13	− 2009.92	+ 2.18	− 2005.59	− 4.33	
		6.9	1850	8 4 49.42	2007.39	3.14	2003.06	4.33	
581	η Virginis . . .	3.5	1755	+ 0 41 48.64	− 2008.19	+ 2.29	− 2005.41	− 2.78	
		4.0	1850	+ 0 10 2.04	2005.56	3.24	2002.78	2.78	
			1900	− 0 6 40.32	2003.82	3.74	2001.04	2.78	
582	c Virginis . . .	5.5	1755	+ 4 40 45.67	− 2013.03	+ 2.38	− 2005.26	− 7.77	
		5.5	1850	4 8 54.51	2010.32	3.31	2002.55	7.77	
583	17 Virginis . . .	6.0	1755	+ 6 40 13.82	− 2011.22	+ 2.81	− 2004.52	− 6.70	
		6.6	1850	6 8 24.57	2008.11	3.74	2001.40	6.71	
584	α¹ Crucis	1.3	1850	− 62 16 1.06	− 2003.38	+ 4.62	− 1999.25	− 4.13	
			1875	62 24 21.76	2002.19	4.91	1998.05	4.14	
			1900	62 32 42.15	2000.92	5.22	1996.78	4.14	
585	q Virginis . . .	5.5	1755	− 8 5 49.32	− 1999.35	+ 5.00	− 1997.91	− 1.44	
		5.7	1850	8 37 26.29	1994.12	5.98	1992.70	1.42	
586	β Corvi	2.5	1755	− 22 2 17.77	− 2004.13	+ 5.11	− 1997.55	− 6.58	− 0.01
		2.0	1850	22 33 59.24	1998.81	6.11	1992.22	6.59	
			1900	22 50 37.86	1995.63	6.63	1989.04	6.59	
587	κ Draconis . . .	3.5	1755	+ 71 8 30.38	− 1995.93	+ 4.72	− 1996.48	+ 0.55	
			1800	70 53 32.70	1993.74	5.01	1994.28	0.54	
		3.3	1850	70 36 56.47	1991.16	5.33	1991.69	0.53	
			1900	70 20 21.57	1988.41	5.64	1988.93	0.52	
588	f Virginis . . .	6.5	1755	− 4 28 39.08	− 1999.46	+ 5.58	− 1995.29	− 4.17	
		6.0	1850	5 0 15.90	1993.70	6.54	1989.53	4.17	
589	B. A. C. 4254 . .	6.1	1850	+ 2 40 52.20	− 1989.46	+ 6.85	− 1987.64	− 1.82	
590	χ Virginis . . .	6.0	1755	− 6 38 34.86	− 1997.19	+ 6.06	− 1992.93	− 4.26	
		5.2	1850	7 10 9.31	1990.98	7.02	1086.70	4.28	
591	γ Virginis . . .	4.0	1755	− 0 6 5.18	− 1990.72	+ 6.43	− 1990.14	− 0.58	
		3.1	1850	0 37 33.32	1984.18	7.34	1983.53	0.65	
592	28 Virginis . . .	6.0	1755	− 6 8 56.90	− 1994.17	+ 6.62	− 1990.10	− 4.07	
		7.0	1850	6 40 28.22	1987.42	7.58	1983.34	4.08	
593	38 Virginis . . .	6.0	1755	− 2 12 59.59	− 1976.34	+ 8.71	− 1975.00	− 1.34	
		6.2	1850	2 44 13.03	1967.62	9.66	1966.23	1.39	
594	ψ Virginis . . .	5.5	1755	− 8 12 9.43	− 1976.87	+ 9.02	− 1973.43	− 3.44	
		5.2	1850	8 43 23.23	1967.84	9.99	1964.40	3.44	

RIGHT ASCENSIONS.

No.	Star.	Epoch.	Number of observations.	Right ascension.	Centennial variation.	Secular variation.	Struve's precession.	Proper motion.	Sec. var. of proper motion.
				h. m. s.	s.	s.	s.	s.	s.
595	32 (H) Camelopardalis	1755	7	12 47 48.42	+ 4.37	+31.42	+ 5.73	— 1.36	
	(foll.)	1775	. .	12 47 49.90	10.45	29.67	11.79	1.34	
		1800	. .	12 47 53.41	17.58	27.61	18.89	1.31	
		1825	. .	12 47 58.64	24.20	25.57	25.49	1.29	
		1850	. .	12 48 5.47	30.37	23.81	31.63	1.26	
		1875	. .	12 48 13.79	36.12	22.21	37.34	1.23	
		1900	. .	12 48 23.51	+ 41.51	+20.76	+ 42.64	— 1.21	
596	α Canum Venaticorum	1755	5	12 44 31.499	+ 283.624	— 1.614	+ 285.597	— 1.973	+0.015
		1850	364	12 49 0.227	282.135	1.520	284.096	1.961	
		1900	. .	12 51 21.107	281.387	1.473	283.340	1.953	
597	*k* Virginis . . .	1755	5	12 47 3.282	+ 307.907	+ 0.577	+ 308.179	— 0.272	
		1850	11	12 51 56.061	308.479	0.627	308.752	0.273	
598	46 Virginis . . .	1755	5	12 48 0.168	+ 307.673	+ 0.560	+ 307.989	— 0.316	
		1850	8	12 52 52.717	308.229	0.610	308.544	0.315	
599	48 Virginis . . .	1755	5	12 51 18.199	+ 307.807	+ 0.593	+ 308.210	— 0.403	
		1850	17	12 56 10.889	308.393	0.641	308.795	0.402	
600	*g* Virginis . . .	1755	5	12 55 5.463	+ 312.336	+ 0.987	+ 312.282	+ 0.054	
		1850	21	13 0 2.635	313.298	1.039	313.244	0.054	
601	B. A. C. 4394 . .	1755	. .	. . . . . .	. . . .	+ 0.890	+ 311.338	. . .	
		1850	. .	13 0 43.5	. . . .	0.945	312.210	. . .	
602	50 Virginis . . .	1755	5	12 56 57.385	+ 312.284	+ 0.977	+ 312.216	+ 0.068	
		1850	20	13 1 54.503	313.235	1.026	313.167	0.068	
603	θ Virginis . . .	1755	5	12 57 17.296	+ 309.093	+ 0.720	+ 309.441	— 0.348	
		1850	364	13 2 11.267	309.801	0.770	310.147	0.346	
		1900	. .	13 4 46.265	310.192	0.794	310.537	0.345	
604	56 Virginis . . .	1755	4	13 1 56.435	+ 312.433	+ 1.000	+ 312.693	— 0.260	
		1850	3	13 6 53.705	313.407	1.050	313.665	0.258	
605	58 Virginis . . .	1755	1	13 4 38.618	+ 312.506	+ 1.021	+ 313.056	— 0.550	
		1850	19	13 9 35.969	313.500	1.072	314.051	0.551	
606	62 Virginis . . .	1755	5	13 7 29.969	+ 312.856	+ 1.072	+ 313.849	— 0.993	
		1850	5	13 12 27.673	313.898	1.122	314.894	0.996	
607	65 Virginis . . .	1755	5	13 10 38.610	+ 309.298	+ 0.750	+ 309.574	— 0.276	
		1850	11	13 15 32.789	310.032	0.796	310.308	0.276	
608	66 Virginis . . .	1755	4	13 11 49.511	+ 310.700	+ 0.770	+ 309.790	+ 0.910	
		1850	11	13 16 45.030	311.453	0.816	310.541	0.912	
609	α Virginis . . .	1755	100	13 12 19.140	+ 313.862	+ 1.088	+ 314.223	— 0.361	
		1850	. .	13 17 17.807	314.919	1.135	315.281	0.362	
		1900	. .	13 19 55.409	315.493	1.160	315.854	0.361	
610	*i* Virginis . . .	1755	5	13 13 48.726	+ 314.554	+ 1.181	+ 315.552	— 0.998	
		1850	11	13 18 48.093	315.701	1.233	316.696	0.995	
611	69 Virginis . . .	1755	4	13 14 25.525	+ 317.236	+ 1.369	+ 318.189	— 0.953	
		1850	7	13 19 27.524	318.560	1.420	319.516	0.956	

DECLINATIONS.

No.	Star.	Mag.	Epoch.	Declination.	Centennial variation.	Secular variation.	Struve's precession.	Proper motion.	Sec. var. of proper motion.
				° ′ ″	″	″	″	″	″
595	32 (H) Camelopardalis	6.0	1755	+ 84 44 45.75	− 1961.64	+ 0.93	− 1962.97	+ 1.33	
	(foll.)		1775	84 38 13.44	1961.43	1.11	1962.75	1.32	
			1800	84 30 3.12	1961.12	1.32	1962.43	1.31	
			1825	84 21 52.88	1960.75	1 53	1962.05	1.30	
		4.7	1850	84 13 42.76	1960.36	1.72	1961.65	1.29	
			1875	84 5 32.73	1959.90	1.91	1961.18	1.28	
			1900	+ 83 57 22.81	− 1959.41	+ 2.08	− 1960.68	+ 1.27	
596	α Canum Venaticorum	2.5	1755	+ 39 38 47.78	− 1963.83	+ 8.79	− 1968.71	+ 4.88	− 0.06
		2.7	1850	39 7 46.21	1955.14	9.51	1959.97	4.83	
			1900	38 51 29.84	1950.29	9.89	1955.09	4.80	
597	*k* Virginis . . .	6.0	1755	− 2 29 3.38	− 1964.36	+ 9.99	1964.31	− 0.05	
		5.9	1850	3 0 4.87	1954.42	10.94	1954.36	0.06	
598	46 Virginis . . .	6.5	1755	− 2 2 42.61	− 1958.53	+ 10.17	− 1962.60	+ 4 07	
		6.1	1850	2 33 38.48	1948.42	11.11	1952.49	4.07	
599	48 Virginis . . .	6.0	1755	− 2 20 20.88	− 1959.70	+ 10.81	− 1956.41	− 3.29	
		6.7	1850	2 51 17.57	1948.98	11.76	1945.68	3.30	
600	*g* Virginis . . .	5.5	1755	− 9 25 25.67	− 1950.08	+ 11.69	− 1948.77	− 1.31	
		5.9	1850	9 56 12.82	1938.51	12.67	1937.20	1.31	
601	B. A. C. 4394 . .	. .	1755	− 7 39 57.94	− 1950.73	+ 11.78	− 1947.32	− 3.41	
		6.0	1850	8 10 45.68	1939.10	12.73	1935.65	3.45	
602	50 Virginis . . .	6.0	1755	− 9 0 56.57	− 1946.13	+ 12.07	− 1944.82	− 1.31	
		6.3	1850	9 31 39.80	1934.21	13.04	1932.90	1.31	
603	θ Virginis . . .	4.5	1755	− 4 13 27.16	− 1948.22	+ 11.99	− 1944.11	− 4.11	− 0.01
		4.7	1850	4 44 12.41	1936.38	12.94	1932.26	4.12	
			1900	5 0 18.96	1929.78	13.44	1925.66	4.12	
604	56 Virginis . . .	7.5	1755	− 9 3 45.17	− 1939.80	+ 12.99	− 1933.62	− 6.18	
		7.0	1850	9 34 21.96	1926.99	13.97	1920.82	6.17	
605	58 Virginis . . .	6.0	1755	− 9 14 51.71	− 1925.89	+ 13.51	− 1927.20	+ 1.31	
		7.0	1850	9 45 15.06	1912.59	14.50	1913.86	1.27	
606	62 Virginis . . .	7.0	1755	− 10 0 32.66	− 1921.99	+ 14.03	− 1920.06	− 1.93	
		7.0	1850	10 30 52.07	1908.19	15.01	1906.22	1.97	
607	65 Virginis . . .	6.0	1755	− 3 38 4.38	− 1914.55	+ 14.50	− 1911.89	− 2.66	
		6.1	1850	4 8 16.52	1900.33	15.44	1897.66	2.67	
608	66 Virginis . . .	6.0	1755	− 3 52 31.42	− 1913.33	+ 14.84	− 1908.73	− 4.60	
		6.0	1850	4 22 42.24	1898.78	15.79	1894.22	4.56	
609	α Virginis . . .	1.0	1755	− 9 52 27.41	− 1911.15	+ 15.01	− 1907.39	− 3.76	− 0.02
		1.5	1850	10 22 36.08	1896.42	15.99	1892.65	3.77	
			1900	10 38 22.27	1888.30	16.51	1884.51	3.79	
610	*i* Virginis . . .	5.0	1755	− 11 25 26.59	− 1907.39	+ 15.34	− 1903.26	− 4.13	
		5.7	1850	11 55 31.54	1892.34	16.34	1888.24	4.10	
611	69 Virginis . . .	5.6	1755	− 14 41 39.28	− 1901.80	+ 15.54	− 1901.59	− 0.21	
		5.0	1850	15 11 38.83	1886.56	16.55	1886.31	0.25	

RIGHT ASCENSIONS.

No.	Star.	Epoch.	Number of observations.	Right ascension.			Centennial variation.	Secular variation.	Struve's precession.	Proper motion.	Sec. var. of proper motion.
				h.	m.	s.	s.	s.	s.	s.	s.
612	l^1 Virginis . . .	1755	4	13	17	40.435	+ 311.221	+ 0.858	+ 311.042	+ 0.179	
		1850	3	13	22	36.489	312.057	0.903	311.879	0.178	
613	l^2 Virginis . . .	1755	5	13	19	15.239	+ 310.160	+ 0.856	+ 310.949	— 0.789	
		1850	36	13	24	10.284	310.994	0.900	311.783	0.789	
614	75 Virginis . . .	1755	3	13	19	48.305	+ 318.161	+ 1.359	+ 318.455	— 0.294	
		1850	13	13	24	51.179	319.476	1.409	319.770	0.294	
615	*h* Virginis . . .	1755	5	13	20	5.761	+ 313.810	+ 1.065	+ 314.164	— 0.354	
		1850	39	13	25	4.369	314.843	1.110	315.198	0.355	
616	77 Virginis . . .	1755	. .	13	20	38.158	+ 311.614	+ 0.930	+ 312.130	— 0.516	
		1850	8	13	25	34.618	312.518	0.974	313.036	0.518	
617	ζ Virginis . . .	1755	5	13	22	13.670	+ 304.486	+ 0.583	+ 306.452	— 1.966	—0.002
		1850	483	13	27	3.201	305.060	0.626	307.027	1.967	
		1900	. .	13	29	35.810	305.378	0.648	307.348	1.970	
618	80 Virginis . . .	1755	5	13	22	48.044	+ 310.457	+ 0.823	+ 310.419	+ 0.038	
		1850	19	13	27	43.356	311.260	0.868	311.221	0.039	
619	81 Virginis . . .	1755	1	13	24	46.735	+ 312.425	+ 0.962	+ 312.601	— 0.176	
		1850	4	13	29	43.979	313.360	1.006	313.538	0.178	
620	*m* Virginis . . .	1755	5	13	28	46.935	+ 312.915	+ 1.016	+ 313.610	— 0.695	
		1850	99	13	33	44.671	313.902	1.062	314.598	0.696	
621	83 Virginis . . .	1755	5	13	31	19.193	+ 320.907	+ 1.451	+ 320.841	+ 0.066	
		1850	16	13	36	24.716	322.306	1.495	322.242	0.064	
622	85 Virginis . . .	1755	4	13	32	26.185	+ 320.090	+ 1.427	+ 320.599	— 0.509	
		1850	15	13	37	30.921	321.467	1.472	321.979	0.512	
623	86 Virginis . . .	1755	5	13	32	55.227	+ 317.207	+ 1.236	+ 317.432	— 0.225	
		1850	37	13	37	57.138	318.401	1.278	318.627	0.226	
624	87 Virginis . . .	1755	5	13	34	8.640	+ 323.153	+ 1.565	+ 322.943	+ 0.210	
		1850	9	13	39	16.349	324.661	1.610	324.451	0.210	
625	B. A. C. 4591 . .	1850	7	13	39	18.0	. . . .	+ 1.133	+ 316.023	. . .	
626	88 Virginis . . .	1755	3	13	35	30.784	+ 311.859	+ 0.948	+ 312.309	— 0.450	
		1850	3	13	40	27.482	312.778	0.987	313.228	0.450	
627	η Ursæ Majoris . .	1755	5	13	37	51.349	+ 238.553	— 1.119	+ 239.696	— 1.143	+0.012
		1850	593	13	41	37.482	237.529	1.038	238.657	1.128	
		1900	. .	13	43	36.119	237.021	0.996	238.145	1.124	
628	89 Virginis . . .	1755	5	13	36	36.388	+ 322.839	+ 1.584	+ 323.628	— 0.789	
		1850	42	13	41	43.807	324.365	1.629	325.155	0.790	
629	B. A. C. 4647 (mean)	1755	1	13	42	9.183	+ 312.439	+ 1.028	+ 313.915	— 1.476	
		1850	19	13	47	6.470	313.435	1.068	314.913	1.478	
630	η Bootis	1755	5	13	43	1.125	+ 285.764	— 0.098	+ 286.260	— 0.496	
		1850	853	13	47	32.564	285.693	0.052	286.177	0.484	
		1900	. .	13	49	55.405	285.673	0.027	286.154	0.481	

DECLINATIONS.

No.	Star.	Mag.	Epoch.	Declination.	Centennial variation.	Secular variation.	Struve's precession.	Proper motion.	Sec. var. of proper motion.
				° ′ ″	″	″	″	″	″
612	l^1 Virginis . . .	7.5	1755	— 5 11 50.72	— 1890.05	+ 15.92	— 1892.32	+ 2.27	
		6.7	1850	5 41 38.94	1874.48	16.87	1876.76	2.28	
613	l^2 Virginis . . .	6.0	1755	— 4 58 56.40	— 1892.28	+ 16.18	— 1887.70	— 4.58	
		5.1	1850	5 28 46.63	1876.47	17.13	1871.88	4.59	
614	75 Virginis . . .	6.0	1755	— 14 5 37.09	— 1887.23	+ 16.65	— 1886.04	— 1.19	
		6.0	1850	14 35 22.30	1870.94	17.66	1869.74	1.20	
615	h Virginis . . .	6.0	1755	— 8 53 38.06	— 1889.01	+ 16.47	— 1885.16	— 3.85	
		5.8	1850	9 23 25.03	1872.90	17.44	1869.04	3.86	
616	77 Virginis . . .	7.0	1755	— 6 21 19.23	— 1883.46	+ 16.45	— 1883.57	+ 0.11	
		7.0	1850	6 51 0.95	1867.38	17.41	1867.43	0.05	
617	ζ Virginis . . .	4.0	1755	+ 0 39 55.50	— 1874.57	+ 16.31	— 1878.70	— 4.13	— 0.10
		3.6	1850	+ 0 10 22.16	1858.65	17.20	1862.69	4.04	
			1900	— 0 5 5.00	1849.93	17.66	1853.91	3.98	
618	80 Virginis . . .	6.0	1755	— 4 8 19.35	— 1870.11	+ 16.85	— 1876.96	+ 6.85	
		6.1	1850	4 37 48.21	1853.66	17.80	1860.50	6.84	
619	81 Virginis . . .	7.5	1755	— 6 36 45.02	— 1875.01	+ 17.27	— 1870.73	— 4.28	
		7.3	1850	7 6 18.34	1858.15	18.23	1853.86	4.29	
620	m Virginis . . .	5.5	1755	— 7 27 25.86	— 1853.78	+ 18.04	— 1857.79	+ 4.01	
		5.7	1850	7 56 38.64	1836.17	19.04	1840.18	4.01	
621	83 Virginis . . .	6.0	1755	— 14 56 10.21	— 1852.39	+ 18.96	— 1849.26	— 3.13	
		6.0	1850	15 25 21.27	1833.89	19.99	1830.78	3.11	
622	85 Virginis . . .	6.0	1755	— 14 31 33.03	— 1849.76	+ 19.09	— 1845.46	— 4.30	
		6.5	1850	15 0 41.54	1831.14	20.11	1826.80	4.34	
623	86 Virginis . . .	6.0	1755	— 11 11 19.09	— 1843.83	+ 19.03	— 1843.79	— 0.04	
		5.9	1850	11 40 21.98	1825.28	20.03	1825.22	0.06	
624	87 Virginis . . .	6.0	1755	— 16 37 20.34	— 1844.32	+ 19.63	— 1839.54	— 4.78	
		5.8	1850	17 6 23.42	1825.17	20.68	1820.41	4.76	
625	B. A. C. 4591 . .	6.0	1850	— 8 57 15.2	. . . .	+ 20.14	— 1820.30	. . .	
626	88 Virginis . . .	7.0	1755	— 5 36 13.04	— 1838.30	+ 19.18	— 1834.72	— 3.58	
		6.8	1850	6 5 10.62	1819.63	20.12	1816.03	3.60	
627	η Ursæ Majoris . .	2.5	1755	+ 50 32 39.21	— 1828.71	+ 15.15	— 1826.34	— 2.37	— 0.07
		2.0	1850	50 3 48.84	1814.11	15.59	1811.68	2.43	
			1900	49 48 43.74	1806.26	15.81	1803.79	2.47	
628	89 Virginis . . .	5.5	1755	— 16 54 10.17	— 1835.92	+ 20.01	— 1830.82	— 5.10	
		5.4	1850	17 23 5.11	1816.42	21.05	1811.28	5.14	
629	B. A. C. 4647 (mean)	7.0	1755	— 6 50 33.32	— 1812.57	+ 20.33	— 1810.44	— 2.13	
		6.4	1850	7 19 5.95	1792.81	21.27	1790.56	2.25	
630	η Bootis	3.0	1755	+ 19 38 8.46	— 1843.22	+ 18.89	— 1807.15	— 36.07	— 0.02
		3.0	1850	19 9 6.04	1824.94	19.60	1788.86	36.08	
			1900	18 53 56.03	1815.04	19.98	1778.95	36.09	

RIGHT ASCENSIONS.

No.	Star.	Epoch.	Number of observations.	Right ascension.	Centennial variation.	Secular variation.	Struve's precession.	Proper motion.	Sec. var. of proper motion.
				h. m. s.	s.	s.	s.	s.	s.
631	W² 13h 825 . .	1850	. .	13 47 51.9	. . . .	+ 1.150	+ 316.587	. . .	
632	β Centauri . . .	1850	. .	13 53 17.20	+ 414.47	+ 8.32	+ 415.50	− 1.03	
		1875	. .	13 55 1.07	416.56	8.40	417.59	1.03	
		1900	. .	13 56 45.48	418.67	8.47	419.70	1.03	
633	94 Virginis . . .	1755	5	13 53 21.442	+ 315.372	+ 1.104	+ 315.594	− 0.222	
		1850	35	13 58 21.550	316.438	1.141	316.660	0.222	
634	95 Virginis . . .	1755	5	13 53 47.368	+ 315.044	+ 1.124	+ 316.103	− 1.059	
		1850	20	13 58 47.173	316.130	1.162	317.189	1.059	
635	α Draconis .	1755	10	13 57 45.97	+ 161.69	+ 0.50	+ 162.36	− 0.67	
		1800	. .	13 58 58.79	161.92	0.50	162.58	0.66	
		1850	. .	14 0 19.82	162.17	0.50	162.83	0.66	
		1900	. .	14 1 40.97	162.43	0.50	163.07	0.64	
636	96 Virginis . . .	1755	5	13 55 59.370	+ 317.418	+ 1.197	+ 317.454	− 0.036	
		1850	5	14 1 1.460	318.567	1.221	318.598	0.031	
637	B. A. C. 4700 . .	1850	34	14 2 39.376	+ 326.485	+ 1.560	+ 326.205	+ 0.280	
638	97 Virginis . . .	1755	1	13 59 31.942	+ 317.587	+ 1.176	+ 317.236	+ 0.351	
		1850	6	14 4 34.185	318.720	1.208	318.367	0.353	
639	κ Virginis . . .	1755	5	13 59 51.625	+ 317.747	+ 1.195	+ 317.719	+ 0.028	
		1850	145	14 4 54.029	318.898	1.229	318.868	0.030	
640	B. A. C. 4720 . .	1755	2	14 1 37.353	+ 310.604	+ 0.973	+ 312.675	− 2.071	
		1850	3	14 6 32.870	311.543	1.004	313.619	2.076	
641	B. A. C. 4722 . .	1755	. .	14 1 56.718	+ 327.538	+ 1.640	+ 327.800	− 0.262	
		1850	20	14 7 8.625	329.114	1.678	329.381	0.267	
642	ι Virginis . . .	1755	5	14 3 11.881	+ 312.560	+ 1.013	+ 312.771	− 0.211	
		1850	28	14 8 9.276	313.538	1.045	313.735	0.197	
643	α Bootis	1755	. .	14 4 29.638	+ 273.209	+ 0.182	+ 281.190	− 7.981	+0.086
		1850	. .	14 8 49.275	273.402	0.224	281.291	7.889	
		1900	. .	14 11 6.005	273.520	0.249	281.357	7.837	
644	λ Virginis . . .	1755	5	14 5 53.608	+ 321.926	+ 1.367	+ 322.134	− 0.208	
		1850	121	14 11 0.059	323.239	1.397	323.449	0.210	
645	2 Libræ	1755	5	14 10 16.899	+ 320.273	+ 1.290	+ 320.448	− 0.175	
		1850	34	14 15 21.745	321.513	1.320	321.686	0.173	
646	B. A. C. 4772 . .	1755	. .	14 11 32.710	+ 320.211	+ 1.287	+ 320.515	− 0.304	
		1850	6	14 16 37.496	321.447	1.316	321.749	0.302	
647	θ Bootis	1755	4	14 16 51.097	+ 204.611	− 0.169	+ 207.219	− 2.608	+0.069
		1850	136	14 20 5.407	204.469	0.130	207.012	2.543	
		1900	. .	14 21 47.626	204.409	0.111	206.912	2.503	
648	106 Virginis . . .	1755	5	14 15 48.250	+ 314.344	+ 1.058	+ 314.567	− 0.223	
		1850	13	14 20 47.357	315.361	1.084	315.582	0.221	

DECLINATIONS.

No.	Star.	Mag.	Epoch.	Declination.	Centennial variation.	Secular variation.	Struve's precession.	Proper motion.	Sec. var. of proper motion.
				° ′ ″	″	″	″	″	″
631	W² 13ʰ 825 . .	6.8	1850	− 8 49 18.2	. . . .	+ 21.71	− 1787.57	. . .	
632	β Centauri . . .	1.2	1850	− 59 38 45.35	− 1770.85	+ 29.37	− 1765.55	− 5.30	
			1875	59 46 7.14	1763.44	29.92	1758.13	5.31	
			1900	59 53 27.05	1755.88	30.47	1750.57	5.31	
633	94 Virginis . . .	6.0	1755	− 7 42 36.88	− 1765.59	+ 22.64	− 1766.00	+ 0.41	
		6.8	1850	8 10 23.83	1743.63	23.61	1744.07	0.44	
634	95 Virginis . . .	6.0	1755	− 8 7 58.48	− 1763.63	+ 22.63	− 1764.17	+ 0.54	
		6.0	1850	8 35 43.56	1741.69	23.55	1742.22	0.53	
635	α Draconis . . .	3.5	1755	+ 65 33 12.43	− 1746.69	+ 12.30	− 1747.34	+ 0.65	
			1800	65 20 7.66	1741.15	12.42	1741.77	0.62	
		3.3	1850	65 5 38.66	1734.88	12.56	1735.48	0.60	
			1900	64 51 12.79	1728.58	12.70	1729.14	0.56	
636	96 Virginis . . .	6.5	1755	− 9 9 40.23	− 1754.95	+ 23.20	− 1754.93	− 0.02	
		6.9	1850	9 37 16.82	1732.45	24.16	1732.43	0.02	
637	B. A. C. 4700 . .	5.6	1850	− 15 35 26.83	− 1726.06	+ 25.05	− 1725.20	− 0.86	
638	97 Virginis . . .	7.0	1755	− 8 44 6.37	− 1743.45	+ 23.86	− 1739.68	− 3.77	
		7.0	1850	9 11 31.74	1720.33	24.81	1716.59	3.74	
639	κ Virginis . . .	4.0	1755	− 9 7 13.80	− 1725.80	+ 24.00	− 1738.28	+12.48	
		4.2	1850	9 34 22.32	1702.52	25.01	1715.10	12.58	
640	B. A. C. 4720 . .	7.5	1755	− 4 47 45.08	− 1724.57	+ 23.52	− 1730.50	+ 5.93	
		6.7	1850	5 14 52.68	1701.80	24.41	1707.57	5.77	
641	B. A. C. 4722 . .	. .	1755	− 17 2 41.05	− 1730.69	+ 25.12	− 1729.12	− 1.57	
		5.8	1850	17 29 53.70	1706.31	26.20	1704.82	1.49	
642	ι Virginis . . .	4.0	1755	− 4 49 9.08	− 1766.07	+ 24.08	− 1723.48	−42.59	
		4.1	1850	5 16 55.87	1742.77	24.98	1700.16	42.61	
643	α Bootis	1.0	1755	+ 20 28 7.65	− 1916.81	+ 20.72	− 1717.61	−199.20	− 0.62
		1.0	1850	19 57 56.12	1896.84	21.34	1696.05	199.79	
			1900	19 42 10.38	1886.08	21.67	1685.99	200.09	
644	λ Virginis . . .	4.0	1755	− 12 13 47.71	− 1709.70	+ 25.33	− 1711.28	+ 1.58	
		5.0	1850	12 40 40.33	1685.17	26.31	1686.82	1.65	
645	2 Libræ	6.0	1755	− 10 34 51.79	− 1698.59	+ 25.87	− 1690.90	− 7.69	
		6.5	1850	11 1 33.63	1673.56	26.82	1665.88	7.68	
646	B. A. C. 4772 . .	7.5	1755	− 10 32 35.70	− 1689.44	+ 26.06	− 1684.88	− 4.56	
		6.6	1850	10 59 8.76	1664.23	27.02	1659.68	4.55	
647	θ Bootis	4.0	1755	+ 52 59 32.22	− 1699.69	+ 17.26	− 1659.23	−40.46	− 0.22
		4.0	1850	52 32 45.35	1683.14	17.59	1642.47	40.67	
			1900	52 18 45.99	1674.30	17.76	1633.53	40.77	
648	106 Virginis . . .	6.0	1755	− 5 47 10.45	− 1671.67	+ 26.31	− 1664.39	− 7.28	
		5.9	1850	6 13 26.53	1646.25	27.20	1638.95	7.30	

RIGHT ASCENSIONS.

No.	Star.	Epoch.	Number of observations.	Right ascension.	Centennial variation.	Secular variation.	Struve's precession.	Proper motion.	Sec. var. of proper motion.
				h. m. s.	s.	s.	s.	s.	s.
649	ρ Bootis	1755	5	14 21 15.916	+ 259.030	— 0.219	+ 259.687	— 0.657	
		1850	204	14 25 21.902	258.841	0.179	259.505	0.664	
		1900	. .	14 27 31.301	258.757	0.157	259.425	0.668	
650	5 Ursæ Minoris . .	1755	5	14 28 22.70	— 36.03	+13.46	— 36.57	+ 0.54	
		1800	. .	14 28 7.82	30.10	12.89	30.63	0.53	
		1850	. .	14 27 54.34	23.81	12.29	24.32	0.51	
		1900	. .	14 27 43.96	17.77	11.78	18.30	0.53	
651	α² Centauri . . .	1850	. .	14 29 27.77	+ 401.70	+ 7.32	+ 448.94	—47.24	—0.710
		1875	. .	14 31 8.43	403.53	7.36	450.95	47.42	
		1900	. .	14 32 49.54	405.37	7.40	452.98	47.61	
652	5 Libræ	1755	4	14 32 29.743	+ 327.980	+ 1.499	+ 328.217	— 0.237	
		1850	37	14 37 42.003	329.413	1.517	329.651	0.238	
653	ε Bootis	1755	4	14 34 17.151	+ 262.162	— 0.045	+ 262.428	— 0.266	—0.001
		1850	839	14 38 26.189	262.135	— 0.011	262.401	0.266	
		1900	. .	14 40 37.256	262.136	+ 0.016	262.405	0.269	
654	μ Libræ	1755	5	14 35 55.819	+ 326.016	+ 1.428	+ 326.598	— 0.582	
		1850	23	14 41 6.181	327.380	1.445	327.962	0.582	
655	α¹ Libræ	1755	7	14 37 10.919	+ 328.702	+ 1.530	+ 329.634	— 0.932	
		1850	61	14 42 23.880	330.165	1.550	331.097	0.932	
656	α² Libræ	1755	7	14 37 22.169	+ 328.884	+ 1.529	+ 329.732	— 0.848	+0.001
		1850	660	14 42 35.303	330.345	1.545	331.193	0.848	
		1900	. .	14 45 20.667	331.120	1.553	331.968	0.848	
657	B. A. C. 4896 . .	1755	1	14 37 55.766	+ 332.202	+ 1.635	+ 332.601	— 0.399	
		1850	22	14 43 12.098	333.763	1.651	334.160	0.397	
658	10 Libræ	1755	3	14 38 9.670	+ 333.170	+ 1.659	+ 333.568	— 0.398	
		1850	3	14 43 26.932	334.753	1.674	335.151	0.398	
659	12 Libræ	1755	4	14 40 9.788	+ 344.606	+ 2.065	+ 344.633	— 0.027	
		1850	14	14 45 38.097	346.573	2.078	346.597	0.024	
660	ξ¹ Libræ	1755	5	14 41 7.165	+ 323.077	+ 1.310	+ 323.622	— 0.545	
		1850	11	14 46 14.681	324.328	1.325	324.875	0.547	
661	ξ² Libræ	1755	5	14 43 30.762	+ 322.974	+ 1.283	+ 323.066	— 0.092	
		1850	80	14 48 38.169	324.200	1.300	324.298	0.098	
662	B. A. C. 4923 . .	1755	. .	. . . . . .	. . . .	+ 1.881	+ 339.320	. . .	
		1850	13	14 48 42.8	. . . .	2.049	341.164	. . .	
663	17 Libræ	1755	4	14 44 59.173	+ 324.271	+ 1.276	+ 322.792	+ 1.479	
		1850	3	14 50 7.808	325.490	1.291	324.008	1.482	
664	18 Libræ	1755	5	14 45 40.629	+ 322.079	+ 1.276	+ 322.826	— 0.747	
		1850	3	14 50 47.182	323.298	1.291	324.043	0.745	
665	β Ursæ Minoris . .	1755	6	14 51 42.50	— 37.51	+11.24	— 36.78	— 0.73	
		1800	. .	14 51 26.78	32.51	10.86	31.79	0.72	
		1850	. .	14 51 11.87	27.18	10.45	26.47	0.71	
		1900	. .	14 50 59.57	— 22.03	+10.06	— 21.35	— 0.68	

DECLINATIONS.

No.	Star.	Mag.	Epoch.	Declination.	Centennial variation.	Secular variation.	Struve's precession.	Proper motion.	Sec. var. of proper motion.
				° ′ ″	″	″	″	″	″
649	ρ Bootis	4.0	1755	+ 31 27 31.54	− 1626.92	+ 22.54	− 1637.20	+10.28	
		3.7	1850	31 1 56.23	1605.26	23.06	1615.55	10.29	
			1900	30 48 36.49	1593.66	23.33	− 1603.95	10.29	
650	5 Ursæ Minoris . .	4.0	1755	+ 76 47 6.13	− 1598.89	− 2.44	− 1600.43	+ 1.54	
			1800	76 35 6.38	1599.88	1.91	1601 44	1.56	
		4.7	1850	76 21 46.22	1600.69	1.36	1602.27	1.58	
			1900	76 8 25.73	1601.24	0.84	1602.84	1.60	
651	α² Centauri . . .	0.7	1850	− 60 12 45.01	− 1550.52	+ 32.06	− 1994.02	+43.50	− 4.30
			1875	60 19 11.63	1542.45	32.51	1584.91	42.46	
			1900	60 25 36.22	1534.27	32.96	1575.65	41.38	
652	5 Libræ	6.0	1755	− 14 24 39.56	− 1579.32	+ 30.20	− 1578.43	− 0.89	
		6.6	1850	14 49 26.13	1550.17	31.16	1549.28	0.89	
653	ε Bootis	3.0	1755	+ 28 7 11.23	− 1567.66	+ 24.50	− 1568.74	+ 1.08	− 0.02
		2.3	1850	27 42 33.09	1544.14	25.02	1545.17	1.03	
			1900	27 29 44.15	1531.56	25.28	1532.57	1.01	
654	μ Libræ	5.5	1755	− 13 6 43.74	− 1562.83	+ 30.54	− 1559.70	− 3.13	
		5.7	1850	13 31 14.51	1533.37	31.48	1530.17	3.20	
655	ι¹ Libræ	6.0	1755	− 14 57 43.72	− 1560.92	+ 31.08	− 1552.78	− 8.14	
		6.3	1850	15 22 12.43	1530.96	32.03	1522.83	8.13	
656	α² Libræ	3.0	1755	− 15 0 26.87	− 1559.53	+ 31.04	− 1551.74	− 7.79	− 0.06
		3.0	1850	15 24 54.27	1529.59	31.99	1521.75	7.84	
			1900	15 37 35.05	1513.47	32.49	1505.59	7.88	
657	B. A. C. 4896 . .	6.0	1755	− 16 45 18.33	− 1560.01	+ 31.44	− 1548.60	−11.41	
		6.6	1850	17 9 46.00	1529.67	32.43	1518.27	11.40	
658	10 Libræ	7.0	1755	− 17 19 45.14	− 1548.15	+ 31.57	− 1547.33	− 0.82	
		6.5	1850	17 44 1.48	1517.69	32.56	1516.84	0.85	
659	12 Libræ	6.0	1755	− 23 37 19.94	− 1541.78	+ 33.00	− 1536.12	− 5.66	
		5.8	1850	24 1 29.58	1509.92	34.08	1504.26	5.66	
660	ξ¹ Libræ	6.0	1755	− 10 52 55.52	− 1533.18	+ 31.08	− 1530.72	− 2.46	
		6.1	1850	11 16 57.88	1503.23	31.98	1500.72	2.51	
661	ξ² Libræ	5.0	1755	− 10 24 15.41	− 1517.86	+ 31.48	− 1517.09	− 0.77	
		5.7	1850	10 48 3.04	1487.53	32.37	1486.75	0.78	
662	B. A. C. 4923 . .	. .	1755	− 20 17 40.08	− 1684.03	+ 34.30	− 1518.76	−165.27	
		7.3	1850	20 44 4.27	1650.92	35.40	1486.30	164.62	
663	17 Libræ	7.0	1755	− 10 9 14.79	− 1510.93	+ 31.98	− 1508.61	− 2.32	
		7.2	1850	10 32 55.60	1480.12	32.88	1478.12	2.00	
664	18 Libræ	7.0	1755	− 10 8 31.71	− 1513.63	+ 31.66	− 1504.62	− 9.01	
		6.3	1850	10 32 15.24	1483.13	32.54	1474.04	9.09	
665	β Ursæ Minoris . .	3.0	1755	+ 75 9 23.15	− 1468.65	− 3.17	− 1469.17	+ 0.52	
			1800	74 58 21.96	1469.97	2.66	1470.46	0.49	
		2.0	1850	74 46 6.67	1471.14	2.13	1471.60	0.46	
			1900	+ 74 33 50.85	− 1472.09	− 1.62	− 1472.52	+ 0.43	

RIGHT ASCENSIONS.

No.	Star.	Epoch.	Number of observations.	Right ascension. h.	m.	s.	Centennial variation. s.	Secular variation. s.	Struve's precession. s.	Proper motion. s.	Sec. var. of proper motion. s.
666	γ Scorpii	1755	5	14	49	47.383	+ 347.132	+ 2.081	+ 347.762	− 0.630	
		1850	52	14	55	18.099	349.112	2.088	349.741	0.629	
667	β Bootis	1755	4	14	52	43.067	+ 226.016	− 0.025	+ 226.391	− 0.375	+0.005
		1850	151	14	56	17.775	226.005	+ 0.001	226.369	0.364	
		1900	. .	14	58	10.778	226.009	+ 0.016	226.372	0.363	
668	ν^1 Libræ	1755	5	14	53	0.366	+ 331.623	+ 1.522	+ 332.052	− 0.429	
		1850	18	14	58	16.096	333.073	1.531	333.501	0.428	
669	ν^2 Libræ	1755	5	14	53	11.216	+ 331.826	+ 1.532	+ 332.468	− 0.642	
		1850	16	14	58	27.142	333.285	1.540	333.927	0.642	
670	ι^1 Libræ	1755	5	14	58	18.339	+ 338.627	+ 1.708	+ 339.043	− 0.416	
		1850	89	15	3	40.804	340.252	1.713	340.663	0.411	
671	ι^2 Libræ	1755	5	14	59	24.687	+ 338.506	+ 1.693	+ 338.942	− 0.436	
		1850	7	15	4	47.032	340.116	1.698	340.516	0.430	
672	26 Libræ	1755	4	15	0	46.977	+ 335.496	+ 1.586	+ 335.715	− 0.219	
		1850	4	15	6	6.415	337.004	1.589	337.222	0.218	
673	β Libræ	1755	10	15	3	51.375	+ 320.595	+ 1.181	+ 321.303	− 0.708	−0.001
		1850	734	15	8	56.473	321.716	1.178	322.420	0.704	
		1900	. .	15	11	37.478	322.305	1.179	323.011	0.706	
674	28 Libræ	1755	2	15	7	2.920	+ 337.123	+ 1.596	+ 337.254	− 0.131	
		1850	20	15	12	23.908	338.640	1.598	338.766	0.126	
675	o^1 Libræ	1755	5	15	7	21.983	+ 332.688	+ 1.449	+ 332.498	+ 0.190	
		1850	5	15	12	38.690	334.065	1.449	333.876	0.189	
676	o^2 Libræ	1755	5	15	9	24.323	+ 331.868	+ 1.421	+ 331.968	− 0.100	
		1850	66	15	14	40.239	333.219	1.424	333.319	0.100	
677	B. A. C. 5070	1850	8	15	15	38.822	+ 328.054	+ 1.294	+ 328.286	− 0.232	
678	μ^1 Bootis	1755	5	15	15	14.291	+ 226.458	+ 0.114	+ 227.649	− 1.191	−0.004
		1850	144	15	18	49.479	226.571	0.123	227.769	1.198	
		1900	. .	15	20	42.781	226.637	0.142	227.837	1.200	
679	ζ^1 Libræ	1755	5	15	14	28.904	+ 335.483	+ 1.495	+ 335.480	+ 0.003	
		1850	104	15	19	48.285	336.901	1.491	336.893	0.008	
680	γ^1 Ursæ Minoris	1755	. .	15	21	19.34	− 23.73	+ 8.02	− 23.69	− 0.04	
		1800	. .	15	21	9.46	20.17	7.82	20.12	0.05	
		1850	. .	15	21	0.34	16.32	7.60	16.27	0.05	
		1900	. .	15	20	53.12	12.58	7.39	12.51	0.07	
681	ζ^2 Libræ	1755	5	15	15	45.875	+ 336.370	+ 1.516	+ 337.020	− 0.650	
		1850	2	15	21	6.110	337.807	1.511	338.460	0.653	
682	ζ^3 Libræ	1755	5	15	16	53.588	+ 335.660	+ 1.478	+ 335.562	+ 0.098	
		1850	13	15	22	13.132	337.064	1.478	336.962	0.102	
683	B. A. C. 5109	1850	17	15	24	0.262	+ 343.124	+ 1.625	+ 343.271	− 0.147	
684	ζ^4 Libræ	1755	5	15	19	7.244	+ 336.091	+ 1.480	+ 336.290	− 0.199	
		1850	17	15	24	27.197	337.494	1.474	337.688	0.194	

DECLINATIONS.

No.	Star.	Mag.	Epoch.	Declination.	Centennial variation.	Secular variation.	Struve's precession.	Proper motion.	Sec. var. of proper motion.
				° ′ ″	″	″	″	″	″
666	γ Scorpii	3.5	1755	+ 24 18 3.62	− 1485.64	+ 34.75	− 1480.56	− 5.08	
		3.5	1850	24 41 19.14	1452.12	35.82	1446.97	5.15	
667	β Bootis	3.0	1755	+ 41 22 8.63	− 1467.61	+ 23.15	− 1463.13	− 4.48	− 0.04
		3.0	1850	40 59 4.90	1445.46	23.50	1440.94	4.52	
			1900	40 47 5.12	1433.66	23.68	1429.12	4.54	
668	ν^1 Libræ	6.0	1755	− 15 17 20.15	− 1465.94	+ 33.73	− 1461.41	− 4.53	
		5.5	1850	15 40 17.43	1433.45	34.67	1428.87	4.58	
669	ν^2 Libræ	6.5	1755	− 15 31 4.58	− 1463.17	+ 33.76	− 1460.32	− 2.85	
		6.9	1850	15 53 59.21	1430.65	34.69	1427.74	2.91	
670	ι^1 Libræ	5.5	1755	− 18 50 46.15	− 1434.36	+ 35.37	− 1429.25	− 5.11	
		5.0	1850	19 13 12.68	1400.27	36.41	1395.25	5.02	
671	ι^2 Libræ	6.5	1755	− 18 42 25.38	− 1425.72	+ 35.40	− 1422.42	− 3.30	
		6.5	1850	19 4 43.70	1391.64	36.36	1388.28	3.36	
672	26 Libræ	7.0	1755	− 16 50 6.43	− 1416.66	+ 35.31	− 1413.92	− 2.74	
		6.5	1850	17 12 16.17	1382.63	36.34	1379.92	2.71	
673	β Libræ	2.5	1755	− 8 27 40.58	− 1397.71	+ 34.17	− 1394.72	− 2.99	− 0.07
		3.1	1850	8 49 32.87	1364.87	34.97	1361.80	3.07	
			1900	9 0 50.92	1347.29	35.39	1344.18	3.11	
674	28 Libræ	6.0	1755	− 17 14 58.23	− 1383.44	+ 36.50	− 1374.49	− 8.95	
		6.0	1850	17 36 35.90	1348.31	37.47	1339.44	8.87	
675	o^1 Libræ	7.0	1755	14 38 47.44	− 1370.04	+ 36.01	− 1372.46	+ 2.42	
		6.4	1850	15 0 12.59	1335.40	36.90	1337.84	2.44	
676	o^2 Libræ	6.0	1755	− 14 14 25.54	− 1359.55	+ 36.23	− 1359.41	− 0.14	
		7.0	1850	14 35 40.65	1324.70	37.13	1324.58	0.12	
677	B. A. C. 5070 . .	6.2	1850	− 11 49 48.61	− 1319.79	+ 36.61	− 1318.15	− 1.64	
678	μ^1 Bootis	4.0	1755	+ 38 14 56.29	− 1312.43	+ 25.34	− 1321.40	+ 8.97	− 0.11
		4.0	1850	37 54 20.97	1288.20	25.67	1297.07	8.87	
			1900	37 43 40.09	1275.32	25.83	1284.13	8.81	
679	ζ^1 Libræ	6.0	1755	− 15 50 33.86	− 1331.53	+ 37.42	− 1326.40	− 5.13	
		5.9	1850	16 11 21.82	1295.56	38.31	1290.50	5.06	
680	γ^2 Ursæ Minoris . .	5.5	1755	+ 72 42 19.77	− 1279.06	− 2.12	− 1280.84	+ 1.78	
			1800	72 32 43.99	1279.94	1.72	1281.71	1.77	
		3.0	1850	72 22 3.83	1280.67	1.28	1282.44	1.77	
			1900	72 11 23.34	1281.21	0.84	1282.98	1.77	
681	ζ^2 Libræ	7.5	1755	− 16 34 34.13	− 1317.89	+ 37.50	− 1317.91	+ 0.02	
		7.0	1850	16 55 9.08	1281.85	38.38	1281.77	− 0.08	
682	ζ^3 Libræ	6.0	1755	− 15 44 56.84	− 1312.52	+ 37.72	− 1310.45	− 2.07	
		6.0	1850	16 5 26.57	1276.24	38.66	1274.27	1.97	
683	B. A. C. 5109 . .	6.2	1850	− 19 9 19.56	− 1264.92	+ 39.43	− 1262.18	− 2.74	
684	ζ^4 Libræ	6.0	1755	− 16 0 7.21	− 1299.09	+ 38.10	− 1295.63	− 3.46	
		5.8	1850	16 20 24.00	1262.47	39.00	1259.11	3.36	

RIGHT ASCENSIONS.

No.	Star.	Epoch.	Number of observations.	Right ascension.			Centennial variation.	Secular variation.	Struve's precession.	Proper motion.	Sec. var. of proper motion.
				h.	m.	s.	s.	s.	s.	s.	s.
685	γ Libræ	1755	5	15	21	51.474	+ 333.101	+ 1.368	+ 332.629	+ 0.472	
		1850	64	15	27	8.536	334.397	1.361	333.927	0.470	
686	α Coronæ Borealis .	1755	10	15	24	19.267	+ 253.615	+ 0.220	+ 252.698	+ 0.917	+0.002
		1850	. .	15	28	20.303	253.832	0.237	252.914	0.918	
		1900	. .	15	30	27.249	253.952	0.244	253.036	0.916	
687	41 Libræ	1755	3	15	24	50.857	+ 342.446	+ 1.587	+ 341.832	+ 0.614	
		1850	7	15	30	16.895	343.948	1.576	343.329	0.619	
688	42 Libræ	1755	2	15	25	50.957	+ 351.188	+ 1.825	+ 351.411	— 0.223	
		1850	26	15	31	25.405	352.912	1.806	353.137	0.225	
689	κ Libræ	1755	5	15	27	52.603	+ 342.550	+ 1.591	+ 343.004	— 0.454	
		1850	34	15	33	18.741	344.056	1.579	344.497	0.441	
690	B. A. C. 5188 . .	1850	13	15	35	0.7	. . . .	+ 1.351	+ 335.180	. . .	
691	B. A. C. 5197 . .	1850	6	15	36	53.6	. . . .	+ 1.822	+ 356.023	. . .	
692	η Libræ	1755	5	15	30	19.691	+ 334.933	+ 1.386	+ 335.209	— 0.276	
		1850	40	15	35	38.499	336.244	1.375	336.514	0.270	
693	α Serpentis . . .	1755	10	15	32	13.070	+ 294.342	+ 0.607	+ 293.466	+ 0.876	—0.004
		1850	. .	15	36	52.970	294.920	0.612	294.047	0.873	
		1900	. .	15	39	20.506	295.226	0.613	294.360	0.866	
694	b Scorpii	1755	2	15	36	18.092	+ 356.943	+ 1.875	+ 357.471	— 0.528	
		1850	15	15	41	58.030	358.712	1.850	359.238	0.526	
695	ε Serpentis . . .	1755	5	15	38	37.301	+ 297.833	+ 0.650	+ 297.003	+ 0.830	—0.005
		1850	228	15	43	20.536	298.452	0.650	297.631	0.821	
		1900	. .	15	45	49.843	298.778	0.653	297.958	0.820	
696	A Scorpii (2d star) .	1755	3	15	38	57.379	+ 356.572	+ 1.829	+ 356.942	— 0.370	
		1850	22	15	44	36.943	358.297	1.803	358.666	0.369	
697	λ Libræ	1755	5	15	39	9.334	+ 345.285	+ 1.548	+ 345.513	— 0.228	
		1850	23	15	44	38.050	346.746	1.528	346.972	0.226	
698	B. A. C. 5253 . .	1755	1	15	39	18.850	+ 354.953	+ 1.780	+ 355.180	— 0.227	
		1850	10	15	44	56.854	356.632	1.754	356.858	0.226	
699	B. A. C. 5254 . .	1850	8	15	45	0.782	+ 355.266	+ 1.721	+ 355.579	— 0.313	
*700	B. A. C. 5255 . .	1850	. .	15	45	12.2	. . . .	+ 1.799	+ 358.956	. . .	
701	θ Libræ	1755	4	15	39	54.862	+ 338.981	+ 1.370	+ 338.387	+ 0.594	
		1850	57	15	45	17.509	340.273	1.351	339.683	0.590	
702	3 Scorpii	1755	1	15	40	0.123	+ 356.692	+ 1.816	+ 356.913	— 0.221	
		1850	6	15	45	39.796	358.404	1.789	358.634	0.230	
703	47 Libræ	1755	2	15	40	53.374	+ 343.798	+ 1.500	+ 344.053	— 0.255	
		1850	5	15	46	20.656	345.213	1.479	345.465	0.252	
704	4 Scorpii	1755	1	15	40	44.802	+ 359.077	+ 1.872	+ 359.425	— 0.348	
		1850	10	15	46	26.765	360.841	1.842	361.194	0.353	

*No. 700. There is some doubt respecting the existence of this star.

DECLINATIONS.

No.	Star.	Mag.	Epoch.	Declination.	Centennial variation.	Secular variation.	Struve's precession.	Proper motion.	Sec. var. of proper motion.
				° ′ ″	″	″	″	″	″
685	γ Libræ	4.5	1755	− 13 57 10.35	− 1277.52	+ 38.07	− 1277.21	− 0.31	
		4.4	1850	14 17 6.68	1240.96	38.91	1240.70	0.26	
686	α Coronæ Borealis	2.0	1755	+ 27 33 14.97	− 1270.62	+ 29.45	− 1260.52	−10.10	+ 0.11
		2.0	1850	27 13 21.23	1242.45	29.85	1232.45	10.00	
			1900	27 3 3.75	1227.52	30.05	1217.58	9.94	
687	41 Libræ	6.0	1755	− 18 28 29.80	− 1264.39	+ 39.55	− 1256.94	− 7.45	
		5.9	1850	18 48 12.99	1226.39	40.44	1219.00	7.39	
688	42 Libræ	5.5	1755	− 23 0 1.15	− 1253.47	+ 40.66	− 1250.12	− 3.35	
		5.7	1850	23 19 33.41	1214.38	41.64	1211.05	3.33	
689	κ Libræ	5.0	1755	− 18 51 50.65	− 1247.56	+ 39.97	− 1236.17	−11.39	
		5.5	1850	19 11 17.65	1209.20	40.80	1197.84	11.36	
690	B. A. C. 5188	6.6	1850	− 14 33 27.71	− 1196.09	+ 39.94	− 1185.87	−10.22	
691	B. A. C. 5197	6.0	1850	− 24 14 27.4	. . . .	+ 42.64	− 1172.55	. . .	
692	η Libræ	4.5	1755	− 14 52 18.79	− 1226.55	+ 39.52	− 1219.23	− 7.32	
		5.9	1850	15 11 26.06	1188.59	40.41	1181.36	7.23	
693	α Serpentis	2.5	1755	+ 7 12 50.82	− 1202.73	+ 34.95	− 1205.99	+ 3.26	+ 0.11
		2.6	1850	6 54 4.08	1169.26	35.51	1172.62	3.36	
			1900	6 44 23.90	1151.44	35.79	1154.86	3.42	
694	b Scorpii	5.0	1755	− 24 59 1.36	− 1183.26	+ 42.61	− 1177.22	− 6.04	
		5.3	1850	25 17 26.08	1142.33	43.55	1136.24	6.09	
695	ε Serpentis	3.0	1755	+ 5 13 58.35	− 1154.05	+ 36.07	− 1160.73	+ 6.68	+ 0.11
		3.7	1850	4 55 58.36	1119.52	36.62	1126.30	6.78	
			1900	4 46 43.19	1101.14	36.92	1107.98	6.84	
696	A Scorpii (2d star)	5.0	1755	− 24 34 24.20	− 1162.18	+ 42.93	− 1158.34	− 3.84	
		5.2	1850	24 52 28.76	1120.96	43.85	1117.06	3.90	
697	λ Libræ	5.0	1755	− 19 24 47.28	− 1160.51	+ 41.63	− 1156.91	− 3.60	
		5.5	1850	19 42 50.85	1120.57	42.47	1116.93	3.64	
698	B. A. C. 5253	6.0	1755	− 23 46 51.61	− 1158.87	+ 42.90	− 1155.82	− 3.05	
		5.8	1850	24 4 53.06	1117.68	43.83	1114.65	3.03	
699	B. A. C. 5254	5.8	1850	− 23 31 35.69	− 1115.89	+ 43.54	− 1114.16	− 1.73	
700	B. A. C. 5255	6.0	1850	− 24 57 37.5	. . . .	+ 44.04	− 1112.80	. . .	
701	θ Libræ	4.5	1755	− 15 59 20.24	− 1140.08	+ 41.26	− 1151.51	+11.43	
		4.8	1850	16 17 4.57	1100.48	42.07	1112.15	11.67	
702	3 Scorpii	6.0	1755	− 24 29 42.80	− 1153.62	+ 43.09	− 1150.82	− 2.80	
		6.7	1850	24 47 39.15	1112.25	44.00	1109.46	2.79	
703	47 Libræ	7.0	1755	− 18 38 15.52	− 1147.86	+ 41.66	− 1144.47	− 3.39	
		6.4	1850	18 56 7.07	1107.90	42.48	1104.46	3.44	
704	4 Scorpii	6.5	1755	− 25 31 17.08	− 1149.19	+ 43.46	− 1145.48	− 3.71	
		6.3	1850	25 49 9.06	1107.47	44.38	1103.73	3.74	

RIGHT ASCENSIONS.

No.	Star.	Epoch.	Number of observations.	Right ascension.			Centennial variation.	Secular variation.	Struve's precession.	Proper motion.	Sec. var. of proper motion.
				h.	m.	s.	s.	s.	s.	s.	s.
705	ζ Ursæ Minoris . .	1755	. .	15	53	22.50	− 253.11	+21.49	− 254.41	+ 1.30	
		1800	. .	15	51	30.74	243.75	21.04	245.04	1.29	
		1850	. .	15	49	31.57	233.16	20.51	234.43	1.27	
		1900	. .	15	47	37.54	223.02	20.00	224.28	1.26	
706	π Scorpii	1755	5	15	44	4.931	+ 359.439	+ 1.838	+ 359.634	− 0.195	
		1850	20	15	49	47.225	361.174	1.814	361.361	0.187	
707	48 Libræ	1755	3	15	44	30.328	+ 333.539	+ 1.254	+ 333.766	− 0.227	
		1850	15	15	49	47.753	334.723	1.239	334.949	0.226	
708	ε Coronæ Borealis .	1755	4	15	47	27.108	+ 247.928	+ 0.286	+ 248.435	− 0.507	+0.003
		1850	57	15	51	22.770	248.204	0.296	248.707	0.503	
		1900	. .	15	53	26.909	248.354	0.303	248.861	0.507	
709	δ Scorpii	1755	5	15	45	53.486	+ 351.708	+ 1.630	+ 351.829	− 0.121	
		1850	158	15	51	28.339	353.242	1.598	353.362	0.120	
		1900	. .	15	54	25.159	354.038	1.582	354.156	0.118	
710	49 Libræ	1755	2	15	46	36.766	+ 334.235	+ 1.390	+ 338.606	− 4.371	
		1850	17	15	51	54.914	335.545	1.368	339.871	4.326	
711	B. A. C. 5314 . .	1850	5	15	54	17.201	+ 361.142	+ 1.750	+ 361.458	− 0.316	
712	β^1 Scorpii	1755	10	15	51	13.855	+ 346.151	+ 1.457	+ 346.222	0.071	
		1850	492	15	56	43.350	347.519	1.429	347.592	0.073	
		1900	. .	15	59	37.287	348.229	1.411	348.301	0.072	
713	ω^1 Scorpii	1755	5	15	52	31.044	+ 348.126	+ 1.487	+ 348.306	− 0.180	
		1850	16	15	58	2.430	349.525	1.459	349.701	0.176	
714	ω^2 Scorpii	1755	5	15	53	4.735	+ 349.023	+ 1.494	+ 348.822	+ 0.201	
		1850	62	15	58	36.977	350.429	1.467	350.225	0.204	
715	Lal. 29314 . . .	1850	. .	15	58	42.7	. . . .	+ 1.178	+ 335.236	. . .	
716	B. A. C. 5347 . .	1850	10	15	58	59.598	+ 364.106	+ 1.719	+ 363.312	+ 0.794	
717	c^1 Scorpii	1755	1	15	57	10.800	+ 367.159	+ 1.860	+ 367.523	− 0.364	
		1850	7	16	3	0.432	368.904	1.815	369.264	0.360	
718	c^2 Scorpii	1755	1	15	57	15.839	+ 366.070	+ 1.828	+ 366.287	− 0.217	
		1850	17	16	3	4.424	367.786	1.784	368.001	0.215	
719	ν^2 Scorpii	1755	5	15	57	47.760	+ 345.987	+ 1.387	+ 346.188	− 0.201	
		1850	78	16	3	17.068	347.292	1.361	347.525	0.233	
720	B. A. C. 5395 . .	1755	. .	15	59	18.630	+ 349.991	+ 1.475	+ 350.818	− 0.827	
		1850	12	16	4	51.782	351.376	1.441	352.205	0.829	
721	Groombridge 2320	1755	. .	16	5	46.03	+ 8.43	+ 4.21	+ 9.39	− 0.96	
		1800	. .	16	5	50.24	10.32	4.16	11.29	0.97	
		1850	. .	16	5	55.91	12.37	4.08	13.35	0.98	
		1900	. .	16	6	2.60	14.40	4.01	15.39	0.99	
722	δ Ophiuchi . . .	1755	5	16	1	31.764	+ 312.842	+ 0.842	+ 313.183	− 0.341	+0.007
		1850	699	16	6	29.342	313.636	0.830	313.967	0.331	
		1900	. .	16	9	6.264	314.050	0.824	314.379	0.329	

DECLINATIONS.

No.	Star.	Mag.	Epoch.	Declination.	Centennial variation.	Secular variation.	Struve's precession.	Proper motion.	Sec. var. of proper motion.
				° ′ ″	″	″	″	″	″
705	ζ Ursæ Minoris . .	4.0	1755	+ 78 32 6.19	— 1053.38	— 30.86	— 1053.03	— 0.35	
			1800	78 24 9.08	1066.96	29.50	1066.68	0.28	
		4.5	1850	78 15 11.97	1081.33	28.03	1081.13	0.20	
			1900	78 6 7.86	1094.98	26.60	1094.86	0.12	
706	π Scorpii	3.5	1755	— 25 23 8.66	— 1125.98	+ 44.06	— 1121.40	— 4.58	
		3.4	1850	25 40 38.33	1083.70	44.97	1079.20	4.50	
707	48 Libræ	5.0	1755	— 13 33 4.39	— 1122.20	+ 40.85	— 1118.32	— 3.88	
		5.4	1850	13 50 31.94	1083.04	41.60	1079.14	3.90	
708	ε Coronæ Borealis .	4.5	1755	+ 27 36 9.05	— 1102.96	+ 30.77	— 1096.85	— 6.11	— 0.03
		4.0	1850	27 18 55.17	1073.57	31.11	1067.43	6.14	
			1900	27 10 2.28	1057.97	31.29	1051.81	6.16	
709	δ Scorpii	3.0	1755	— 21 54 7.77	— 1111.90	+ 43.31	— 1108.27	— 3.63	— 0.02
		2.3	1850	22 11 24.42	1070.38	44.08	1066.73	3.65	
		.	1900	22 20 14.08	1048.24	44.49	1044.59	3.65	
710	49 Libræ	5.5	1755	— 15 47 27.45	— 1141.78	+ 40.74	— 1102.94	—38.84	
		5.9	1850	16 5 13.64	1102.72	41.50	1063.46	39.26	
711	B. A. C. 5314 . .	5.7	1850	— 25 26 31.76	— 1048.59	+ 45.37	— 1045.78	— 2.81	
712	β^1 Scorpii	2.0	1755	— 19 6 45.40	— 1072.76	+ 43.23	— 1068.97	— 3.79	— 0.02
		2.5	1850	19 23 24.90	1031.35	43.94	1027.56	3.79	
			1900	19 31 55.06	1009.29	44.32	1005.48	3.81	
713	ω^1 Scorpii	4.5	1755	— 19 58 59.26	— 1063.40	+ 43.56	— 1059.42	— 3.98	
		4.6	1850	20 15 29.71	1021.64	44.36	1017.62	4.02	
714	ω^2 Scorpii	4.5	1755	— 20 11 2.56	— 1062.10	+ 43.91	— 1055.27	— 6.83	
		4.6	1850	20 27 31.62	1020.02	44.70	1013.30	6.72	
715	Lal. 29314 . . .	6.8	1850	— 13 39 49.8	. . . .	+ 42.65	— 1012.56	. . .	
716	B. A. C. 5347 . .	6.0	1850	— 25 55 13.74	— 999.09	+ 46.42	— 1010.45	+11.36	
717	c^1 Scorpii	6.0	1755	— 27 45 20.47	— 1030.85	+ 46.45	— 1024.50	— 6.35	
		6.1	1850	28 1 18.68	986.30	47.33	979.94	6.36	
718	c^2 Scorpii	5.0	1755	— 27 16 0.14	— 1027.48	+ 46.34	— 1023.94	— 3.54	
		5.3	1850	27 31 55.21	983.04	47.21	979.43	3.61	
719	ν^2 Scorpii	4.0	1755	— 18 48 5.24	— 1024.29	+ 44.13	— 1019.95	— 4.34	
		4.5	1850	19 3 58.28	982.05	44.78	977.83	4.22	
720	B. A. C. 5395 . .	. . .	1755	— 20 45 9.16	— 1005.57	+ 44.63	— 1008.51	+ 2.94	
		7.0	1850	21 0 44.22	962.81	45.40	965.74	2.93	
721	Groombridge 2320	. . .	1755	+ 68 27 24.09	— 952.14	+ 1.37	— 959.19	+ 7.05	
			1800	68 20 15.76	951.48	1.61	958.47	6.99	
		5.7	1850	68 12 20.22	950.60	1.87	957.53	6.93	
			1900	68 4 25.17	949.60	2.16	956.41	6.81	
722	δ Ophiuchi . . .	3.0	1755	— 3 2 36.92	— 1005.90	+ 40.08	— 991.62	—14.28	— 0.05
		2.7	1850	3 18 14.36	967.57	40.61	953.24	14.33	
			1900	3 26 13.06	947.19	40.89	932.84	14.35	

RIGHT ASCENSIONS.

No.	Star.	Epoch.	Number of observations.	Right ascension.	Centennial variation.	Secular variation.	Struve's precession.	Proper motion.	Sec. var. of proper motion.
				h. m. s.	s.	s.	s.	s.	s.
723	W^2 16h 140 . .	1850	. .	16 7 23.8	. . . .	+ 1.154	+ 337.598	. . .	
724	B. A. C. 5429 . .	1850	3	16 9 0.281	+ 370.403	+ 1.770	+ 370.721	— 0.318	
725	19 Scorpii	1755	3	16 5 56.293	+ 358.007	+ 1.545	+ 358.236	— 0.229	
		1850	18	16 11 37.090	359.455	1.504	359.684	0.229	
726	σ Scorpii	1755	5	16 6 20.507	+ 361.586	+ 1.608	+ 361.811	— 0.225	
		1850	76	16 12 4.733	363.094	1.566	363.319	0.225	
727	τ Herculis . . .	1755	. .	16 12 23.416	+ 179.408	+ 0.521	+ 179.487	— 0.079	—0.002
		1850	159	16 15 14.087	179.900	0.516	179.977	0.077	
		1900	. .	16 16 44.101	180.157	0.512	180.232	0.075	
728	ψ Ophiuchi . . .	1755	5	16 9 48.077	+ 348.690	+ 1.334	+ 348.855	— 0.165	
		1850	40	16 15 19.928	349.939	1.297	350.101	0.162	
729	ρ Ophiuchi (south star)	1755	5	16 10 56.129	+ 356.948	+ 1.467	+ 357.124	— 0.176	
		1850	11	16 16 35.885	358.322	1.425	358.492	0.170	
730	χ Ophiuchi . . .	1755	5	16 12 51.584	+ 345.296	+ 1.294	+ 345.515	— 0.219	
		1850	28	16 18 20.186	346.483	1.204	346.675	0.192	
731	α Scorpii	1755	20	16 14 25.669	+ 364.953	+ 1.573	+ 365.085	— 0.132	+0.002
		1850	520	16 20 13.076	366.423	1.521	366.550	0.127	
		1900	. .	16 23 16.476	367.176	1.491	367.306	0.130	
732	22 Scorpii	1755	5	16 15 21.776	+ 361.736	+ 1.500	+ 361.846	— 0.110	
		1850	16	16 21 6.095	363.137	1.450	363.243	0.106	
733	η Draconis . . .	1755	. .	16 20 43.073	+ 78.139	+ 1.875	+ 77.952	+ 0.187	—0.020
		1850	264	16 21 58.144	79.909	1.851	79.757	0.152	
		1900	. .	16 22 38.329	80.828	1.824	80.691	0.137	
734	φ Ophiuchi . . .	1755	5	16 17 8.855	+ 341.268	+ 1.146	+ 341.682	— 0.414	
		1850	30	16 22 33.572	342.341	1.112	342.751	0.410	
735	ω Ophiuchi . . .	1755	5	16 17 38.966	+ 353.218	+ 1.314	+ 353.112	+ 0.106	
		1850	26	16 23 15.108	354.445	1.271	354.343	0.102	
736	β Herculis . . .	1755	5	16 19 41.845	+ 257.260	+ 0.362	+ 257.946	— 0.686	
		1850	48	16 23 46.405	257.604	0.363	258.289	0.685	
		1900	. .	16 25 55.252	257.786	0.364	258.473	0.687	
737	τ Scorpii	1755	5	16 20 40.296	+ 370.688	+ 1.556	+ 370.687	+ 0.001	
		1850	46	16 26 33.148	372.155	1.532	372.166	— 0.011	
738	A Draconis . . .	1755	. .	16 28 34.13	— 19.08	+ 4.23	— 19.17	+ 0.09	
		1800	. .	16 28 25.96	17.18	4.18	17.27	0.09	
		1850	. .	16 28 17.89	15.11	4.13	15.19	0.08	
		1900	. .	16 28 10.85	13.07	4.05	13.14	0.07	
739	ζ Ophiuchi . . .	1755	5	16 23 41.529	+ 328.711	+ 0.906	+ 328.637	+ 0.074	—0.003
		1850	199	16 28 54.209	329.559	0.879	329.489	0.070	
		1900	. .	16 31 39.098	329.995	0.863	329.928	0.067	
740	α Trianguli Australis	1850	. .	16 32 50.05	+ 626.32	+ 9.32	+ 626.32	0.00	
		1875	. .	16 35 26.92	628.62	9.13	628.62	0.00	
		1900	. .	16 38 4.36	630.88	8.92	630.87	+ 0.01	

DECLINATIONS.

No.	Star.	Mag.	Epoch.	Declination.	Centennial variation.	Secular variation.	Struve's precession.	Proper motion.	Sec. var. of proper motion.
				° ′ ″	″	″	″	″	″
723	W² 16h 140 . .	6.3	1850	− 14 28 8.5	. . . .	+ 43.83	− 946.26	. . .	
724	B. A. C. 5429 . .	6.0	1850	− 28 14 8.26	− 945.53	+ 48.18	− 933.80	−11.73	
725	19 Scorpii	5.5	1755	− 23 33 15.20	− 964.36	+ 46.33	− 957.87	− 6.49	
		5.1	1850	23 48 10.31	919.95	47.17	913.52	6.43	
726	σ Scorpii	4.0	1755	− 24 58 51.75	− 957.45	+ 46.88	− 954.78	− 2.67	
		3.4	1850	25 13 40.05	912.52	47.71	909.93	2.59	
727	τ Herculis . . .	4.0	1755	+ 46 54 32.35	− 905.55	+ 23.67	− 907.86	+ 2.31	− 0.04
		3.3	1850	46 40 22.79	882.97	23.86	885.24	2.27	
			1900	46 33 4.29	871.01	23.96	873.26	2.25	
728	ψ Ophiuchi . . .	5.0	1755	− 19 26 24.84	− 935.50	+ 45.48	− 928.02	− 7.48	
		4.8	1850	19 40 52.94	891.96	46.18	884.47	7.49	
729	ρ Ophiuchi (south star)	5.0	1755	− 22 51 31.45	- 923.40	+ 46.66	− 919.20	− 4.20	
		5.0	1850	23 5 47.52	878.72	47.40	874.51	4.21	
730	χ Ophiuchi . . .	5.0	1755	− 17 52 37.65	− 908.62	+ 45.44	− 904.20	− 4.42	
		4.6	1850	18 6 40.23	865.14	46.08	860.80	4.34	
731	α Scorpii	1.0	1755	− 25 51 49.30	− 895.63	+ 48.15	− 891.97	− 3.66	− 0.01
		1.4	1850	26 5 38.31	849.57	48.82	845.90	3.67	
			1900	26 12 36.98	825.07	49.18	821.40	3.67	
732	22 Scorpii	6.0	1755	− 24 33 5.64	− 888.47	+ 47.86	− 884.60	− 3.87	
		5.5	1850	24 46 47.99	842.68	48.55	838.87	3.81	
733	η Draconis	3.0	1755	+ 62 4 27.52	− 836.85	+ 10.76	− 842.27	+ 5.42	+ 0.03
		2.7	1850	61 51 17.40	826.52	11.01	831.97	5.45	
			1900	61 44 25.52	820.98	11.13	826.45	5.47	
734	φ Ophiuchi . . .	4.5	1755	− 16 3 20.65	− 874.89	+ 45.18	− 870.53	− 4.36	
		4.6	1850	16 16 51.32	831.68	45.80	827.26	4.42	
735	ω Ophiuchi . . .	5.0	1755	− 20 55 6.28	− 863.74	+ 46.86	− 866.58	+ 2.84	
		4.7	1850	21 8 25.58	818.90	47.53	821.74	2.84	
736	β Herculis . . .	2.5	1755	+ 22 2 25.95	− 852.12	+ 34.29	− 850.34	− 1.78	
		2.3	1850	21 49 11.98	819.40	34.60	817.58	1.82	
			1900	21 42 26.61	802.06	34.76	800.21	1.85	
737	τ Scorpii	3.5	1755	− 27 40 55.63	− 847.07	+ 49.57	− 842.62	− 4.45	
		3.2	1850	27 53 57.87	799.63	50.32	795.30	4.33	
738	A Draconis . . .	4.5	1755	+ 69 17 52.50	− 777.32	− 2.22	− 779.40	+ 2.08	
			1800	69 12 2.50	778.27	1.96	780.35	2.08	
		5.0	1850	69 5 33.14	779.17	1.68	781.26	2.09	
			1900	68 59 3.35	779.96	1.44	782.04	2.08	
739	ζ Ophiuchi . . .	3.5	1755	− 10 2 55.76	− 816.47	+ 44.18	− 818.58	+ 2.11	− 0.01
		2.7	1850	10 15 31.39	774.26	44.67	776.38	2.12	
			1900	10 21 52.93	751.86	44.93	753.98	2.12	
740	α Trianguli Australis	2.2	1850	− 68 44 34.71	− 750.19	+ 85.16	− 744.54	− 5.65	
			1875	68 47 39.59	728.82	85.85	723.17	5.65	
			1900	68 50 39.11	707.28	86.52	701.63	5.65	

RIGHT ASCENSIONS.

No.	Star.	Epoch.	Number of observations.	Right ascension.	Centennial variation.	Secular variation.	Struve's precession.	Proper motion.	Sec. var. of proper motion.
				h. m. s.	s.	s.	s	s.	s.
741	24 Scorpii	1755	5	16 27 25.938	+ 345.043	+ 1.101	+ 345.177	— 0.134	
		1850	33	16 32 54.219	346.069	1.059	346.201	0.132	
742	B. A. C. 5580 . .	1755	. .	16 27 31.671	+ 350.379	+ 1.167	+ 350.392	— 0.013	
		1850	11	16 33 5.051	351.466	1.122	351.488	0.022	
743	15 Ophiuchi . . .	1755	. .	16 30 26.741	+ 358.261	+ 1.262	+ 358.745	— 0.484	
		1850	2	16 36 7.650	359.434	1.208	359.913	0.479	
744	25 Scorpii	1755	1	16 31 53.459	+ 364.929	+ 1.339	+ 364.972	— 0.043	
		1850	26	16 37 40.736	366.173	1.279	366.217	0.044	
745	η Herculis . . .	1755	5	16 34 30.552	+ 204.859	+ 0.387	+ 204.671	+ 0.188	+0.005
		1850	139	16 37 45.343	205.226	0.385	205.031	0.195	
		1900	. .	16 39 28.004	205.418	0.385	205.220	0.198	
746	18 Ophiuchi . . .	1850	22	16 40 37.024	+ 363.874	+ 1.216	+ 364.148	— 0.274	
747	22 Ophiuchi . . .	1755	2	16 40 4.286	+ 360.471	+ 1.178	+ 360.576	— 0.105	
		1850	13	16 45 47.256	361.563	1.120	361.664	0.101	
748	24 Ophiuchi . . .	1755	4	16 42 3.185	+ 359.828	+ 1.138	+ 359.834	— 0.006	
		1850	8	16 47 45.527	360.882	1.079	360.893	0.011	
749	κ Ophiuchi . . .	1755	5	16 46 5.048	+ 283.148	+ 0.453	+ 285.139	— 1.991	0.000
		1850	367	16 50 34.241	283.572	0.439	285.564	1.992	
		1900	. .	16 52 56.082	283.790	0.433	285.780	1.990	
750	B. A. C. 5709 . .	1755	1	16 44 59.446	+ 365.317	+ 1.168	+ 365.237	+ 0.080	
		1850	9	16 50 47.014	366.395	1.103	366.327	0.068	
751	26 Ophiuchi . . .	1755	1	16 45 11.209	+ 365.129	+ 1.174	+ 364.976	+ 0.153	
		1850	11	16 50 58.601	366.213	1.109	366.061	0.152	
752	29 Ophiuchi . . .	1755	3	16 47 33.056	+ 349.056	+ 0.952	+ 349.538	— 0.482	
		1850	24	16 53 5.081	349.936	0.900	350.420	0.484	
753	31 Ophiuchi . . .	1755	2	16 49 40.795	+ 367.200	+ 1.144	+ 367.194	+ 0.006	
		1850	7	16 55 30.141	368.254	1.075	368.245	0.009	
754	*d* Herculis . . .	1755	1	16 52 34.454	+ 220.691	+ 0.328	+ 220.849	— 0.158	—0.002
		1850	39	16 56 4.258	221.001	0.324	221.165	0.164	
		1900	. .	16 57 54.799	221.163	0.323	221.329	0.166	
755	B. A. C. 5758 . .	1755	4	16 51 36.012	+ 356.163	+ 1.000	+ 356.584	— 0.421	
		1850	7	16 57 14.809	357.085	0.942	357.497	0.412	
756	ε Ursæ Minoris . .	1755	. .	17 11 57.10	— 671.73	+25.91	— 673.23	+ 1.50	
		1775	. .	17 9 43.27	666.48	26.82	667.98	1.50	
		1800	. .	17 6 57.51	659.63	27.87	661.13	1.50	
		1825	. .	17 4 13.48	652.53	28.88	654.02	1.49	
		1850	. .	17 1 31.26	645.19	29.86	646.68	1.49	
		1875	. .	16 58 50.91	637.61	30.68	639.10	1.49	
		1900	. .	16 56 12.47	— 629.85	+31.49	— 631.34	+ 1.49	
757	η Ophiuchi . . .	1755	5	16 56 21.014	+ 342.542	+ 0.785	+ 342.424	+ 0.118	
		1850	188	17 1 46.776	343.265	0.738	343.151	0.114	

DECLINATIONS.

No.	Star.	Mag.	Epoch.	Declination.	Centennial variation.	Secular variation.	Struve's precession.	Proper motion.	Sec. var. of proper motion.
				° ′ ″	″	″	″	″	″
741	24 Scorpii	5.0	1755	− 17 14 39.97	− 790.32	+ 46.61	− 788.55	− 1.77	
		5.5	1850	17 26 49.65	745.77	47.18	744.00	1.77	
742	B. A. C. 5580 . .	7.5	1755	− 19 25 50.65	− 785.01	+ 47.34	− 787.79	+ 2.78	
		6.0	1850	19 37 54.95	739.75	47.94	742.58	2.83	
743	15 Ophiuchi . . .	7.5	1755	− 22 42 14.03	− 764.68	+ 48.59	− 764.23	− 0.45	
		7.3	1850	22 53 58.44	718.21	49.21	717.68	0.53	
744	25 Scorpii	6.0	1755	− 25 3 27.54	− 752.87	+ 49.66	− 752.57	− 0.30	
		7.0	1850	25 15 0.24	705.38	50.30	705.02	0.36	
745	η Herculis . . .	3.0	1755	+ 39 24 7.92	− 740.32	+ 28.21	− 731.24	− 9.08	+ 0.04
		3.3	1850	39 12 37.37	713.43	28.39	704.39	9.04	
			1900	39 6 44.21	699.21	28.49	690.19	9.02	
746	18 Ophiuchi . . .	6.7	1850	− 24 22 19.59	− 685.30	+ 50.18	− 680.92	− 4.38	
747	22 Ophiuchi . . .	6.5	1755	− 23 5 5.12	− 690.87	+ 49.70	− 685.65	− 5.22	
		6.7	1850	23 15 38.93	643.39	50.26	638.16	5.23	
748	24 Ophiuchi . . .	6.5	1755	− 22 44 10.12	− 669.88	+ 49.77	− 669.33	− 0.55	
		5.9	1850	22 54 23.96	622.34	50.32	621.76	0.58	
749	κ Ophiuchi . . .	4.0	1755	+ 9 46 30.55	− 636.14	+ 39.15	− 635.95	− 0.19	− 0.31
		3.4	1850	9 36 43.93	598.83	39.40	598.35	0.48	
			1900	9 31 49.44	579.09	39.54	578.45	0.64	
750	B. A. C. 5709 . .	6.0	1755	− 24 41 43.68	− 643.91	+ 50.75	− 645.01	+ 1.10	
		6.3	1850	24 51 32.41	595.44	51.30	596.58	1.14	
751	26 Ophiuchi . . .	6.0	1755	− 24 35 22.37	− 650.59	+ 50.75	− 643.35	− 7.24	
		6.1	1850	24 45 17.44	602.12	51.30	594.96	7.16	
752	29 Ophiuchi . . .	6.0	1755	− 18 30 3.29	− 625.01	+ 48.71	− 623.77	− 1.24	
		6.8	1850	18 39 35.01	578.54	49.12	577.30	1.24	
753	31 Ophiuchi . . .	7.5	1755	− 25 16 12.75	− 614.34	+ 51.33	− 606.05	− 8.29	
		6.7	1850	25 25 33.13	565.33	51.85	556.96	8.37	
754	d Herculis . . .	5.0	1755	+ 33 56 17.26	− 581.57	+ 31.01	− 581.83	+ 0.26	− 0.05
		5.0	1850	33 47 18.79	552.04	31.16	552.26	0.22	
			1900	33 42 46.67	536.44	31.24	536.64	0.20	
755	B. A. C. 5758 . .	6.0	1755	− 21 11 56.52	− 600.62	+ 49.86	− 589.98	−10.64	
		6.6	1850	21 21 4.54	553.03	50.33	542.34	10.69	
756	ε Ursæ Minoris . .	4.0	1755	+ 82 23 51.57	− 418.12	− 95.45	− 417.58	− 0.54	
			1775	82 22 26.04	437.13	94.50	436.63	0.50	
			1800	82 20 33.80	460.60	93.26	460.15	0.45	
			1825	82 18 35.76	483.74	91.96	483.35	0.39	
		4.3	1850	82 16 31.95	506.56	90.65	506.22	0.34	
			1875	82 14 22.49	529.06	89.26	528.77	0.29	
			1900	+ 82 12 7.45	− 551.20	− 87.88	− 550.96	− 0.24	
757	η Ophiuchi . . .	2.5	1755	− 15 23 49.43	− 542.32	+ 48.31	− 550.11	+ 7.79	
		2.4	1850	15 32 2.78	496.24	48.70	504.04	7.80	

RIGHT ASCENSIONS.

No.	Star.	Epoch.	Number of observations.	Right ascension.			Centennial variation.	Secular variation.	Struve's precession.	Proper motion.	Sec. var. of proper motion.
				h.	m.	s.	s.	s.	s.	s.	s.
758	B. A. C. 5800 . .	1755	1	16	59	0.616	+ 371.598	+ 1.069	+ 371.801	− 0.203	
		1850	10	17	4	54.105	372.576	0.991	372.773	0.197	
759	A Ophiuchi . . .	1755	5	17	0	18.672	+ 366.921	+ 1.193	+ 370.645	− 3.724	
		1850	38	17	6	7.775	368.019	1.120	371.661	3.642	
760	B. A. C. 5813 . .	1755	. .	17	1	11.242	+ 366.882	+ 1.181	+ 370.567	− 3.685	
		1850	23	17	7	0.302	367.969	1.108	371.571	3.602	
761	B. A. C. 5815 . .	1755	. .	17	1	25.657	+ 366.552	+ 0.970	+ 367.353	− 0.801	
		1850	13	17	7	14.308	367.439	0.898	368.241	0.802	
762	α Herculis . . .	1755	10	17	3	29.196	+ 272.878	+ 0.358	+ 272.995	− 0.117	−0.003
		1850	. .	17	7	48.592	273.213	0.347	273.331	0.118	
		1900	. .	17	10	5.242	273.385	0.341	273.508	0.123	
763	38 Ophiuchi . . .	1850	2	17	8	20.460	+ 371.416	+ 0.928	+ 372.036	− 0.620	
764	39 Ophiuchi (south star)	1755	5	17	3	5.755	+ 364.111	+ 0.929	+ 364.709	− 0.598	
		1850	11	17	8	52.068	364.959	0.858	365.555	0.596	
765	B. A. C. 5831 . .	1755	1	17	3	10.666	+ 364.846	+ 0.935	+ 364.123	+ 0.723	
		1850	10	17	8	57.681	365.701	0.866	364.969	0.732	
766	ξ Ophiuchi . . .	1755	5	17	6	20.512	+ 358.139	+ 0.843	+ 356.487	+ 1.652	
		1850	51	17	12	1.116	358.909	0.777	357.242	1.667	
767	B. A. C. 5846 . .	1755	1	17	6	41.647	+ 366.135	+ 0.899	+ 366.679	− 0.544	
		1850	9	17	12	29.870	366.954	0.826	367.500	0.546	
768	θ Ophiuchi . . .	1755	5	17	6	59.265	+ 366.791	+ 0.901	+ 366.962	− 0.171	
		1850	200	17	12	48.112	367.612	0.827	367.779	0.167	
		1900	. .	17	15	52.020	368.016	0.788	368.181	0.165	
769	43 Ophiuchi . . .	1755	3	17	7	57.998	+ 375.812	+ 0.963	+ 375.940	− 0.128	
		1850	16	17	13	55.441	376.686	0.877	376.814	0.128	
770	B. A. C. 5868 . .	1755	5	17	10	9.085	+ 365.207	+ 0.835	+ 365.130	+ 0.077	
		1850	12	17	15	56.397	365.966	0.763	365.887	0.079	
771	b Ophiuchi . . .	1755	5	17	11	25.859	+ 364.859	+ 0.833	+ 365.011	− 0.152	+0.010
		1850	163	17	17	12.838	365.610	0.750	365.761	0.151	
		1900	. .	17	20	15.735	365.977	0.717	366.128	0.151	
772	d Ophiuchi . . .	1755	. .	17	11	44.435	+ 381.065	+ 0.975	+ 381.358	− 0.293	
		1850	30	17	17	46.873	381.947	0.882	382.231	0.284	
773	c^2 Ophiuchi . . .	1755	5	17	16	29.272	+ 364.677	+ 0.752	+ 364.786	− 0.109	
		1850	49	17	22	16.043	365.356	0.679	365.465	0.109	
774	52 Ophiuchi . . .	1755	5	17	20	35.360	+ 359.623	+ 0.669	+ 359.848	− 0.225	
		1850	5	17	26	17.293	360.226	0.601	360.448	0.222	
775	β Draconis . . .	1755	10	17	24	54.634	+ 134.635	+ 0.534	+ 134.750	− 0.115	+0.001
		1850	299	17	27	2.776	135.134	0.517	135.255	0.121	
		1900	. .	17	28	10.407	135.390	0.507	135.510	0.120	

DECLINATIONS.

No.	Star.	Mag.	Epoch.	Declination.	Centennial variation.	Secular variation.	Struve's precession.	Proper motion.	Sec. var. of proper motion.
				° ′ ″	″	″	″	″	″
758	B. A. C. 5800 . .	6.5	1755	− 26 39 49.53	− 538.08	+ 52.50	− 527.67	− 10.41	
		7.5	1850	26 47 56.94	487.98	52.96	477.57	10.41	
759	A Ophiuchi . . .	4.5	1755	− 26 12 59.29	− 631.78	+ 51.44	− 516.66	−115.12	
		4.9	1850	26 22 36.20	582.70	51.90	467.12	115.58	
760	B. A. C. 5813 . .	7.0	1755	− 26 9 59.32	− 624.54	+ 51.49	− 509.26	−115.28	
		6.8	1850	26 19 29.33	575.41	51.94	459.64	115.77	
761	B. A. C. 5815 . .	7.5	1755	. . . . . .	. . . .	+ 51.84	− 507.23	. . .	
		7.3	1850	− 25 7 48.5	. . . .	52.27	457.79	. . .	
762	α Herculis . . .	3.5	1755	+ 14 41 20.30	− 487.11	+ 38.82	− 489.77	+ 2.66	− 0.02
		3.3	1850	14 33 55.08	450.14	39.00	452.79	2.65	
			1900	14 30 14.89	430.62	39.10	433.25	2.63	
763	38 Ophiuchi . . .	6.7	1850	− 26 27 28.30	− 455.63	+ 52.92	− 448.24	− 7.39	
764	39 Ophiuchi (south star)	6.0	1755	− 23 59 36.53	− 496.55	+ 51.62	− 493.09	− 3.46	
		5.5	1850	24 7 4.89	447.32	52.03	443.77	3.55	
765	B. A. C. 5831 . .	6.0	1755	− 23 46 32.11	− 502.85	+ 51.93	− 492.48	− 10.37	
		6.9	1850	23 54 6.33	453.39	52.20	442.97	10.42	
766	ξ Ophiuchi . . .	4.5	1755	− 20 49 29.98	− 484.94	+ 51.38	− 465.51	− 19.43	
		5.1	1850	20 56 47.44	435.98	51.70	416.84	19.14	
767	B. A. C. 5846 . .	7.5	1755	− 24 37 57.23	− 466.40	+ 52.10	− 462.48	− 3.92	
		6.8	1850	24 44 56.71	416.72	52.49	412.72	4.00	
768	θ Ophiuchi . . .	3.5	1755	− 24 43 40.56	− 465.45	+ 52.39	− 459.99	− 5.46	
		3.6	1850	24 50 39.04	415.49	52.78	410.13	5.36	
			1900	24 54 0.17	389.05	52.97	383.76	5.29	
769	43 Ophiuchi . . .	6.0	1755	− 27 52 38.42	− 458.20	+ 53.71	− 451.65	− 6.55	
		5.8	1850	27 59 29.42	406.98	54.12	400.52	6.46	
770	B. A. C. 5868 . .	7.0	1755	− 23 59 34.40	− 433.71	+ 52.22	− 432.98	− 0.73	
		7.0	1850	24 6 2.81	383.93	52.58	383.21	0.72	
771	b Ophiuchi . . .	5.5	1755	− 23 55 24.59	− 435.15	+ 52.28	− 422.07	− 13.08	− 0.03
		4.5	1850	24 1 54.35	385.36	52.53	372.27	13.09	
			1900	24 5 0.46	359.06	52.67	345.94	13.12	
772	d Ophiuchi . . .	5.0	1755	− 29 37 2.98	− 434.26	+ 54.50	− 419.41	− 14.85	
		4.6	1850	29 43 30.88	382.30	54.89	367.40	14.90	
773	c^3 Ophiuchi . . .	5.0	1755	− 23 44 48.49	− 382.30	+ 52.39	− 378.65	− 3.65	
		5.2	1850	23 50 27.98	332.38	52.70	328.74	3.64	
774	52 Ophiuchi . . .	7.0	1755	− 21 51 6.24	− 348.32	+ 51.80	− 343.34	− 4.98	
		6.5	1850	21 56 13.73	298.98	52.08	293.97	5.01	
775	β Draconis . . .	2.0	1755	+ 52 29 33.71	− 305.99	+ 19.49	− 305.98	− 0.01	− 0.04
		2.7	1850	52 24 51.82	287.44	19.57	287.39	0.05	
			1900	52 22 30.54	277.64	19.61	277.57	0.07	

RIGHT ASCENSIONS.

No.	Star.	Epoch.	Number of observations.	Right ascension.	Centennial variation.	Secular variation.	Struve's precession.	Proper motion.	Sec. var. of proper motion.
				h. m. s.	s.	s.	s.	s.	s.
776	α Ophiuchi . . .	1755	10	17 23 34.319	+ 277.828	+ 0.348	+ 277.092	+ 0.736	+0.014
		1850	. .	17 27 58.411	278.153	0.336	277.402	0.751	
		1900	. .	17 30 17.529	278.319	0.328	277.565	0.755	
777	ξ Serpentis . . .	1755	4	17 23 34.390	+ 342.529	+ 0.545	+ 342.930	− 0.401	
		1850	33	17 29 0.031	343.023	0.494	343.419	0.396	
778	B. A. C. 5954 . .	1755	1	17 24 2.649	+ 359.373	+ 0.624	+ 359.690	− 0.317	
		1850	7	17 29 44.324	359.933	0.555	360.239	0.306	
779	σ Octantis . . .	1800	. .	16 8 4.77	+ 8628.31	+5460.7	+ 8621.89	+ 6.42	
		1825	. .	16 46 46.30	9904.91	4517.9	9895.20	9.71	
		1850	. .	17 30 2.49	10764.86	+2138.1	10751.76	13.10	
		1875	. .	18 15 27.61	10899.75	−1102.1	10883.89	15.86	
		1900	. .	18 59 46.38	+10253.76	−3884.7	+10236.40	+17.36	
780	58 Ophiuchi . . .	1755	5	17 28 45.687	+ 358.660	+ 0.566	+ 359.306	− 0.646	
		1850	57	17 34 26.658	359.164	0.497	359.800	0.636	
781	ω Draconis . . .	1755	3	17 38 24.54	− 36.81	+ 1.04	− 37.56	+ 0.75	
		1800	. .	17 38 8.08	36.34	1.05	37.02	0.68	
		1850	. .	17 37 50.04	35.82	1.06	36.41	0.59	
		1900	. .	17 37 32.32	35.26	1.10	35.80	0.54	
782	3 Sagittarii . . .	1755	4	17 32 9.343	+ 376.519	+ 0.597	+ 376.736	− 0.217	
		1850	34	17 38 7.291	377.042	0.506	377.258	0.216	
783	μ Herculis . . .	1755	5	17 36 52.825	+ 231.122	+ 0.386	+ 236.590	− 2.468	+0.029
		1850	367	17 40 35.414	234.486	0.380	236.899	2.413	
		1900	. .	17 42 32.704	234.675	0.375	237.059	2.384	
784	ψ^1 Draconis . . .	1755	. .	17 46 21.05	− 110.41	+ 1.78	− 110.43	+ 0.02	
		1800	. .	17 45 31.55	109.59	1.84	109.70	0.11	
		1850	. .	17 44 36.99	108.65	1.90	108.86	0.21	
		1900	. .	17 43 42.90	107.70	1.94	107.99	0.29	
785	63 Ophiuchi . . .	1755	5	17 39 49.979	+ 368.556	+ 0.448	+ 368.604	− 0.048	
		1850	11	17 45 40.297	368.945	0.370	368.990	0.045	
786	B. A. C. 6060 . .	1850	. .	17 47 5.7	. . . .	+ 0.319	+ 352.544	. . .	
787	B. A. C. 6066 . .	1755	1	17 42 9.692	+ 366.017	+ 0.411	+ 366.026	− 0.009	
		1850	9	17 47 57.581	366.370	0.333	366.380	0.010	
788	4 Sagittarii . . .	1755	5	17 44 50.619	+ 365.702	+ 0.381	+ 365.773	− 0.071	
		1850	50	17 50 38.196	366.027	0.303	366.092	0.065	
789	5 Sagittarii . . .	1755	1	17 45 10.686	+ 367.404	+ 0.374	+ 367.091	+ 0.313	
		1850	12	17 50 59.876	367.721	0.294	367.407	0.314	
790	6 Sagittarii . . .	1755	5	17 47 9.517	+ 348.113	+ 0.308	+ 348.141	− 0.028	
		1850	7	17 52 40.354	348.377	0.249	348.407	0.030	
791	γ Draconis . . .	1755	9	17 50 55.566	+ 138.707	+ 0.341	+ 138.795	− 0.088	+0.006
		1850	550	17 53 7.479	139.023	0.324	139.107	0.084	
		1900	. .	17 54 17.031	139.183	0.314	139.268	0.085	

DECLINATIONS.

No.	Star.	Mag.	Epoch.	Declination.	Centennial variation.	Secular variation.	Struve's precession.	Proper motion.	Sec. var. of proper motion.
				° ′ ″	″	″	″	″	″
776	α Ophiuchi . . .	2.0	1755	+ 12 45 30.11	− 341.43	+ 40.30	− 317.59	−23.84	+ 0.11
		2.0	1850	12 40 23.95	303.10	40.41	279.36	23.74	
			1900	12 37 57.46	282.88	40.47	259.20	23.68	
777	ξ Serpentis . . .	5.0	1755	− 15 13 10.63	− 323.93	+ 49.42	− 317.57	− 6.36	
		3.7	1850	15 17 56.03	276.88	49.64	270.45	6.43	
778	B. A. C. 5954 . .	6.0	1755	− 21 44 27.55	− 317.99	+ 51.87	− 313.49	− 4.50	
		6.8	1850	21 49 6.20	268.59	52.12	264.07	4.52	
779	σ Octantis . . .	. .	1800	− 89 11 10.36	− 944.75	+1112.8	− 941.19	− 3.56	
			1825	89 14 28.99	− 633.36	1373.4	− 630.06	3.30	
		5.8	1850	89 16 22.18	− 264.32	1559.0	− 261.43	2.89	
			1875	89 16 38.77	+ 132.82	1588.7	+ 135.18	2.36	
			1900	89 15 17.01	+ 515.31	1447.5	+ 517.07	1.76	
780	58 Ophiuchi . . .	5.0	1755	− 21 32 15.46	− 278.82	+ 51.86	− 272.64	− 6.18	
		5.4	1850	21 36 16.90	229.45	52.07	223.19	6.26	
781	ω Draconis . . .	5.0	1755	+ 68 52 7.20	− 156.45	− 5.16	− 188.72	+32.27	
			1800	68 50 56.26	158.79	5.10	191.10	32.31	
		5.0	1850	68 49 36.23	161.32	5.03	193.67	32.35	
			1900	68 48 14.94	163.82	4.96	196.21	32.39	
782	3 Sagittarii . . .	5.0	1755	− 27 42 35.12	− 246.14	+ 54.60	− 243.17	− 2.97	
		4–6	1850	27 46 4.28	194.17	54.80	191.20	2.97	
783	μ Herculis . . .	4.0	1755	+ 27 52 50.89	− 277.51	+ 33.70	− 202.05	−75.46	− 0.36
		3.3	1850	27 48 42.48	245.46	33.78	169.66	75.80	
			1900	27 46 43.97	228.56	33.82	152.58	75.98	
784	ψ¹ Draconis . . .	5.5	1755	+ 72 15 41.81	− 146.80	− 16.02	− 119.45	−27.35	
			1800	72 14 34.15	153.95	15.89	126.62	27.33	
		4.3	1850	72 13 15.20	161.85	15.73	134.53	27.32	
			1900	72 11 52.32	169.66	15.58	142.36	27.30	
785	63 Ophiuchi . . .	6.5	1755	− 24 48 42.26	− 176.14	+ 53.63	− 176.33	+ 0.19	
		6.6	1850	24 51 5.37	125.13	53.76	125.30	0.17	
786	B. A. C. 6060 . .	6.7	1850	− 18 46 10.7	. . . .	+ 51.22	− 112.88	. . .	
787	B. A. C. 6066 . .	7.5	1755	− 23 52 36.91	− 158.18	+ 53.30	− 156.00	− 2.18	
		7.3	1850	23 54 43.11	107.49	53.42	105.31	2.18	
788	4 Sagittarii . . .	5.0	1755	− 23 45 59.70	− 139.19	+ 53.28	− 132.59	− 6.60	
		5.4	1850	23 47 47.88	88.53	53.38	81.93	6.60	
789	5 Sagittarii . . .	7.0	1755	− 24 14 17.20	− 133.23	+ 53.59	− 129.75	− 3.48	
		7.0	1850	24 15 59.57	82.28	53.68	78.74	3.54	
790	6 Sagittarii . . .	7.0	1755	− 17 7 20.85	− 111.42	+ 50.75	− 112.37	+ 0.95	
		6.9	1850	17 8 43.80	63.18	50.82	64.12	0.94	
791	γ Draconis . . .	2.0	1755	+ 51 31 39.79	− 82.49	+ 20.32	− 79.43	− 3.06	+ 0.03
		2.3	1850	51 30 30.60	63.17	20.36	60.14	3.03	
			1900	51 30 1.56	52.99	20.38	49.98	3.01	

RIGHT ASCENSIONS.

No.	Star.	Epoch.	Number of observations.	Right ascension.			Centennial variation.	Secular variation.	Struve's precession.	Proper motion.	Sec. var. of proper motion.
				h.	m.	s.	s.	s.	s.	s.	s.
792	7 Sagittarii . . .	1755	3	17	47	50.929	+ 367.003	+ 0.334	+ 367.179	— 0.176	
		1850	8	17	53	39.720	367.282	0.254	367.458	0.176	
793	B. A. C. 6098 . .	1850	7	17	53	40.520	+ 357.469	+ 0.250	+ 357.767	— 0.298	
794	Piazzi 17^h 330 . .	1850	. .	17	54	31.1	. . . .	+ 0.238	+ 364.291	. . .	
795	9 Sagittarii . . .	1755	2	17	48	51.705	+ 367.223	+ 0.322	+ 367.430	— 0.207	
		1850	24	17	54	40.700	367.491	0.242	367.696	0.205	
796	Piazzi 17^h 334 . .	1850	6	17	54	50.465	+ 363.238	+ 0.233	+ 363.469	— 0.231	
797	γ^1 Sagittarii . . .	1850	20	17	55	26.314	+ 383.383	+ 0.232	+ 383.062	+ 0.321	
798	γ^2 Sagittarii . . .	1755	4	17	50	4.776	+ 384.758	+ 0.365	+ 385.395	— 0.637	+0.019
		1850	95	17	56	10.445	385.055	0.260	385.676	0.621	
		1900	. .	17	59	23.003	385.172	0.206	385.787	0.615	
799	B. A. C. 6127 . .	1850	14	17	58	35.045	+ 379.913	+ 0 183	+ 379.675	+ 0.238	
800	B. A. C. 6161 . .	1755	1	17	56	46.605	+ 365.791	+ 0.220	+ 365.772	+ 0.019	
		1850	11	18	2	34.194	365.964	0.144	365.938	0.026	
801	μ Sagittarii . . .	1755	5	17	59	7.010	+ 358.486	+ 0.177	+ 358.606	. . .	
		1900	752	18	4	47.641	358.621	0.107	358.741	. . .	
		1850	. .	18	7	46.963	358.666	0.071	358.786	. . .	
802	14 Sagittarii . .	1755	4	17	59	33.093	+ 359.986	+ 0.178	+ 360.368	— 0.382	
		1850	2	18	5	15.149	360.122	0.108	360.502	0.380	
803	15 Sagittarii . . .	1755	5	18	0	36.168	+ 357.691	+ 0.163	+ 357.742	— 0.051	
		1850	14	18	6	16.036	357.812	0.092	357.865	0.053	
804	16 Sagittarii . . .	1755	5	18	0	38.568	+ 356.729	+ 0.165	+ 356.835	— 0.106	
		1850	4	18	6	17.525	356.858	0.107	356.958	0.100	
805	17 Sagittarii . . .	1755	1	18	2	0.087	+ 356.979	+ 0.150	+ 357.269	— 0.290	
		1850	5	18	7	39.273	357.087	0.079	357.377	0.290	
806	B. A. C. 6194 . .	1850	18	18	8	40.018	+ 374.668	+ 0.023	+ 375.526	— 0.858	
807	B. A. C. 6201 . .	1850	. .	18	9	47.	. . . .	+ 0.060	352.307	. . .	
808	δ Sagittarii . . .	1755	5	18	5	18.567	+ 384.122	+ 0.072	+ 383.906	+ 0.216	
		1850	38	18	11	23.499	384.141	— 0.031	383.927	0.214	
809	B. A. C. 6210 . .	1755	3	18	6	2.546	+ 345.297	+ 0.112	+ 345.108	+ 0.189	
		1850	14	18	11	30.620	345.377	0.056	345.188	0.189	
810	η Serpentis . . .	1755	5	18	8	38.435	+ 309.967	+ 0.218	+ 313.866	— 3.899	
		1850	261	18	13	32.997	310.160	0.188	314.016	3.856	
		1900	. .	18	16	8.100	+ 310.250	+ 0.172	+ 314.085	— 3.835	
811	21 Sagittarii . . .	1755	5	18	10	45.640	+ 357.264	+ 0.040	+ 357.346	— 0.082	
		1850	23	18	16	25.048	357.272	— 0.024	357.351	0.079	
812	λ Sagittarii . . .	1755	5	18	12	51.007	+ 370.324	+ 0.013	+ 370.783	— 0.459	
		1850	116	18	18	42.807	370.295	— 0.071	370.741	0.446	

DECLINATIONS.

No.	Star.	Mag.	Epoch.	Declination.	Centennial variation.	Secular variation.	Struve's precession.	Proper motion.	Sec. var. of proper motion.
				° ′ ″	″	″	″	″	″
792	7 Sagittarii . . .	7.0	1755	− 24 15 13.38	− 107.52	+ 53.48	− 106.32	− 1.20	
		5.9	1850	24 16 31.38	56.66	53 60	55.36	1.30	
793	B. A. C. 6098 . .	6.0	1850	− 20 43 50.42	− 58.40	+ 52.11	− 55.34	− 3.06	
794	Piazzi 17ʰ 330 . .	5.3	1850	− 23 8 8.3	. . . .	+ 53.15	− 47.98	. . .	
795	9 Sagittarii . . .	6.5	1755	− 24 20 17.88	− 100.11	+ 53.52	− 97.46	− 2.65	
		6.0	1850	− 24 21 28.82	49.23	53.58	46.58	2.65	
796	Piazzi 17ʰ 334 . .	5.3	1850	− 22 50 6.45	− 45.18	+ 52.96	− 45.14	− 0.04	
797	γ^1 Sagittarii . . .	5–6.5	1850	− 29 34 50.56	− 39.12	+ 55.98	− 39.88	+ 0.76	
798	γ^2 Sagittarii . . .	5.0	1755	− 30 23 53.59	− 108.62	+ 56.12	− 86.87	− 21.75	− 0.07
		2.8	1850	30 25 11.46	55.31	56.12	33.48	21.83	
			1900	30 25 32.10	27.25	56.12	5.38	21.87	
799	B. A. C. 6127 . .	5.1	1850	− 28 28 4.80	− 13.11	+ 55.46	− 12.40	− 0.71	
800	B. A. C. 6161 . .	6.0	1755	− 23 43 25.17	− 35.22	+ 53.37	− 28.22	− 7.00	
		5.7	1850	23 43 34.54	+ 15.49	53.38	+ 22.48	6.99	
801	μ Sagittarii . . .	3.5	1755	− 21 5 48.62	− 8.81	+ 52.24	− 7.70	− 1.11	− 0.06
		4.3	1850	21 5 33.43	+ 40.79	52.18	+ 41.95	1.16	
			1900	21 5 6.51	66.87	52.14	68.07	1.20	
802	14 Sagittarii . . .	6.0	1755	− 21 45 7.82	− 7.28	+ 52.46	− 3.93	− 3.35	
		6.0	1850	21 44 51.07	+ 42.55	52.44	+ 45.99	3.44	
803	15 Sagittarii . . .	6.0	1755	− 20 46 31.05	+ 4.95	+ 52.17	+ 5.29	− 0.34	
		5.8	1850	20 46 2.82	54.50	52.15	54.86	0.36	
804	16 Sagittarii . . .	6.0	1755	− 20 26 3.95	+ 3.12	+ 52.02	+ 5.63	− 2.51	
		6.6	1850	20 25 37.52	52.53	52.00	55.07	2.54	
805	17 Sagittarii . . .	7.0	1755	− 20 35 54.73	+ 14.52	+ 52.02	+ 17.51	− 2.99	
		7.0	1850	20 35 17.46	63.94	52.02	66.98	3.04	
806	B. A. C. 6194 . .	5.1	1850	− 27 5 28.14	+ 78.79	+ 54.46	+ 75.83	+ 2.96	
807	B. A. C. 6201 . .	. .	1850	− 18 40	. . . .	+ 51.31	+ 85.59	. . .	
808	δ Sagittarii . . .	3.5	1755	− 29 54 16.39	+ 43.74	+ 56.03	+ 46.48	− 2.74	
		2.8	1850	29 53 9.57	96.93	55.96	99.63	2.70	
809	B. A. C. 6210 . .	6.0	1755	. . . . . .	. . . .	+ 50.36	+ 52.90	. . .	
		6.0	1850	− 15 53 17.7	. . . .	50.30	100.71	. . .	
810	η Serpentis . . .	4.0	1755	− 2 56 28.41	+ 9.02	+ 44.56	+ 75.64	− 66.62	− 0.58
		3.5	1850	2 55 59.74	51.33	44.50	118.51	67.18	
			1900	2 55 28.52	73.57	44.46	141.03	67.46	
811	21 Sagittarii . . .	6.0	1755	− 20 38 50.30	+ 91.73	+ 52.09	+ 94.19	− 2.46	
		5.1	1850	20 36 59.63	141.20	52.06	143.54	2.34	
812	λ Sagittarii . . .	4.0	1755	− 25 31 45.85	+ 89.98	+ 53.99	+ 112.42	− 22.44	
		2.7	1850	25 29 56.03	141.21	53.88	163.60	22.39	

RIGHT ASCENSIONS.

No.	Star.	Epoch.	Number of observations.	Right ascension.			Centennial variation.	Secular variation.	Struve's precession.	Proper motion.	Sec. var. of proper motion.
				h.	m.	s.	s.	s.	s.	s.	s.
813	δ Ursæ Minoris . .	1755	. .	18	50	35.73	—1831.68	—139.45	—1835.35	+ 3.67	
		1775	. .	18	44	26.68	1858.19	125.30	1861.73	3.54	
		1800	. .	18	36	38.41	1887.13	105.91	1890.49	3.36	
		1825	. .	18	28	43.53	1911.02	84.80	1914.18	3.16	
		1850	. .	18	20	43.36	1929.39	62.28	1932.35	2.96	
		1875	. .	18	12	39.32	1942.03	38.72	1944.77	2.74	
		1900	. .	18	4	32.87	—1948.72	— 14.49	—1951.22	+ 2.50	
814	B. A. C. 6287 . .	1755	. .	. .	. .	. .	. . . .	+ 0.005	+ 352.578	. . .	
		1850	4	18	21	22.6	. . . .	— 0.060	352.545	. . .	
815	B. A. C. 6294 . .	1850	6	18	22	39.006	+ 351.707	— 0.075	+ 351.698	+ 0.009	
816	B. A. C. 6304 . .	1755	1	18	18	15.600	+ 366.958	— 0.085	+ 367.090	— 0.132	
		1850	8	18	24	4.159	366.839	0.165	366.976	0.137	
817	24 Sagittarii . . .	1755	5	18	18	55.296	+ 366.752	— 0.093	+ 366.874	— 0.122	
		1850	11	18	24	43.657	366.626	0.173	366.746	0.120	
818	25 Sagittarii . . .	1755	1	18	19	32.717	+ 367.900	— 0.108	+ 367.407	+ 0.493	
		1850	5	18	25	22.161	367.759	0.188	367.266	0.493	
819	1 Aquilæ (3 H. Scuti.)	1755	5	18	21	52.574	+ 326.421	+ 0.051	+ 326.640	— 0.219	+0.020
		1850	170	18	27	2.691	326.451	+ 0.012	326.650	0.199	
		1900	. .	18	29	45.917	326.452	— 0.008	326.641	0.189	
820	B. A. C. 6336 . .	1755	2	18	23	14.293	+ 359.331	— 0.108	+ 359.613	— 0.282	
		1850	7	18	28	55.595	359.186	0.198	359.472	0.286	
821	B. A. C. 6343 . .	1755	3	18	23	36.389	+ 365.230	— 0.149	+ 365.370	— 0.140	
		1850	23	18	29	23.279	365.053	0.224	365.190	0.137	
822	B. A. C. 6347 . .	1755	2	18	24	16.636	+ 358.024	— 0.109	+ 358.634	— 0.610	
		1850	5	18	29	56.699	357.888	0.178	358.520	0.632	
823	α Lyræ	1755	. .	18	28	38.734	+ 202.987	+ 0.111	+ 201.162	+ 1.825	—0.028
		1850	. .	18	31	51.621	203.090	0.105	201.292	1.798	
		1900	. .	18	33	33.179	203.142	0.101	201.356	1.786	
824	26 Sagittarii . . .	1755	5	18	26	54.679	+ 366.357	— 0.198	+ 366.241	+ 0.116	
		1850	8	18	32	42.618	366.133	0.275	366.015	0.118	
825	B. A. C. 6369 . .	1850	3	18	35	36.211	+ 369.109	— 0.336	+ 369.221	— 0.112	
826	φ Sagittarii . . .	1755	5	18	30	20.586	+ 375.329	— 0.300	+ 375.184	+ 0.145	
		1850	74	18	36	17.000	375.003	0.387	374.859	0.144	
827	28 Sagittarii . . .	1755	5	18	31	33.733	+ 362.270	— 0.240	+ 362.212	+ 0.058	
		1850	6	18	37	17.770	362.009	0.310	361.946	0.063	
828	B. A. C. 6386 . .	1755	1	18	33	19.838	+ 356.439	— 0.233	+ 356.511	— 0.072	
		1850	3	18	38	58.332	356.190	0.291	356.269	0.079	
829	29 Sagittarii . . .	1755	5	18	35	7.382	+ 356.548	— 0.255	+ 356.627	— 0.079	
		1850	24	18	40	45.978	356.276	0.318	356.354	0.078	
830	30 Sagittarii . . .	1755	5	18	36	6.677	+ 360.985	— 0.290	+ 361.492	— 0.507	
		1850	3	18	41	49.471	360.677	0.359	361.181	0.504	

DECLINATIONS.

No.	Star.	Mag.	Epoch.	Declination.	Centennial variation.	Secular variation.	Struve's precession.	Proper motion.	Sec. var. of proper motion.
				° ′ ″	″	″	″	″	″
813	δ Ursæ Minoris . .	3.0	1755	+ 86 30 49.15	+ 443.90	—260.39	+ 439.38	+ 4.52	
			1775	86 32 12.68	391.26	265.64	386.65	4.61	
			1800	86 33 42.12	324.09	271.49	319.36	4.73	
			1825	86 34 54.61	255.60	276.25	250.75	4.85	
		4.3	1850	86 35 49.84	186.06	279.95	181.10	4.96	
			1875	86 36 27.58	115.74	282.52	110.68	5.06	
			1900	+ 86 36 47.67	+ 44.95	—283.87	+ 39.79	+ 5.16	
814	B. A. C. 6287 . .	. .	1755	— 18 51 32.56	+ 128.70	+ 51.11	+ 138.25	— 9.55	
		6.0	1850	18 49 7.22	177.19	50.97	186.80	9.61	
815	B. A. C. 6294 . .	5.5	1850	— 18 30 0.39	+ 191.78	+ 50.97	+ 197.87	— 6.09	
816	B. A. C. 6304 . .	7.0	1755	— 24 15 41.82	+ 157.61	+ 53.27	+ 159.70	— 2.09	
			1850	24 12 48.08	208.14	53.11	210.22	2.08	
817	24 Sagittarii . . .	6.5	1755	— 24 11 18.37	+ 164.60	+ 53.23	+ 165.47	— 0.87	
		5.9	1850	24 8 18.01	215.09	53.06	215.99	0.90	
818	25 Sagittarii . . .	7.5	1755	— 24 22 58.92	+ 171.76	+ 53.47	+ 170.86	+ 0.90	
		6.3	1850	24 19 51.65	222.48	53.30	221.59	0.89	
819	1 Aquilæ (3 H. Scuti.)	5.5	1755	— 8 23 30.35	+ 157.96	+ 47.55	+ 191.08	—33.12	+ 0.08
		3.6	1850	8 20 38.85	203.05	47.37	236.09	33.04	
			1900	8 18 51.41	226.71	47.29	259.71	33.00	
820	B. A. C. 6336 . .	6.5	1755	— 21 34 24.70	+ 192.39	+ 52.02	+ 203.10	—10.71	
		6.2	1850	21 30 58.49	241.71	51.83	252.46	10.75	
821	B. A. C. 6343 . .	6.0	1755	— 23 41 15.22	+ 203.03	+ 52.89	+ 206.30	— 3.27	
		6.3	1850	23 37 38.51	253.19	52.70	256.46	3.27	
822	B. A. C. 6347 . .	6.5	1755	— 21 13 44.34	+ 196.68	+ 51.75	+ 212.16	—15.48	
		6.0	1850	21 10 14.18	245.75	51.56	261.31	15.56	
823	α Lyræ	1.0	1755	+ 38 34 12.44	+ 277.05	+ 29.56	+ 250.12	+26.93	+ 0.26
		1.0	1850	38 38 48.96	305.10	29.48	277.92	27.18	
			1900	38 41 25.19	319.83	29.44	292.53	27.30	
824	26 Sagittarii . . .	6.0	1755	— 24 2 7.78	+ 232.00	+ 52.99	+ 235.06	— 3.06	
		6.6	1850	23 58 3.51	282.23	52.76	285.28	3.05	
825	B. A. C. 6369 . .	6.2	1850	— 25 9 19.58	+ 307.31	+ 53.05	+ 310.34	— 3.03	
826	φ Sagittarii . . .	4.5	1755	— 27 12 54.85	+ 262.02	+ 54.46	+ 264.82	— 2.80	
		3.7	1850	27 8 21.43	313.57	54.07	316.20	2.63	
827	28 Sagittarii . . .	6.0	1755	— 22 37 21.32	+ 273.30	+ 52.24	+ 275.49	— 2.19	
		5.6	1850	22 32 38.15	322.80	51.98	324.96	2.16	
828	B. A. C. 6386 . .	7.5	1755	— 20 30 50.02	+ 287.37	+ 51.31	+ 290.76	— 3.39	
		7.3	1850	20 25 53.90	335.99	51.06	339.44	3.45	
829	29 Sagittarii . . .	6.0	1755	— 20 34 38.80	+ 307.63	+ 51.37	+ 306.24	+ 1.39	
		5.5	1850	20 29 23.40	356.32	51.14	354.88	1.44	
830	30 Sagittarii . . .	6.0	1755	— 22 25 2.00	+ 311.02	+ 51.80	+ 314.84	— 3.82	
		6.6	1850	22 19 43.20	360.10	51.52	363.88	3.78	

RIGHT ASCENSIONS.

No.	Star.	Epoch.	Number of observations.	Right ascension.			Centennial variation.	Secular variation.	Struve's precession.	Proper motion.	Sec. var. of proper motion.
				h.	m.	s.	s.	s.	s.	s.	s.
831	31 Sagittarii . . .	1755	5	18	37	25.140	+ 360.785	− 0.303	+ 360.814	− 0.029	
		1850	9	18	43	7.739	360.465	0.371	360.491	0.026	
832	β Lyræ	1755	5	18	41	2.353	+ 221.219	+ 0.154	+ 221.214	+ 0.005	
		1850	1058	18	44	32.580	221.363	0.149	221.355	0.008	
		1900	. .	18	46	23.280	221.437	0.147	221.429	0.008	
833	33 Sagittarii . . .	1755	5	18	39	21.022	+ 359.225	− 0.331	+ 359.273	− 0.048	
		1850	8	18	45	2.128	358.880	0.396	358.935	0.055	
834	ν^1 Sagittarii . . .	1755	5	18	39	22.206	+ 362.797	− 0.344	+ 362.996	− 0.199	
		1850	27	18	45	6.698	362.437	0.414	362.632	0.195	
835	σ Sagittarii . . .	1755	5	18	40	3.795	+ 372.811	− 0.422	+ 372.884	− 0.073	
		1850	126	18	45	57.762	372.371	0.504	372.437	0.066	
		1900	. .	18	49	3.883	372.108	0.545	372.178	0.070	
836	ν^2 Sagittarii . . .	1755	5	18	40	17.912	+ 363.389	− 0.358	+ 362.770	+ 0.619	
		1850	19	18	46	2.960	363.017	0.427	362.393	0.624	
837	B. A. C. 6447 . .	1850	. .	18	46	55.	. . . .	− 0.312	+ 346.081	. . .	
838	B. A. C. 6448 . .	1850	12	18	46	55.635	+ 363.686	− 0.450	+ 363.716	− 0.030	
839	ξ^1 Sagittarii . . .	1755	5	18	42	46.480	+ 357.124	− 0.351	+ 357.324	− 0.200	
		1850	12	18	48	25.579	356.760	0.416	356.957	0.197	
840	ξ^2 Sagittarii . . .	1755	5	18	43	6.191	+ 358.687	− 0.363	+ 358.507	+ 0.180	
		1850	40	18	48	46.771	358.317	0.416	358.128	0.189	
841	50 Draconis . . .	1755	2	18	54	7.57	− 183.19	− 5.81	− 182.86	− 0.33	
		1800	.	18	52	44.55	185.79	5.72	185.43	0.36	
		1850	. .	18	51	10.95	188.63	5.62	188.23	− 0.40	
		1900	. .	18	49	35.95	191.38	5.48	190.97	0.41	
842	ζ Sagittarii . . .	1755	5	18	47	0.430	+ 382.888	− 0.630	+ 383.226	− 0.338	
		1850	18	18	53	3.876	382.246	0.722	382.583	0.337	
843	Lal. 35497 . . .	1850	. .	18	54	14.6	. . . .	− 0.447	+ 353.097	. . .	
844	o Sagittarii . . .	1755	5	18	49	59.435	+ 360.367	− 0.457	+ 359.958	+ 0.409	
		1850	58	18	55	41.567	359.902	0.521	359.496	0.406	
845	A. Oe.[2] 19053 . .	1850	. .	18	57	5.8	. . . .	− 0.397	+ 344.043	. . .	
846	τ Sagittarii . . .	1755	5	18	51	37.717	+ 375.731	− 0.601	376.334	− 0.603	
		1850	30	18	57	34.377	375.120	0.685	375.704	0.584	
847	ζ Aquilæ	1755	5	18	54	9.125	+ 275.627	+ 0.053	+ 275.733	− 0.106	
		1850	1049	18	58	30.993	275.672	0.042	275.772	0.100	
		1900	. .	19	0	48.834	275.692	0.037	275.792	0.100	
848	B. A. C. 6536 . .	1755	1	18	53	52.323	+ 353.331	− 0.448	+ 353.444	− 0.113	
		1850	9	18	59	27.777	352.879	0.504	352.991	0.112	
849	π Sagittarii . . .	1755	5	18	55	10.873	+ 357.745	− 0.510	+ 357.876	− 0.131	
		1850	143	19	0	50.492	357.233	0.568	357.371	0.138	
850	ψ Sagittarii . . .	1755	5	19	0	30.025	+ 369.156	− 0.693	+ 369.017	+ 0.139	
		1850	78	19	6	20.399	368.464	0.764	368.327	0.137	

DECLINATIONS.

No.	Star.	Mag.	Epoch.	Declination.	Centennial variation.	Secular variation.	Struve's precession.	Proper motion.	Sec. var. of proper motion.
				° ′ ″	″	″	″	″	″
831	31 Sagittarii . . .	6.0	1755	− 22 11 0.75	+ 321.78	+ 51.79	+ 326.14	− 4.36	
		7.0	1850	22 5 31.74	370.84	51.49	375.21	4.37	
832	β Lyræ	3.0	1755	+ 33 5 37.70	+ 355.64	+ 31.63	+ 357.36	− 1.72	+ 0.01
		4.0	1850	33 11 29.81	385.64	31.53	387.35	1.71	
			1900	33 14 46.57	401.39	31.47	403.10	1.71	
833	33 Sagittarii . . .	6.0	1755	− 21 38 5.54	+ 342.03	+ 51.67	+ 342.77	− 0.74	
		6.0	1850	21 32 17.32	390.96	51.35	391.58	0.62	
834	ν^1 Sagittarii . . .	5.0	1755	− 23 1 12.40	+ 339.93	+ 51.97	+ 342.98	− 3.05	
		5.0	1850	22 55 26.07	389.15	51.66	392.23	3.08	
835	σ Sagittarii . . .	3.0	1755	− 26 34 26.85	+ 341.21	+ 53.46	+ 348.93	− 7.72	0.00
•		2.4	1850	26 28 38.64	391.81	53.07	399.53	7.72	
			1900	26 25 16.11	418.29	52.85	426.01	7.72	
836	ν^2 Sagittarii . . .	5.0	1755	− 22 57 6.20	+ 348.44	+ 52.13	+ 350.99	− 2.55	
		5.1	1850	22 51 11.72	397.81	51.81	400.28	2.47	
837	B. A. C. 6447 . .	5.8	1850	− 16 33	. . . .	+ 49.26	+ 407.72	. . .	
838	B. A. C. 6448 . .	6.4	1850	− 23 21 33.04	+ 406.03	+ 51.77	+ 407.83	− 1.80	
839	ξ^1 Sagittarii . . .	6.0	1755	− 20 57 3.61	+ 369.34	+ 51.09	+ 372.29	− 2.95	
		5.7	1850	20 50 49.73	417.70	50.75	420.65	2.95	
840	ξ^2 Sagittarii . . .	5.0	1755	− 21 24 11.60	+ 372.87	+ 51.30	+ 375.14	− 2.27	
		3.5	1850	21 17 54.27	421.44	50.97	423.64	2.20	
841	50 Draconis . . .	5.5	1755	+ 75 7 54.42	+ 477.00	− 26.23	+ 469.48	+ 7.52	
			1800	75 11 26.40	465.09	26.64	457.60	7.49	
		6.0	1850	75 15 15.60	451.66	27.08	444.20	7.46	
			1900	75 18 58.03	438.02	27.52	430.59	7.43	
842	ζ Sagittarii . . .	3.5	1755	− 30 12 11.79	+ 408.16	+ 54.46	+ 408.66	− 0.50	
		3.1	1850	30 5 19.59	459.51	53.65	460.14	0.63	
843	Lal. 35497 . . .	6.4	1850	− 19 27 26.8	. . . .	+ 49.87	+ 470.27	. . .	
844	o Sagittarii . . .	4.5	1755	− 22 4 29.76	+ 426.86	+ 51.25	+ 434.22	− 7.36	
		3.8	1850	21 57 21.18	475.34	50.84	482.60	7.26	
845	A. Oe.[2] 19053 . .	5.9	1850	− 15 52 52.1	. . . .	+ 48.43	+ 494.52	. . .	
846	τ Sagittarii . . .	4.0	1755	− 28 0 7.32	+ 421.63	+ 53.29	+ 448.17	−26.54	
		3.6	1850	27 53 2.76	472.07	52.88	498.55	26.48	
847	ζ Aquilæ	6.0	1755	+ 13 31 5.80	+ 459.43	+ 38.84	+ 469.71	−10.28	− 0.04
		3.0	1850	13 38 39 75	496.22	38.62	506.54	10.32	
			1900	13 42 52.68	515.50	38.50	525.84	10.34	
848	B. A. C. 6536 . .	6.5	1755	. . . . . .	. . . .	+ 49.91	+ 467.30	. . .	
		5.8	1850	− 19 31 12.6	. . . .	49.52	514.56	. . .	
849	π Sagittarii . . .	4.5	1755	− 21 23 17.83	+ 473.98	+ 50.50	+ 478.43	− 4.45	
		3.1	1850	21 15 24.83	521.78	50.14	526.19	4.41	
850	ψ Sagittarii . . .	6.0	1755	− 25 39 12.33	+ 519.37	+ 51.88	+ 523.52	− 4.15	
		5.4	1850	25 30 35.57	568.43	51.40	572.50	4.07	

RIGHT ASCENSIONS.

No.	Star.	Epoch.	Number of observations.	Right ascension.			Centennial variation.	Secular variation.	Struve's precession.	Proper motion.	Sec. var. of proper motion.
				h.	m.	s.	s.	s.	s.	s.	s.
851	d Sagittarii	1755	5	19	3	17.230	+ 352.013	− 0.547	+ 352.233	− 0.220	
		1850	82	19	8	51.387	351.468	0.600	351.691	0.223	
		1900	. .	19	11	47.045	351.161	0.626	351.390	0.229	
852	B. A. C. 6591 . .	1755	1	19	5	6.748	+ 344.495	− 0.481	+ 344.617	− 0.122	
		1850	2	19	10	33.794	344.016	0.527	344.138	0.122	
853	B. A. C. 6607 . .	1755	. .	19	5	56.095	+ 360.901	− 0.655	+ 360.998	− 0.097	
		1850	14	19	11	38.645	360.247	0.720	360.338	0.091	
854	δ Draconis . . .	1755	5	19	12	25.70	+ 6.02	− 2.26	+ 4.08	+ 1.94	
		1800	. .	19	12	28.17	5.00	2.28	3.07	1.93	
		1850	. .	19	12	30.38	3.86	2.31	1.94	1.92	
		1900	. .	19	12	32.02	2.70	2.32	0.78	1.92	
855	ρ¹ Sagittarii . . .	1755	4	19	7	26.917	+ 349.047	− 0.563	+ 349.295	− 0.248	
		1850	52	19	12	58.250	348.488	0.613	348.744	0.256	
856	ρ² Sagittarii . . .	1755	5	19	7	32.491	+ 351.133	− 0.566	+ 350.422	+ 0.711	
		1850	3	19	13	5.804	350.571	0.617	349.861	0.710	
857	υ Sagittarii . . .	1755	5	19	7	40.927	+ 344.594	− 0.504	+ 344.639	− 0.045	
		1850	20	19	13	8.056	344.091	0.555	344.139	0.048	
858	B. A. C. 6628 . .	1755	. .	19	9	12.320	+ 375.802	− 0.891	+ 375.799	+ 0.003	
		1850	10	19	15	8.917	374.916	0.976	374.909	0.007	
859	χ¹ Sagittarii . . .	1755	5	19	10	20.670	+ 366.636	− 0.782	+ 366.346	+ 0.290	
		1850	18	19	16	8.612	365.863	0.847	365.567	0.296	
860	χ² Sagittarii . . .	1850	3	19	16	15.366	+ 365.420	− 0.847	+ 365.308	+ 0.112	
861	χ³ Sagittarii . . .	1755	5	19	10	38.702	+ 364.606	− 0.774	+ 364.864	− 0.258	
		1850	13	19	16	24.720	363.842	0.835	364.094	0.252	
862	50 Sagittarii . . .	1755	5	19	11	41.392	+ 359.087	− 0.714	+ 359.033	+ 0.054	
		1850	8	19	17	22.193	358.381	0.772	358.336	0.045	
863	B. A. C. 6643 . .	1850	5	19	17	38.958	+ 342.190	− 0.562	+ 341.786	+ 0.404	
864	δ Aquilæ	1755	5	19	13	8.558	+ 302.761	− 0.165	+ 301.126	+ 1.635	
		1850	956	19	17	56.104	302.597	0.180	300.967	1.630	
		1900	. .	19	20	27.380	302.505	0.187	300.880	1.625	
865	τ Draconis . . .	1755	5	19	20	5.55	− 103.88	− 5.78	− 101.35	− 2.53	
		1800	. .	19	19	18.22	106.47	5.81	103.89	2.58	
		1850	. .	19	18	24.25	109.41	5.84	106.77	2.64	
		1900	. .	19	17	28.82	112.32	5.87	109.64	2.68	
866	B. A. C. 6658 . .	1755	. .	. .	. .	. .	. . . .	− 0.633	+ 350.268	. . .	
		1850	3	19	19	21.5	. . . .	0.680	349.646	. . .	
867	B. A. C. 6666 . .	1850	21	19	20	35.249	+ 371.825	− 1.006	+ 371.934	− 0.109	
868	h¹ Sagittarii . . .	1755	5	19	21	7.549	+ 366.129	− 0.930	+ 366.125	+ 0.004	
		1850	14	19	26	54.942	365.217	0.990	365.209	0.008	
869	h² Sagittarii . . .	1755	5	19	21	46.384	+ 366.876	− 0.946	+ 366.513	+ 0.363	
		1850	120	19	27	34.479	365.949	1.006	365.583	0.366	

DECLINATIONS.

No.	Star.	Mag.	Epoch.	Declination.	Centennial variation.	Secular variation.	Struve's precession.	Proper motion.	Sec. var. of proper motion.
				° ′ ″	″	″	″	″	″
851	*d* Sagittarii . . .	5.0	1755	− 19 21 53.76	+ 545.34	+ 49.17	+ 547.04	− 1.70	− 0.03
		5.6	1850	19 12 53.57	591.81	48.68	593.54	1.73	
			1900	19 7 51.59	616.09	48.42	617.84	1.75	
852	B. A. C. 6591 . .	5.0	1755	. . . . .	. . . .	+ 47.98	+ 562.41	. . .	
		8.0	1850	− 16 10 32.8	. . . .	47.55	607.85	. . .	
853	B. A. C. 6607 . .	. .	1755	− 22 49 55.84	+ 567.53	+ 50.03	+ 569.39	− 1.86	
		5.9	1850	22 40 34.19	614.80	49.50	616.80	2.00	
854	δ Draconis . . .	3.0	1755	+ 67 13 51.36	+ 631.83	+ 0.84	+ 623.60	+ 8.23	
			1800	67 18 35.76	632.17	0.69	623.83	8.34	
		3.0	1850	67 23 51.92	632.46	0.53	623.99	8.47	
			1900	67 29 8.21	632.69	0.36	624.08	8.61	
855	ρ^1 Sagittarii . . .	5.0	1755	− 18 17 2.76	+ 581.38	+ 48.60	+ 582.01	− 0.63	
		4.2	1850	18 7 28.59	627.33	48.15	627.84	0.51	
856	ρ^2 Sagittarii . . .	5.5	1755	− 18 44 20.08	+ 573.55	+ 48.98	+ 582.77	− 9.22	
		6.5	1850	18 34 53.19	619.81	48.40	628.90	9.09	
857	*v* Sagittarii . . .	5.5	1755	− 16 23 28.71	+ 582.15	+ 47.84	+ 583.96	− 1.81	
		4.9	1850	16 13 54.16	627.38	47.38	629.20	1.82	
858	B. A. C. 6628 . .	. .	1755	− 28 18 50.40	+ 594.99	+ 52.22	+ 596.66	− 1.67	
		5.9	1850	28 9 1.69	644.30	51.60	645.93	1.63	
859	χ^1 Sagittarii . . .	6.0	1755	− 24 57 32.08	+ 599.32	+ 50.77	+ 606.23	− 6.91	
		5.4	1850	24 47 39.91	647.29	50.21	654.14	6.85	
860	χ^2 Sagittarii . . .	6.3	1850	− 24 42 2.84	+ 649.41	+ 50.12	+ 655.09	− 5.68	
861	χ^3 Sagittarii . . .	6.0	1755	− 24 25 2.98	+ 607.08	+ 50.39	+ 608.74	− 1.66	
		5.6	1850	24 15 3.60	654.69	49.84	656.38	1.69	
862	50 Sagittarii . . .	6.5	1755	− 22 14 14.98	+ 616.59	+ 49.58	+ 617.45	− 0.86	
		5.9	1850	22 4 6.93	663.44	49.05	664.30	0.86	
863	B. A. C. 6643 . .	5.9	1850	− 15 20 44.23	+ 664.42	+ 46.85	+ 666.56	− 2.14	
864	δ Aquilæ	3.5	1755	+ 2 38 47.43	+ 636.93	+ 41.89	+ 629.55	+ 7.38	+ 0.21
		3.4	1850	2 49 11.36	676.55	41.52	668.97	7.58	
			1900	2 54 54.81	697.25	41.32	689.57	7.68	
865	τ Draconis . . .	4.5	1755	+ 72 53 35.05	+ 698.31	− 14.88	+ 687.02	+ 11.29	
			1800	72 58 47.77	691.53	15.26	680.40	11.13	
		4.7	1850	73 4 31.60	683.78	15.69	672.83	10.95	
			1900	73 10 11.52	675.84	16.10	665.06	10.78	
866	B. A. C. 6658 . .	. .	1755	− 18 49 55.13	+ 637.76	+ 48.20	+ 635.14	+ 2.62	
		7.3	1850	18 39 27.61	683.25	47.56	680.70	2.55	
867	B. A. C. 6666 . .	5.8	1850	− 27 17 14.56	+ 686.98	+ 50.61	+ 690.76	− 3.78	
868	h^1 Sagittarii . . .	6.0	1755	− 25 13 54.47	+ 693.16	+ 49.80	+ 695.51	− 2.35	
		5½–6¾	1850	25 2 33.59	740.17	49.18	742.51	2.34	
869	h^2 Sagittarii . . .	4.5	1755	− 25 23 59.95	+ 698.07	+ 49.90	+ 700.83	− 2.76	
		4.7	1850	25 12 34.36	745.17	49.26	747.86	2.69	

RIGHT ASCENSIONS.

No.	Star.	Epoch.	Number of observations.	Right ascension.			Centennial variation.	Secular variation.	Struve's precession.	Proper motion.	Sec. var. of proper motion.
				h.	m.	s.	s.	s.	s.	s	s.
870	B. A. C. 6707 . .	1850	5	19	27	41.259	+ 350.479	— 0.775	+ 350.322	+ 0.157	
871	B. A. C. 6710 . .	1850	2	19	28	20.928	+ 348.990	— 0.754	+ 348.773	+ 0.217	
872	κ Aquilæ	1755	5	19	23	42.109	+ 323.466	— 0.410	+ 323.560	— 0.094	
		1850	132	19	28	49.213	323.065	0.435	323.159	0.094	
		1900	. .	19	31	30.691	322.844	0.448	322.941	0.097	
873	53 Sagittarii . . .	1755	5	19	25	4.709	+ 362.144	— 0.920	+ 362.399	— 0.255	
		1850	10	19	30	48.322	361.244	0.974	361.484	0.240	
874	B. A. C. 6727 . .	1755	5	19	25	21.884	+ 362.486	— 0.930	+ 362.382	+ 0.104	
		1850	11	19	31	5.817	361.577	0.984	361.465	0.112	
875	c^1 Sagittarii . . .	1755	5	19	26	40.130	+ 345.021	— 0.681	+ 344.606	+ 0.415	
		1850	31	19	32	7.587	344.356	0.720	343.936	0.420	
876	c^2 Sagittarii . . .	1755	5	19	28	29.180	+ 344.521	— 0.696	+ 344.117	+ 0.404	
		1850	65	19	33	56.156	343.843	0.733	343.434	0.409	
877	B. A. C. 6746 . .	1850	8	19	35	0.022	+ 342.860	— 0.697	+ 341.841	+ 1.019	
878	f Sagittarii . . .	1755	5	19	32	2.920	+ 351.545	— 0.851	+ 352.615	— 1.070	
		1850	52	19	37	36.497	350.716	0.895	351.777	1.061	
879	γ Aquilæ	1755	50	19	34	36.666	+ 285.359	— 0.097	+ 285.317	+ 0.042	
		1850	. .	19	39	7.712	285.265	0.102	285.222	0.043	
		1900	. .	19	41	30.332	285.213	0.105	285.173	0.040	
880	α Aquilæ	1755	. .	19	38	49.590	+ 293.008	— 0.177	+ 289.399	+ 3.609	—0.023
		1850	. .	19	43	27.867	292.837	0.183	289.247	3.590	
		1900	. .	19	45	54.263	292.745	0.184	289.168	3.577	
881	57 Sagittarii . . .	1755	1	19	37	56.168	+ 350.491	— 0.876	+ 350.453	+ 0.038	
		1850	30	19	43	28.733	349.640	0.916	349.596	0.044	
882	ω Sagittarii . . .	1755	5	19	40	47.877	+ 369.882	— 1.268	+ 368.474	+ 1.408	
		1850	26	19	46	38.686	368.655	1.316	367.256	1.399	
883	b Sagittarii . . .	1755	5	19	41	52.794	+ 370.503	— 1.293	+ 370.655	— 0.151	
		1850	26	19	47	44.179	369.248	1.349	369.397	0.149	
884	β Aquilæ	1755	50	19	43	16.619	+ 294.897	— 0.136	+ 294.738	+ 0.159	+0.028
		1850	. .	19	47	56.708	294.764	0.144	294.576	0.188	
		1900	. .	19	50	24.072	294.690	0.147	294.493	0.197	
885	ε Draconis . . .	1755	2	19	48	52.76	— 12.15	— 4.20	— 13.59	+ 1.44	
		1800	. .	19	48	46.86	14.06	4.26	15.50	1.44	
		1850	. .	19	48	39.29	16.21	4.34	17.65	1.44	
		1900	. .	19	48	30.64	18.38	4.42	19.85	1.47	
886	g Sagittarii . . .	1755	5	19	44	2.121	+ 341.696	— 0.790	+ 341.742	— 0.046	
		1850	15	19	49	26.369	340.927	0.829	340.971	0.044	
887	A Sagittarii . . .	1755	5	19	43	59.585	+ 367.893	— 1.275	+ 367.821	+ 0.072	
		1850	30	19	49	48.500	366.658	1.326	366.587	0.071	
888	c Sagittarii . . .	1755	5	19	47	33.266	+ 371.624	— 1.401	+ 371.415	+ 0.209	
		1850	136	19	53	25.669	370.269	1.452	370.063	0.206	

DECLINATIONS.

No.	Star.	Mag.	Epoch.	Declination.	Centennial variation.	Secular variation.	Struve's precession.	Proper motion.	Sec. var. of proper motion.
				° ′ ″	″	″	″	″	″
870	B. A. C. 6707 . .	6.4	1850	— 19 10 44.52	+ 748.45	+ 47.13	+ 748.80	— 0.35	
871	B. A. C. 6710 . .	5.8	1850	— 18 33 34.20	+ 748.54	+ 46.88	+ 754.14	— 5.60	
872	κ Aquilæ	4.0	1755	— 7 33 4.48	+ 716.69	+ 43.79	+ 716.61	+ 0.08	0.00
		5.4	1850	7 21 23.94	758.05	43.30	757.97	0.08	
			1900	7 14 59.52	779.63	43.04	779.55	0.08	
873	53 Sagittarii . . .	7.0	1755	— 23 57 38.94	+ 722.82	+ 48.89	+ 727.86	— 5.04	
		6.7	1850	23 45 50.33	768.86	48.03	774.05	5.19	
874	B. A. C. 6727 . .	7.0	1755	— 23 57 56.31	+ 728.69	+ 48.96	+ 730.21	— 1.52	
		6.2	1850	23 46 2.06	774.89	48.32	776.40	1.51	
875	e^1 Sagittarii . . .	5.5	1755	— 16 49 56.74	+ 735.34	+ 46.51	+ 740.83	— 5.49	
		5.5	1850	16 37 57.26	779.26	45.95	784.66	5.40	
876	e^2 Sagittarii . . .	5.0	1755	— 16 40 31.36	+ 753.54	+ 46.29	+ 755.58	— 2.04	
		5.4	1850	16 28 14.70	797.24	45.72	799.23	1.99	
877	B. A. C. 6746 . .	5.8	1850	— 15 48 47.81	+ 787.11	+ 45.57	+ 807.73	—20.62	
878	*f* Sagittarii . . .	6.0	1755	— 20 19 38.38	+ 774.89	+ 46.77	+ 784.39	— 9.50	
		5.2	1850	20 7 1.20	819.00	46.10	828.60	9.60	
879	γ Aquilæ	3.0	1755	+ 10 2 4.25	+ 804.21	+ 37.67	+ 805.06	— 0.85	+ 0.01
		3.0	1850	10 15 5.17	839.86	37.39	840.70	0.84	
			1900	10 22 9.77	858.52	37.25	859.36	0.84	
880	α Aquilæ	1.5	1755	+ 8 14 24.72	+ 875.09	+ 38.96	+ 838.64	+36.45	+ 0.47
		1.1	1850	8 28 33.57	911.90	38.52	875.02	36.88	
			1900	8 36 14.32	931.10	38.29	893.97	37.13	
881	57 Sagittarii . . .	5.5	1755	— 19 38 41.18	+ 824.97	+ 46.18	+ 831.56	— 6.59	
		6.1	1850	19 25 16.72	868.53	45.52	875.11	6.58	
882	ω Sagittarii . . .	6.0	1755	— 26 55 35.52	+ 863.74	+ 48.74	+ 854.25	+ 9.49	
		5.1	1850	26 41 33 11	909.64	47.91	899.96	9.68	
883	*b* Sagittarii . . .	5.0	1755	— 27 47 43.87	+ 859.28	+ 48.40	+ 862.85	— 3.57	
		4.6	1850	27 33 45.82	904.89	47.61	908.47	3.58	
884	β Aquilæ	3.5	1755	+ 5 48 48.24	+ 824.76	+ 38.38	+ 873.87	—49.11	+ 0.02
		3.9	1850	6 2 9.02	861.02	37.94	910.12	49.10	
			1900	6 9 24.26	879.94	37.71	929.02	49.08	
885	ε Draconis . . .	5.5	1755	+ 69 38 35.51	+ 920.25	— 1.78	+ 917.76	+ 2.49	
			1800	69 45 29.44	919.40	2.03	916.83	2.57	
		3.7	1850	69 53 8.87	918.31	2.31	915.65	2.66	
			1900	70 0 47.73	917.09	2.60	914.32	2.77	
886	*g* Sagittarii . . .	6.0	1755	— 16 7 13.67	+ 870.94	+ 44.42	+ 879.85	— 8.91	
		5.3	1850	15 53 6.33	912.83	43.77	921.75	8.92	
887	A Sagittarii . . .	5.5	1755	— 26 50 6.92	+ 881.34	+ 47.87	+ 879.53	+ 1.81	
		5.3	1850	26 35 48.16	926.44	47.08	924.59	1.85	
888	*c* Sagittarii . . .	4.5	1755	— 28 22 4.41	+ 909.03	+ 47.99	+ 907.44	+ 1.59	
		4.7	1850	28 7 19.31	954.22	47.15	952.60	1.62	

RIGHT ASCENSIONS.

No.	Star.	Epoch.	Number of observations.	Right ascension.			Centennial variation.	Secular variation.	Struve's precession.	Proper motion.	Sec. var. of proper motion.
				h.	m.	s.	s.	s.	s.	s.	s.
889	63 Sagittarii . . .	1755	5	19	48	14.009	+ 337.467	− 0.766	+ 337.350	+ 0.117	
		1850	15	19	53	34.253	336.727	0.792	336.608	0.119	
890	τ Aquilæ	1755	5	19	52	9.886	+ 293.575	− 0.198	+ 293.326	+ 0.249	
		1850	63	19	56	48.692	293.385	0.201	293.137	0.248	
		1900	. .	19	59	15.360	293.285	0.201	293.041	0.244	
891	65 Sagittarii . . .	1755	5	19	51	48.016	+ 334.812	− 0.747	+ 335.011	− 0.199	
		1850	5	19	57	5.747	334.092	0.769	334.284	0.192	
892	ξ^1 Capricorni . . .	1755	5	19	58	22.156	+ 333.837	− 0.781	+ 334.062	− 0.225	
		1850	12	20	3	38.944	333.082	0.810	333.309	0.227	
893	ξ^2 Capricorni . . .	1755	5	19	58	45.700	+ 335.641	− 0.774	+ 334.497	+ 1.144	
		1850	12	20	4	4.207	334.896	0.794	333.741	1.155	
894	3 Capricorni . . .	1755	5	20	2	47.733	+ 333.619	− 0.806	+ 333.694	− 0.075	
		1850	5	20	8	4.304	332.845	0.824	332.915	0.070	
895	4 Capricorni . . .	1755	5	20	3	35.883	+ 354.715	− 1.232	+ 354.644	+ 0.071	
		1850	15	20	9	12.301	353.530	1.263	353.456	0.074	
896	α^1 Capricorni . . .	1755	9	20	4	2.904	+ 334.010	− 0.823	+ 333.982	+ 0.028	
		1850	114	20	9	19.839	333.221	0.839	333.193	0.028	
897	ι^2 Capricorni . . .	1755	9	20	4	26.473	+ 334.347	− 0.827	+ 334.037	+ 0.310	
		1850	. .	20	9	43.726	333.554	0.844	333.242	0.312	
		1900	. .	20	12	30.397	333.130	0.853	332.820	0.310	
898	σ Capricorni . . .	1755	5	20	5	13.697	+ 348.244	− 1.116	+ 348.285	− 0.041	
		1850	29	20	10	44.022	347.172	1.140	347.211	0.039	
899	ν Capricorni . . .	1755	5	20	7	3.208	+ 334.227	− 0.855	+ 334.322	− 0.095	
		1850	8	20	12	20.337	333.411	0.864	333.509	0.098	
900	B. A. C. 6992 . .	1755	4	20	6	59.277	+ 338.743	− 0.932	+ 338.624	+ 0.119	
		1850	12	20	12	20.660	337.849	0.951	337.727	0.122	
901	β Capricorni . . .	1755	5	20	7	13.359	+ 338.788	− 0.937	+ 338.602	+ 0.186	
		1850	159	20	12	34.782	337.888	0.957	337.702	0.186	
902	λ Ursæ Minoris . .	1755	. .	21	17	40.87	−3033.51	−1806.85	−3030.19	− 3.32	
		1775	. .	21	6	56.11	3419.71	2055.82	3415.94	3.77	
		1800	. .	20	51	33.20	3975.33	2388.14	3970.91	4.42	
		1825	. .	20	33	41.18	4612.77	2702.72	4607.60	5.17	
		1850	. .	20	13	0.78	5320.43	2936.22	5314.42	6.01	
		1875	. .	19	49	18.01	6064.97	2975.83	6058.09	6.88	
		1900	. .	19	22	31.37	−6781.07	2689.28	−6773.33	− 7.74	
903	α Pavonis . . .	1850	. .	20	13	45.15	+ 480 5	− 5.89	+ 480.38	+ 0.37	
		1875	. .	20	15	45.16	479 27	5.92	478.90	0.37	
		1900	. .	20	17	44.79	477.78	5.96	477.41	0.37	
904	κ Cephei	1755	. .	20	16	39.92	− 170.68	−15.56	− 170.95	+ 0.27	
		1800	. .	20	15	21.50	177.77	15.90	178.03	0.26	
		1850	. .	20	13	50.60	185.84	16.35	186.09	0.25	
		1900	. .	20	12	15.63	194.08	16.77	194.35	0.27	

902. The reductions to past epochs are somewhat uncertain.

DECLINATIONS.

No.	Star.	Mag.	Epoch.	Declination.	Centennial variation.	Secular variation.	Struve's precession.	Proper motion.	Sec. var. of proper motion.
				° ′ ″	″	″	″	″	″
889	63 Sagittarii . . .	6.0	1755	− 14 17 42.61	+ 914.30	+ 43.46	+ 912.73	+ 1.57	
		5.7	1850	14 2 54.51	955.28	42.82	953.70	1.58	
890	τ Aquilæ	5.5	1755	+ 6 36 16.00	+ 943.96	+ 37.45	+ 943.24	+ 0.72	+ 0.03
		5.9	1850	6 51 29.59	979.31	36.97	978.56	0.75	
			1900	6 59 43.86	997.73	36.71	996.96	0.77	
891	65 Sagittarii . . .	6.0	1755	− 13 20 13.43	+ 935.37	+ 42.72	+ 940.43	− 5.06	
		6.7	1850	13 5 5.66	975.64	42.07	980.72	5.08	
892	ξ^1 Capricorni . . .	6.5	1755	− 13 5 59.74	+ 988.67	+ 41.89	+ 990.84	− 2.17	
		6.8	1850	12 50 1.70	1028.15	41.22	1030.36	2.21	
893	ξ^2 Capricorni . . .	6.0	1755	− 13 18 54.88	+ 974.84	+ 42.16	+ 993.82	−18.98	
		6.3	1850	13 3 9.86	1014.61	41.57	1033.52	18.91	
894	3 Capricorni . . .	6.5	1755	− 13 4 3.74	+ 1023.83	+ 41.40	+ 1024.35	− 0.52	
		6.8	1850	12 47 32.52	1062.83	40.70	1063.37	0.54	
895	4 Capricorni . . .	6.0	1755	− 22 32 42.48	+ 1026.61	+ 43.97	+ 1030.37	− 3.76	
		6.1	1850	22 16 7.48	1067.99	43.14	1071.76	3.77	
896	α^1 Capricorni . . .	4.0	1755	− 13 14 44.83	+ 1034.11	+ 41.32	+ 1033.76	+ 0.35	
		4.5	1850	12 58 3.89	1073.03	40.62	1072.67	0.36	
897	α^2 Capricorni . . .	3.0	1755	− 13 17 14.05	+ 1036.75	+ 41.31	+ 1036.73	+ 0.02	+ 0.01
		3.6	1850	13 0 20.61	1075.65	40.58	1075.62	0.03	
			1900	12 51 17.73	1095.85	40.20	1095.82	0.03	
898	σ Capricorni . . .	5.5	1755	− 19 51 45.81	+ 1041.73	+ 42.95	+ 1042.60	− 0.87	
		5.6	1850	19 34 56.91	1082.16	42.16	1082.99	0.83	
899	ν Capricorni . .	5.0	1755	− 13 30 37.71	+ 1055.45	+ 40.99	+ 1056.22	− 0.77	
		5.2	1850	13 13 36.55	1094.05	40.27	1094.84	0.79	
900	B. A. C. 6992 . .	7.0	1755	− 15 32 14.63	+ 1055.33	+ 41.63	+ 1055.73	− 0.40	
		6.7	1850	15 15 13.39	1094.50	40.83	1094.88	0.38	
901	β Capricorni . . .	3.5	1755	− 15 32 6.54	+ 1057.08	+ 41.62	+ 1057.45	− 0.37	
		3.2	1850	15 15 3.57	1096.32	40.93	1096.60	0.28	
902	λ Ursæ Minoris . .	. .	1755	+ 88 30 23.06	+ 1525.75	−288.84	+ 1523.91	+ 1.84	
			1775	88 35 22.10	1462.77	343.02	1461.00	1.77	
			1800	88 41 16.25	1366.95	425.97	1365.29	1.66	
			1825	88 46 43.64	1248.05	528.34	1246.52	1.53	
		6.3	1850	88 51 37.91	1101.13	650.27	1099.77	1.36	
			1875	88 55 51.48	921.72	787.33	920.56	1.16	
			1900	+ 88 59 15.81	+ 707.44	−927.16	+ 706.52	+ 0.92	
903	α Pavonis . . .	2.1	1850	− 57 12 34.96	− 1096.09	+ 58.09	+ 1105.16	− 9.07	
			1875	57 7 59.13	1110.55	57.57	1119.62	9.07	
			1900	57 3 19.70	1124.88	57.04	1133.94	9.06	
904	κ Cephei	4.5	1755	+ 76 57 42.66	+ 1129.61	− 21.05	+ 1126.82	+ 2.79	
		. .	1800	77 6 8.82	1119.94	21.99	1117.13	2.81	
		4.3	1850	77 15 25.99	1108.66	23.06	1105.84	2.82	
			1900	77 24 37.40	1096.87	24.16	1094.03	2.84	

RIGHT ASCENSIONS.

No.	Star.	Epoch.	Number of observations.	Right ascension.			Centennial variation.	Secular variation.	Struve's precession.	Proper motion.	Sec. var. of proper motion.
				h.	m.	s.	s.	s.	s.	s.	s.
905	γ Cygni	1755	5	20	13	26.352	+ 215.127	+ 0.174	+ 214.949	+ 0.178	
		1850	87	20	16	50.803	215.298	0.186	215.119	0.179	
		1900	. .	20	18	38.475	215.392	0.191	215.212	0.180	
906	π Capricorni . . .	1755	5	20	13	16.138	+ 345.485	− 1.121	+ 345.476	+ 0.009	
		1850	117	20	18	43.840	344.409	1.145	344.390	0.019	
		1900	. .	20	21	35.901	343.834	1.155	343.814	0.020	
907	ρ Capricorni . . .	1755	5	20	14	51.397	+ 344.313	− 1.117	+ 344.451	− 0.138	
		1850	209	20	20	17.987	343.243	1.136	343.377	0.134	
908	B. A. C. 7043 . .	1850	. .	20	20	26.2	. . . .	− 1.120	+ 342.560	. . .	
909	B. A. C. 7044 . .	1755	1	20	14	59.470	+ 344.616	− 1.106	+ 344.553	+ 0.063	
		1850	13	20	20	26.353	343.556	1.125	343.484	0.072	
910	B. A. C. 7049 . .	1755	. .	20	15	6.754	+ 354.476	− 1.387	+ 354.609	− 0.133	
		1850	14	20	20	42.881	353.161	1.382	353.308	0.147	
911	o Capricorni . . .	1755	5	20	15	49.466	+ 346.013	− 1.155	+ 346.015	− 0.002	
		1850	6	20	21	17.654	344.906	1.175	344.901	+ 0.005	
912	B. A. C. 7063 . .	1850	. .	20	22	42.	. . . .	− 1.019	+ 337.401	. . .	
913	B. A. C. 7077 . .	1755	. .	20	18	14.420	+ 360.336	− 1.532	+ 360.127	+ 0.209	
		1850	18	20	23	56.047	358.877	1.545	358.659	0.218	
914	B. A. C. 7087 . .	1850	5	20	25	50.229	+ 334.433	− 0.979	+ 334.454	− 0.021	
915	ε Delphini . . .	1755	5	20	21	30.305	+ 286.894	− 0.134	+ 286.809	+ 0.085	
		1850	284	20	26	2.794	286.769	0.128	286.683	0.086	
		1900	. .	20	28	26.163	286.707	0.122	286.620	0.087	
916	τ¹ Capricorni . . .	1755	5	20	23	35.101	+ 338.539	− 1.030	+ 338.025	+ 0.514	
		1850	8	20	28	56.244	337.546	1.060	337.028	0.518	
917	Groombridge 3241	1755	. .	20	30	52.39	− 13.23	− 6.28	− 12.95	− 0.28	
		1800	. .	20	30	45.80	16.09	6.43	15.81	0.28	
		1850	. .	20	30	36.94	19.35	6.61	19.07	0.28	
		1900	. .	20	30	26.42	22.70	6.79	22.42	0.28	
918	τ² Capricorni . . .	1755	5	20	25	32.685	+ 337.470	− 1.041	+ 337.459	+ 0.011	
		1850	35	20	30	52.809	336.476	1.052	336.463	0.013	
919	ν Capricorni . . .	1755	5	20	26	4.298	+ 343.766	− 1.205	+ 343.977	− 0.211	
		1850	45	20	31	30.330	342.616	1.217	342.825	0.209	
920	α Cygni	1755	⊙	20	33	5.146	+ 204.153	+ 0.198	+ 204.109	+ 0.044	
		1850	. .	20	36	19.182	204.349	0.214	204.299	0.050	
		1900	. .	20	38	1.384	204.458	0.224	204.406	0.052	
921	ψ Capricorni . . .	1755	5	20	31	32.739	+ 358.304	− 1.641	+ 358.788	− 0.484	
		1850	43	20	37	12.384	356.736	1.659	357.209	0.473	
922	17 Capricorni . . .	1755	5	20	31	55.542	+ 350.496	− 1.443	+ 350.425	+ 0.071	
		1850	15	20	37	27.861	349.123	1.448	349.057	0.066	
923	B. A. C. 7237 . .	1755	. .	20	38	36.516	+ 354.806	− 1.587	+ 354.329	+ 0.477	
		1850	12	20	44	12.862	353.284	1.617	352.790	0.494	

DECLINATIONS.

No.	Star.	Mag.	Epoch.	Declination.	Centennial variation.	Secular variation.	Struve's precession.	Proper motion.	Sec. var. of proper motion.
				° ′ ″	″	″	″	″	″
905	γ Cygni	3.0	1755	+ 39 29 5.18	+ 1102.65	+ 25.73	+ 1103.34	− 0.69	0.00
		2.3	1850	39 46 44.28	1126.97	25.46	1127.66	0.69	
			1900	39 56 10.94	1139.67	25.32	1140.36	0.69	
906	π Capricorni . . .	5.0	1755	− 18 59 41.81	+ 1100.35	+ 41.67	+ 1102.08	− 1.73	0.00
		5.5	1850	18 41 57.81	1139.53	40.82	1141.26	1.73	
			1900	18 32 22.96	1159.83	40.38	1161.56	1.73	
907	ρ Capricorni . . .	6.0	1755	− 18 36 14.47	+ 1111.70	+ 41.29	+ 1113.69	− 1.99	
		5.3	1850	18 18 19.86	1150.53	40.47	1152.54	2.01	
908	B. A. C. 7043 . .	6.7	1850	− 17 55 32.0	. . . .	+ 40.31	+ 1153.52	. . .	
909	B. A. C. 7044 . .	7.5	1755	− 18 39 31.83	+ 1099.85	+ 41.33	+ 1114.68	−14.83	
		7.0	1850	18 21 48.45	1138.72	40.51	1153.55	14.83	
910	B. A. C. 7049 . .	. .	1755	− 23 10 59.00	+ 1111.14	+ 42.49	+ 1115.54	− 4.40	
		6.5	1850	22 53 4.38	1151.08	41.58	1155.50	4.42	
911	o Capricorni . . .	6.0	1755	− 19 22 27.34	+ 1112.41	+ 41.39	+ 1120.72	− 8.31	
		6.2	1850	19 4 32.00	1151.33	40.56	1159.65	8.32	
912	B. A. C. 7063 . .	6.4	1850	− 15 33	. . . .	+ 39.48	+ 1169.66	. . .	
913	B. A. C. 7077 . .	. .	1755	− 25 44 56.14	+ 1128.73	+ 42.51	+ 1138.42	− 9.69	
		6.4	1850	25 26 44.81	1168.66	41.56	1178.40	9.74	
914	B. A. C. 7087 . .	6.3	1850	− 14 13 56.56	+ 1197.06	+ 38.72	+ 1191.88	+ 5.18	
915	ε Delphini . . .	4.0	1755	+ 10 29 11.78	+ 1159.46	+ 33.61	+ 1161.64	− 2.18	− 0.01
		4.0	1850	10 47 48.35	1191.13	33.08	1193.32	2.19	
			1900	10 57 48.04	1207.60	32.80	1209.79	2.19	
916	τ^1 Capricorni . . .	6.0	1755	− 15 58 38.63	+ 1172.34	+ 39.56	+ 1176.43	− 4.09	
		7.0	1850	15 39 47.21	1209.53	38.75	1213.56	4.03	
917	Groombridge 3241	6.0	1755	+ 71 42 0.00	+ 1225.59	− 2.09	+ 1227.51	− 1.92	
			1800	71 51 11.29	1224.58	2.42	1226.52	1.94	
		6.0	1850	72 1 23.26	1223.27	2.80	1225.22	1.95	
			1900	72 11 34.53	1221.78	3.18	1223.74	1.96	
918	τ^2 Capricorni . . .	6.0	1755	− 15 47 43.01	+ 1187.39	+ 39.17	+ 1190.27	− 2.88	
		5.6	1850	15 28 37.45	1224.18	38.28	1227.05	2.87	
919	υ Capricorni . . .	5.0	1755	− 18 58 58.49	+ 1193.83	+ 39.77	+ 1193.99	− 0.16	
		5.7	1850	18 39 46.55	1231.19	38.90	1231.37	0.18	
920	α Cygni	1.0	1755	+ 44 24 57.28	+ 1241.92	+ 22.87	+ 1242.77	− 0.85	+ 0.01
		1.7	1850	44 44 47.38	1263.52	22.60	1264.36	0.84	
			1900	44 55 21.96	1274.78	22.46	1275.62	0.84	
921	ψ Capricorni . . .	4.5	1755	− 26 7 54.78	+ 1215.99	+ 40.67	+ 1232.16	−16.17	
		4.3	1850	25 48 21.39	1254.15	39.67	1270.37	16.22	
922	17 Capricorni . . .	6.0	1755	− 22 23 7.66	+ 1231.90	+ 39.90	+ 1234.76	− 2.86	
		6.0	1850	22 3 19.49	1269.38	39.00	1272.12	2.74	
923	B. A. C. 7237 . .	. .	1755	− 24 40 54.23	+ 1271.48	+ 39.50	+ 1280.27	− 8.79	
		6.9	1850	24 20 28.66	1308.49	38.43	1317.24	8.75	

17

RIGHT ASCENSIONS.

No.	Star.	Epoch.	Number of observations.	Right ascension.			Centennial variation.	Secular variation.	Struve's precession.	Proper motion.	Sec. var. of proper motion.
				h.	m.	s.	s.	s.	s.	s.	s.
924	μ Aquarii	1755	5	20	39	25.136	+ 325.088	— 0.820	+ 324.881	+ 0.207	
		1850	163	20	44	33.598	324.307	0.825	324.106	0.201	
		1900	. .	20	47	15.648	323.895	0.824	323.692	0.203	
925	19 Capricorni . . .	1755	5	20	40	55.262	+ 341.322	— 1.278	+ 341.832	— 0.510	
		1850	13	20	46	18.940	340.107	1.281	340.620	0.513	
926	7 Aquarii	1755	5	20	43	38.368	+ 325.833	— 0.872	+ 325.952	— 0.119	
		1850	6	20	48	47.517	325.007	0.869	325.125	0.118	
927	B. A. C. 7263 . .	1850	6	20	49	16.323	+ 337.121	— 1.084	+ 336.661	+ 0.460	
928	Lal. 40522 . . .	1850	. .	20	50	23.6	. . . . .	— 1.108	+ 333.659	. . .	
929	20 Capricorni . . .	1755	5	20	45	38.621	+ 343.559	— 1.356	+ 343.445	+ 0.114	
		1850	13	20	51	4.389	342.270	1.358	342.156	0.114	
930	ν Cygni	1755	4	20	48	3.056	+ 222.930	+ 0.343	+ 222.905	+ 0.025	
		1850	128	20	51	34.998	223.267	0.366	223.246	0.021	
		1900	. .	20	53	26.678	223.453	0.379	223.428	0.025	
931	8 Aquarii	1755	5	20	46	25.388	+ 331.609	— 1.039	+ 331.910	— 0.301	
		1850	3	20	51	39.948	330.624	1.035	330.921	0.297	
932	21 Capricorni . . .	1755	5	20	47	2.441	+ 340.025	— 1.277	+ 340.323	— 0.298	
		1850	19	20	52	24.888	338.811	1.278	339.108	0.297	
933	9 Aquarii	1755	5	20	47	36.645	+ 332.530	— 1.076	+ 332.692	— 0.162	
		1850	5	20	52	52.064	331.509	1.074	331.677	0.168	
934	12 Year Cat. 1879 .	1755	. .	20	57	50.64	— 215.36	—27.53	— 214.21	— 1.15	
		1775	. .	20	57	7.01	220.95	28.07	219.80	1.15	
		1800	. .	20	56	10.89	228.04	28.75	226.89	1.15	
		1825	. .	20	55	12.96	235.33	29.44	234.18	1.15	
		1850	. .	20	54	13.21	242.78	30.15	241.63	1.15	
		1875	. .	20	53	11.55	250.41	30.88	249.26	1.15	
		1900	. .	20	52	7.99	— 258.23	—31.62	— 257.08	— 1.15	
935	η Capricorni . . .	1755	5	20	50	25.490	+ 344.056	— 1.430	+ 344.411	— 0.355	
		1850	39	20	55	51.699	342.698	1.428	343.054	0.356	
936	θ Capricorni . . .	1755	5	20	52	8.580	+ 339.597	— 1.284	+ 339.125	+ 0.472	
		1850	156	20	57	30.617	338.378	1.282	337.908	0.470	
937	B. A. C. 7325 . .	1755	1	20	52	41.879	+ 344.628	— 1.454	+ 344.718	— 0.090	
		1850	3	20	58	8.620	343.247	1.454	343.337	0.090	
938	χ Capricorni . . .	1755	5	20	54	29.125	+ 346.567	— 1.541	+ 346.439	+ 0.128	
		1850	11	20	59	57.670	345.106	1.536	344.985	0.121	
939	61¹ Cygni	1755	2	20	55	56.104	+ 267.779	+ 0.370	+ 232.986	+34.793	+0.029
		1850	483	21	0	10.670	268.152	0.415	233.317	34.835	
		1900	. .	21	2	24.797	268.366	0.442	233.525	34.841	
940	26 Capricorni . . .	1755	3	20	55	15.793	+ 344.395	— 1.470	+ 344.371	+ 0.024	
		1850	3	21	0	42.306	343.000	1.466	342.973	0.027	

DECLINATIONS.

No.	Star.	Mag.	Epoch.	Declination.	Centennial variation.	Secular variation.	Struve's precession.	Proper motion.	Sec. var. of proper motion.
				° ′ ″	″	″	″	″	″
924	μ Aquarii	4.5	1755	− 9 53 7.01	+ 1281.49	+ 35.87	+ 1285.83	− 4.34	+ 0.01
		5.0	1850	9 32 33.53	1315.18	35.06	1319.51	4.33	
			1900	9 21 31.58	1332.60	34.62	1336.93	4.33	
925	19 Capricorni . . .	6.0	1755	− 18 50 3.16	+ 1294.18	+ 37.51	+ 1295.88	− 1.70	
		6.1	1850	18 29 16.91	1329.38	36.57	1331.04	1.66	
926	7 Aquarii	6.0	1755	− 10 37 12.10	+ 1312.69	+ 35.34	+ 1313.97	− 1.28	
		5.9	1850	10 16 9.21	1345.88	34.53	1347.16	1.28	
927	B. A. C. 7263 . .	5.9	1850	− 16 36 18.75	+ 1347.42	+ 35.83	+ 1350.29	− 2.87	
928	Lal. 40522 . . .	6.1	1850	− 15 3 33.2	. . . .	+ 35.24	+ 1357.52	. . .	
929	20 Capricorni . . .	6.0	1755	− 19 58 2.02	+ 1324.69	+ 37.03	+ 1327.18	− 2.49	
		6.3	1850	19 36 47.01	1359.37	36.00	1361.91	2.54	
930	ν Cygni	4.0	1755	+ 40 14 6.01	+ 1341.09	+ 23.59	+ 1342.92	− 1.83	0.00
		4.0	1850	40 35 30.64	1363.34	23.26	1365.17	1.83	
			1900	40 46 55.21	1374.94	23.10	1376.77	1.83	
931	8 Aquarii	6.0	1755	− 13 59 14.42	+ 1331.15	+ 35.57	+ 1332.29	− 1.14	
		6.8	1850	13 37 53.91	1364.53	34.70	1365.70	1.17	
932	21 Capricorni . . .	6.0	1755	− 18 28 10.13	+ 1336.51	+ 36.39	+ 1336.33	+ 0.18	
		6.4	1850	18 6 44.17	1370.64	35.47	1370.50	0.14	
933	9 Aquarii	6.0	1755	− 14 28 15.24	+ 1338.74	+ 35.59	+ 1340.03	− 1.29	
		6.8	1850	14 6 47.51	1372.12	34.72	1373.40	1.28	
934	12 Year Cat. 1879 .	. .	1755	+ 79 37 11.00	+ 1402.46	− 23.15	+ 1405.37	− 2.91	
			1775	79 41 51.02	1397.78	23.81	1400.71	2.93	
			1800	79 47 39.70	1391.74	24.63	1394.70	2.96	
			1825	79 53 26.86	1385.46	25.50	1388.45	2.99	
		5.3	1850	79 59 12.40	1378.98	26.39	1382.00	3.02	
			1875	80 4 56.31	1372.27	27.29	1375.32	3.05	
			1900	+ 80 10 38.52	+ 1365.33	− 28.22	+ 1368.42	− 3.09	
935	η Capricorni . . .	5.0	1755	− 20 48 21.42	+ 1352.39	+ 36.42	+ 1358.30	− 5.91	
		5.1	1850	20 26 40.38	1386.53	35.47	1392.42	5.89	
936	θ Capricorni . . .	5.5	1755	− 18 11 21.26	+ 1361.71	+ 35.76	+ 1369.29	− 7.58	
		4.1	1850	17 49 31.62	1395.23	34.80	1402.70	7.47	
937	B. A. C. 7325 . .	7.0	1755	− 21 8 32.99	+ 1369.39	+ 36.06	+ 1372.87	− 3.48	
		6.9	1850	20 46 35.95	1403.18	35.08	1406.69	3.51	
938	χ Capricorni . . .	5.5	1755	− 22 9 38.96	+ 1377.79	+ 36.08	+ 1384.22	− 6.43	
		5.4	1850	21 47 33.91	1411.65	35.21	1417.96	6.31	
939	61[1] Cygni	5.5	1755	+ 37 33 31.55	+ 1712.64	+ 30.35	+ 1393.41	+319.23	+ 2.94
		5.0	1850	38 0 52.21	1741.31	30.00	1419.30	322.01	2.94
			1900	38 15 26.60	1756.27	29.82	1432.78	323.49	2.94
940	26 Capricorni . . .	7.5	1755	− 21 10 1.89	+ 1388.23	+ 35.66	+ 1389.16	− 0.93	
		7.0	1850	20 47 47.14	1421.63	34.66	1422.62	0.99	

RIGHT ASCENSIONS.

No.	Star.	Epoch.	Number of observations.	Right ascension.			Centennial variation.	Secular variation.	Struve's precession.	Proper motion.	Sec. var. of proper motion.
				h.	m.	s.	s.	s.	s.	s.	s.
941	27 Capricorni . . .	1755	5	20	55	30.128	+ 345.855	− 1.494	+ 345.025	+ 0.830	
		1850	3	21	0	58.018	344.438	1.490	343.609	0.829	
942	ν Aquarii	1755	5	20	56	13.423	+ 328.594	− 0.986	+ 328.040	+ 0.554	
		1850	70	21	1	25.144	327.660	0.979	327.107	0.553	
943	ζ Cygni	1755	5	21	2	31.393	+ 254.456	+ 0.356	+ 254.646	− 0.190	+0.003
		1850	899	21	6	33.291	254.808	0.384	254.994	0.186	
		1900	. .	21	8	40.744	255.004	0.400	255.190	0.186	
944	φ Capricorni . . .	1755	5	21	1	38.953	+ 344.185	− 1.532	+ 344.276	− 0.091	
		1850	17	21	7	5.238	342.733	1.524	342.829	0.096	
945	29 Capricorni . . .	1755	5	21	2	9.324	+ 334.325	− 1.200	+ 334.170	+ 0.155	
		1850	27	21	7	26.392	333.190	1.190	333.034	0.156	
946	14 Aquarii	1755	4	21	3	7.268	+ 323.697	− 0.893	+ 323.812	− 0.115	
		1850	7	21	8	14.378	322.854	0.882	322.972	0.118	
947	30 Capricorni . . .	1755	5	21	4	10.749	+ 338.990	− 1.367	+ 338.956	+ 0.034	
		1850	3	21	9	32.174	337.696	1.357	337.660	0.036	
948	31 Capricorni . . .	1755	3	21	4	30.844	+ 338.399	− 1.340	+ 337.969	+ 0.430	
		1850	3	21	9	51.721	337.131	1.330	336.699	0.432	
949	ι Capricorni . . .	1755	5	21	8	34.254	+ 336.469	− 1.312	+ 336.318	+ 0.151	
		1850	153	21	13	53.309	335.228	1.300	335.075	0.153	
950	B. A. C. 7408 . .	1755	2	21	8	48.312	+ 323.601	− 0.909	+ 323.594	+ 0.007	
		1850	10	21	13	55.325	322.745	0.894	322.745	0.000	
951	17 Aquarii	1755	5	21	9	46.987	+ 323.104	− 0.916	+ 323.516	− 0.412	
		1850	6	21	14	53.526	322.246	0.891	322.661	0.415	
952	α Cephei	1755	3	21	12	42.732	+ 144.524	− 0.627	+ 142.340	+ 2.184	+0.029
		1850	653	21	14	59.742	143.917	0.651	141.703	2.214	
		1900	. .	21	16	11.618	143.588	0.664	141.360	2.228	
953	1 Pegasi	1755	4	21	10	45.846	+ 276.990	+ 0.151	+ 276.400	+ 0.590	−0.002
		1850	64	21	15	9.059	277.147	0.180	276.560	0.587	
		1900	. .	21	17	27.656	277.241	0.195	276.654	0.587	
954	33 Capricorni . . .	1755	5	21	10	13.595	+ 343.061	− 1.560	+ 343.244	− 0.183	
		1850	14	21	15	38.802	341.587	1.543	341.769	0.182	
955	18 Aquarii	1755	5	21	10	46.528	+ 329.907	− 1.095	+ 329.301	+ 0.606	
		1850	8	21	15	59.448	328.874	1.080	328.266	0.608	
956	19 Aquarii	1755	5	21	12	1.825	+ 323.935	− 0.920	+ 324.030	− 0.095	
		1850	7	21	17	9.153	323.076	0.888	323.157	0.081	
957	ζ Capricorni . . .	1755	5	21	12	38.070	+ 345.690	− 1.680	+ 345.715	− 0.025	
		1850	41	21	18	5.719	344.101	1.667	344.123	0.022	
958	35 Capricorni . . .	1755	5	21	13	18.871	+ 343.115	− 1.602	+ 343.365	− 0.250	
		1850	10	21	18	44.110	341.600	1.587	341.852	0.252	

DECLINATIONS.

No.	Star.	Mag.	Epoch.	Declination.	Centennial variation.	Secular variation.	Struve's precession.	Proper motion.	Sec. var. of proper motion.
				° ′ ″	″	″	″	″	″
941	27 Capricorni . . .	7.5	1755	− 21 31 21.63	+ 1375.28	+ 35.94	+ 1390.62	−15.34	
		6.5	1850	21 9 19.06	1408.95	34.95	1424.17	15.22	
942	ν Aquarii	5.0	1755	− 12 20 51.82	+ 1393.55	+ 33.91	+ 1395.21	− 1.66	
		4.7	1850	11 58 32.78	1425.35	33.03	1426.94	1.59	
943	ζ Cygni	4.0	1755	+ 29 14 3.01	+ 1427.58	+ 25.36	+ 1434.30	− 6.72	− 0.01
		3.0	1850	29 36 50.58	1451.45	24.89	1458.18	6.73	
			1900	29 48 59.40	1463.83	24.65	1470.56	6.73	
944	φ Capricorni . . .	6.0	1755	− 21 39 7.08	+ 1427.33	+ 34.66	+ 1428.95	− 1.62	
		5.5	1850	21 16 15.65	1459.77	33.65	1461.37	1.60	
945	29 Capricorni . . .	5.0	1755	− 16 10 24.51	+ 1431.78	+ 33.56	+ 1432.06	− 0.28	
		5.7	1850	15 47 29.31	1463.22	32.62	1463.48	0.26	
946	14 Aquarii	7.5	1755	− 10 13 10.45	+ 1436.74	+ 32.30	+ 1437.97	− 1.23	
		6.6	1850	9 50 11.10	1467.02	31.44	1468.29	1.27	
947	30 Capricorni . . .	6.0	1755	− 18 59 44.45	+ 1443.25	+ 33.71	+ 1444.41	− 1.16	
		5.5	1850	18 36 38.31	1474.80	32.72	1475.97	1.17	
948	31 Capricorni . . .	6.5	1755	− 18 28 28.01	+ 1447.76	+ 33.64	+ 1446.44	+ 1.32	
		6.7	1850	18 5 17.62	1479.25	32.66	1477.90	1.35	
949	ι Capricorni . . .	5.0	1755	− 17 51 44.69	+ 1471.20	+ 32.77	+ 1470.84	+ 0.36	
		4.4	1850	17 28 12.42	1501.87	31.79	1501.50	0.37	
950	B. A. C. 7408 . .	7.0	1755	. . . . .	. . . .	+ 31.44	+ 1472.24	. . .	
		6.9	1850	− 9 57 44.0	. . . .	30.56	1501.69	. . .	
951	17 Aquarii	6.0	1755	− 10 20 55.53	+ 1475.10	+ 31.20	+ 1478.07	− 2.97	
		6.2	1850	9 57 20.24	1504.32	30.32	1507.31	2.99	
952	α Cephei	3.0	1755	+ 61 33 14.23	+ 1499.19	+ 13.63	+ 1495.25	+ 3.94	+ 0.20
		2.7	1850	61 57 4.58	1512.03	13.40	1507.90	4.13	
			1900	62 9 42.26	1518.69	13.28	1514.46	4.23	
953	1 Pegasi	4.0	1755	+ 18 46 8.78	+ 1488.82	+ 26.67	+ 1483.81	+ 5.01	+ 0.08
		4.3	1850	19 9 55.08	1513.85	26.09	1508.76	5.09	
			1900	19 22 35.27	1526.81	25.77	1521.68	5.13	
954	33 Capricorni . . .	6.0	1755	− 21 52 38.81	+ 1466.53	+ 33.14	+ 1480.67	−14.14	
		5.7	1850	21 29 10.81	1497.52	32.08	1511.64	14.12	
955	18 Aquarii	6.0	1755	− 13 54 50.38	+ 1483.06	+ 31.82	+ 1483.90	− 0.84	
		5.7	1850	13 31 7.26	1512.84	30.88	1513.64	0.80	
956	19 Aquarii	6.0	1755	− 10 46 37.25	+ 1474.23	+ 30.97	+ 1491.27	−17.04	
		5.8	1850	10 23 2.89	1503.23	30.08	1520.27	17.04	
957	ζ Capricorni . . .	4.0	1755	− 23 27 22.97	+ 1495.08	+ 33.00	+ 1494.80	+ 0.28	
		3.7	1850	23 3 27.92	1525.92	31.93	1525.64	0.28	
958	35 Capricorni . . .	6.0	1755	− 22 14 25.10	+ 1494.28	+ 32.70	+ 1498.76	− 4.48	
		6.2	1850	21 50 30.94	1524.82	31.62	1529.27	4.45	

RIGHT ASCENSIONS.

No.	Star.	Epoch.	Number of observations.	Right ascension.			Centennial variation.	Secular variation.	Struve's precession.	Proper motion.	Sec. var. of proper motion.
				h.	m.	s.	s.	s.	s.	s.	s.
959	*b* Capricorni . . .	1755	5	21	14	42.640	+ 345.136	— 1.649	+ 344.231	+ 0.905	
		1850	9	21	20	9.777	343.576	1.634	342.670	0.906	
960	β Aquarii	1755	5	21	18	38.612	+ 317.147	— 0.739	+ 317.069	+ 0.078	
		1850	874	21	23	39.573	316.459	0.710	316.377	0.082	
		1900	. .	21	26	17.714	316.104	0.708	316.021	0.083	
961	37 Capricorni . . .	1755	5	21	21	2.977	+ 339.947	— 1.562	+ 340.099	— 0.152	
		1850	8	21	26	25.226	338.480	1.526	338.639	0.159	
962	38 Capricorni . . .	1755	2	21	21	5.468	+ 340.725	— 1.551	+ 340.370	+ 0.355	
		1850	5	21	26	28.460	339.261	1.532	338.903	0.358	
963	β Cephei	1755	5	21	25	24.07	+ 83.86	— 3.17	+ 83.68	+ 0.18	
		1800	. .	21	26	1.48	82.41	3.27	82.23	0.18	
		1850	. .	21	26	42.28	80.75	3.38	80.57	0.18	
		1900	. .	21	27	22.23	79.04	3.47	78.85	0.19	
964	ε Capricorni . . .	1755	5	21	23	19.482	+ 338.666	— 1.511	+ 338.671	— 0.005	
		1850	34	21	28	40.535	337.238	1.495	337.246	0.008	
965	ξ Aquarii	1755	5	21	24	41.321	+ 320.914	— 0.848	+ 320.191	+ 0.723	0.000
		1850	231	21	29	45.810	320.119	0.826	319.392	0.727	
		1900	. .	21	32	25.767	319.709	0.814	318.982	0.727	
966	γ Capricorni . . .	1755	5	21	26	29.022	+ 334.792	— 1.343	+ 333.593	+ 1.199	
		1850	83	21	31	46.471	333.527	1.322	332.329	1.198	
967	42 Capricorni . . .	1755	5	21	28	11.847	+ 328.318	— 1.149	+ 329.234	— 0.916	
		1850	9	21	33	23.233	327.237	1.127	328.137	0.900	
968	κ Capricorni . . .	1755	5	21	28	56.439	+ 337.637	— 1.483	+ 336.765	+ 0.872	
		1850	21	21	34	16.528	336.238	1.463	335.368	0.870	
969	B. A. C. 7550 . .	1850	10	21	34	49.642	+ 337.311	— 1.505	+ 336.460	+ 0.851	
970	44 Capricorni . . .	1755	5	21	29	40.600	+ 329.465	— 1.203	+ 329.630	— 0.165	
		1850	9	21	34	53.052	328.333	1.179	328.500	0.167	
971	45 Capricorni . . .	1755	5	21	30	36.409	+ 329.804	— 1.216	+ 330.043	— 0.239	
		1850	8	21	35	49.177	328.660	1.193	328.901	0.241	
972	B. A. C. 7558 . .	1755	. .	. .	. .	. .	. . . .	— 1.304	+ 331.898	. . .	
		1850	. .	21	36	4.1	. . . .	1.270	330.667	. . .	
973	ε Pegasi	1755	5	21	32	9.163	+ 294.766	— 0.093	+ 294.599	+ 0.167	
		1850	882	21	36	49.153	294.692	0.062	294.523	0.169	
		1900	. .	21	39	16.492	294.665	0.046	294.498	0.167	
974	B. A. C. 7562 . .	1755	. .	21	31	49.568	+ 321.896	— 0.909	+ 321.418	+ 0.478	
		1850	6	21	36	54.963	321.044	0.886	320.572	0.472	
975	c^1 Capricorni . . .	1755	5	21	31	55.216	+ 321.429	— 0.912	+ 321.481	— 0.052	
		1850	12	21	37	0.166	320.574	0.889	320.627	0.053	
976	c^2 Capricorni . . .	1755	5	21	33	10.814	+ 321.576	— 0.920	+ 321.660	— 0.084	
		1850	5	21	38	15.900	320.714	0.894	320.798	0.084	

DECLINATIONS.

No.	Star.	Mag.	Epoch.	Declination.	Centennial variation.	Secular variation.	Struve's precession.	Proper motion.	Sec. var. of proper motion.
				° ′ ″	″	″	″	″	″
959	*b* Capricorni . . .	5.5	1755	− 22 51 29.81	+ 1505.11	+ 32.68	+ 1506.86	− 1.75	
		4.7	1850	22 27 25.37	1535.65	31.62	1537.34	1.69	
960	β Aquarii	3.0	1755	− 6 38 6.16	+ 1527.69	+ 29.33	+ 1529.36	− 1.67	+ 0.02
		2.6	1850	6 13 41.75	1555.14	28.46	1556.79	1.65	
			1900	6 0 40.64	1569.26	28.00	1570.90	1.64	
961	37 Capricorni . . .	7.0	1755	− 21 9 41.52	+ 1545.46	+ 31.02	+ 1542.93	+ 2.53	
		6.0	1850	20 44 59.49	1574.44	29.98	1571.92	2.52	
962	38 Capricorni . . .	7.0	1755	− 21 19 26.16	+ 1537.40	+ 31.14	+ 1543.16	− 5.76	
		6.9	1850	20 54 51.74	1566.48	30.09	1572.21	5.73	
963	β Cephei	3.0	1755	+ 69 29 19.06	+ 1566.51	+ 6.98	+ 1567.00	− 0.49	
			1800	69 41 4.70	1569.64	6.82	1570.12	0.48	
		3.0	1850	69 54 10.36	1572.99	6.64	1573.46	0.47	
			1900	70 7 17.67	1576.26	6.46	1576.74	0.48	
964	ε Capricorni . . .	5.0	1755	− 20 32 56.40	+ 1553.86	+ 30.61	+ 1555 58	− 1.72	
		4.7	1850	20 8 6.58	1582.45	29.60	1584.10	1.65	
965	ξ Aquarii	5.0	1755	− 8 56 21.21	+ 1559.51	+ 28.73	+ 1563.10	− 3.59	+0.05
		5.0	1850	8 31 26.85	1586.38	27.82	1589.92	3.54	
			1900	8 18 10.20	1600.16	27.34	1603.68	3.52	
966	γ Capricorni . . .	4.0	1755	− 17 45 22.40	+ 1573.99	+ 29.88	+ 1572.88	+ 1.11	
		3.7	1850	17 20 13.78	1601.86	28.82	1600.60	1.26	
967	42 Capricorni . . .	6.0	1755	− 15 7 36.28	+ 1551.85	+ 28.70	+ 1582.20	−30.35	
		5.6	1850	14 42 49.22	1578.65	27.73	1609.05	30.40	
968	κ Capricorni . . .	5.0	1755	− 19 58 7.52	+ 1583.92	+ 29.61	+ 1586.18	− 2.26	
		5.0	1850	19 32 49.59	1611.54	28.54	1613.67	2.13	
969	B. A. C. 7550 . .	6.3	1850	− 20 18 9.71	+ 1614.52	+ 28.50	+ 1616.53	− 2.01	
970	44 Capricorni . . .	6.0	1755	− 15 30 23.28	+ 1592.28	+ 28.61	+ 1590.10	+ 2.18	
		6.1	1850	15 4 57.85	1619.00	27.63	1616.84	2.16	
971	45 Capricorni . . .	6.0	1755	− 15 51 23.90	+ 1589.20	+ 28.48	+ 1595.06	− 5.86	
		6.3	1850	15 26 1.46	1615.79	27.49	1621.68	5.89	
972	B. A. C. 7558 . .	6.0	1755	− 17 4 43.86	+ 1594.18	+ 28.65	+ 1596.21	− 2.03	
		8.0	1850	16 39 16.61	1620.92	27.64	1622.95	2.03	
973	ε Pegasi	2.5	1755	+ 8 45 49.08	+ 1602.60	+ 25.22	+ 1603.23	− 0.63	+ 0.02
		2.3	1850	9 11 22.83	1626.20	24.47	1626.79	0.59	
			1900	9 24 58.98	1638.34	24.09	1638.92	0.58	
974	B. A. C. 7562 . .	7.5	1755	− 10 8 57.73	+ 1601.45	+ 27.64	+ 1601.51	− 0.06	
		5.5	1850	9 43 24.02	1627.27	26.72	1627.32	0.05	
975	c^1 Capricorni . . .	6.0	1755	− 10 11 41.20	+ 1601.44	+ 27.60	+ 1602.00	− 0.56	
		5.5	1850	9 46 7.51	1627.23	26.72	1627.74	0.51	
976	c^2 Capricorni . . .	6.5	1755	− 10 23 35.40	+ 1607.89	+ 27.34	+ 1608.66	− 0.77	
		6.4	1850	9 57 55.71	1633.42	26.41	1634.17	0.75	

RIGHT ASCENSIONS.

No.	Star.	Epoch.	Number of observations.	Right ascension.	Centennial variation.	Secular variation.	Struve's precession.	Proper motion.	Sec. var. of proper motion.
				h. m. s.	s.	s.	s.	s.	s.
977	λ Capricorni . . .	1755	5	21 33 19.327	+ 324.771	— 1.034	+ 324.683	+ 0.088	
		1850	24	21 38 27.397	323.800	1.010	323.713	0.087	
978	50 Capricorni . . .	1755	3	21 33 28.199	+ 325.224	— 1.038	+ 325.140	+ 0.084	
		1850	3	21 38 36.696	324.249	1.014	324.159	0.090	
979	δ Capricorni . . .	1755	5	21 33 29.175	+ 333.484	— 1.295	+ 331.741	+ 1.743	
		1850	159	21 38 45.403	332.265	1.272	330.519	1.746	
980	11 Cephei . . .	1755	1	21 38 14.12	+ 94.33	— 2.99	+ 91.71	+ 2.62	
		1800	. .	21 38 56.26	92.96	3.09	90.32	2.64	
		1850	. .	21 39 42.33	91.39	3.21	88.74	2.65	
		1900	. .	21 40 27.63	89.76	3.33	87.09	2.67	
981	μ Capricorni . . .	1755	5	21 39 54.641	+ 329.139	— 1.167	+ 327.134	+ 2.005	—0.006
		1850	126	21 45 6.801	328.044	1.139	326.040	2.004	
		1900	. .	21 47 50.681	327.479	1.122	325.477	2.002	
982	B. A. C. 7620 . .	1755	. .	. . . . . .	. . . .	— 0.072	+ 322.493	. . .	
		1850	5	21 45 35.0	. . . .	0.945	321.582	. . .	
983	B. A. C. 7650 . .	1755	1	21 45 21.921	+ 315.556	— 0.706	+ 315.539	+ 0.017	
		1850	9	21 50 21.386	314.900	0.676	314.878	0.022	
984	79 Draconis . . .	1755	5	21 49 47.11	+ 78.90	— 4.21	+ 78.08	+ 0.82	
		1800	. .	21 50 22.18	76.97	4.37	76.15	0.82	
		1850	. .	21 51 0.11	74.74	4.56	73.91	0.83	
		1900	. .	21 51 36.90	72.42	4.73	71.57	0.85	
985	29 Aquarii (mean) .	1755	5	21 48 59.089	+ 330.709	— 1.362	+ 330.696	+ 0.013	
		1850	22	21 54 12.652	329.431	1.328	329.416	0.015	
986	30 Aquarii	1755	5	21 50 22.220	+ 316.837	— 0.765	+ 316.682	+ 0.155	
		1850	21	21 55 22.875	316.125	0.734	315.971	0.154	
987	B. A. C. 7680 . .	1755	1	21 51 46.707	+ 314.247	— 0.677	+ 314.483	— 0.236	
		1850	5	21 56 44.941	313.619	0.645	313.850	0.231	
988	α Aquarii	1755	10	21 53 11.492	+ 308.859	— 0.459	+ 308.837	+ 0.022	
		1850	. .	21 58 4.706	308.438	0.426	308.418	0.020	
		1900	. .	22 0 38.872	308.229	0.409	308.206	0.023	
989	B. A. C. 7690 . .	1755	1	21 53 13.540	+ 315.457	— 0.701	+ 315.047	+ 0.410	
		1850	6	21 58 12.912	314.806	0.669	314.397	0.409	
990	ι Aquarii	1755	5	21 53 10.622	+ 326.080	— 1.163	+ 325.895	+ 0.185	
		1850	77	21 58 19.878	324.991	1.130	324.804	0.187	
991	α Gruis	1755	. .	21 52 39.906	+ 386.833	— 4.699	+ 385.721	+ 1.112	+0.004
		1850	86	21 58 45.292	382.414	4.603	381.297	1.118	
		1900	. .	22 1 55.927	380.128	4.546	379.007	1.121	
992	B. A. C. 7704 . .	1755	1	21 54 50.654	+ 315.328	— 0.722	+ 315.545	— 0.217	
		1850	3	21 59 49.894	314.658	0.688	314.874	0.216	
993	35 Aquarii	1755	5	21 55 30.586	+ 331.684	— 1.458	+ 331.761	— 0.077	
		1850	12	22 0 45.033	330.316	1.422	330.392	0.076	

DECLINATIONS.

No.	Star.	Mag.	Epoch.	Declination.	Centennial variation.	Secular variation.	Struve's precession.	Proper motion.	Sec. var. of proper motion.
				° ′ ″	″	″	″	″	″
977	λ Capricorni . . .	5.5	1755	− 12 28 57.60	+ 1607.10	+ 27.66	+ 1609.36	− 2.26	
		5.7	1850	12 3 18.50	1632.92	26.72	1635.14	2.22	
978	50 Capricorni . . .	7.5	1755	− 12 48 25.08	+ 1595.93	+ 27.62	+ 1610.14	−14.21	
		6.9	1850	12 22 56.64	1621.72	26.67	1635.86	14.14	
979	δ Capricorni . . .	3.5	1755	− 17 13 31.37	+ 1578.79	+ 28.51	+ 1610.22	−31.43	
		2.8	1850	16 48 18.80	1605.39	27.48	1636.66	31.27	
980	11 Cephei	4.5	1755	+ 70 11 11.86	+ 1644.06	+ 7.50	+ 1634.68	+ 9.38	
		. .	1800	70 23 32.45	1647.42	7.35	1637.94	9.48	
		5.0	1850	70 37 17.06	1651.04	7.18	1641.44	9.60	
			1900	70 51 3.48	1654.59	7.00	1644.86	9.73	
981	μ Capricorni . . .	5.0	1755	− 14 41 31.42	+ 1642.69	+ 27.04	+ 1643.12	− 0.43	+ 0.17
		5.4	1850	14 15 18.81	1667.90	26.03	1668.17	0.27	
			1900	14 1 21.64	1680.78	25.49	1680.97	0.19	
982	B. A. C. 7620 . .	. .	1755	− 11 27 3.32	+ 1639.32	+ 26.19	+ 1646.04	− 6.72	
		6.5	1850	11 0 54.31	1663.73	25.22	1670.46	6.73	
983	B. A. C. 7650 . .	6.5	1755	− 6 34 24.82	+ 1656.33	+ 24.80	+ 1670.11	−13.78	
		6.5	1850	6 8 0.25	1679.46	23.90	1693.23	13.77	
984	79 Draconis . . .	6.0	1755	+ 72 32 42.93	+ 1694.26	+ 5.53	+ 1691.22	+ 3.04	
		. .	1800	72 45 25.90	1696.72	5.35	1693.65	3.07	
		6.5	1850	72 59 34.92	1699.35	5.15	1696.24	3.11	
			1900	73 13 45.22	1701.86	4.95	1698.71	3.15	
985	29 Aquarii (mean) .	6.0	1755	− 18 8 1.48	+ 1688.39	+ 25.37	+ 1687.52	+ 0.87	
		6.5	1850	17 41 6.22	1712.00	24.33	1711.13	0.87	
986	30 Aquarii	5.5	1755	− 7 41 42.04	+ 1694.08	+ 24.05	+ 1693.97	+ 0.11	
		5.8	1850	7 14 41.95	1716.49	23.13	1716.38	0.11	
987	B. A. C. 7680 . .	8.0	1755	. . . . .	. . . .	+ 23.58	+ 1700.54	. . .	
		8.0	1850	− 5 33 53.7	. . . .	22.67	1722.56	. . .	
988	α Aquarii	3.0	1755	− 1 29 57.77	+ 1705.69	+ 22.95	+ 1707.08	− 1.39	0.00
		2.7	1850	1 2 47.14	1727.08	22.08	1728.47	1.39	
			1900	0 48 20.86	1738.00	21.62	1739.39	1.39	
989	B. A. C. 7690 . .	7.0	1755	. . . . .	. . . .	+ 23.47	+ 1707.24	. . .	
		7.0	1850	− 6 4 57.0	. . . .	22.56	1729.07	. . .	
990	ι Aquarii	4.5	1755	− 15 2 48.49	+ 1700.65	+ 24.28	+ 1707.00	− 6.35	
		4.4	1850	14 35 42.05	1723.24	23.27	1729.60	6.36	
991	α Gruis	. .	1755	− 48 7 59.47	+ 1687.38	+ 29.06	+ 1704.66	−17.28	+ 0.07
		1.9	1850	47 41 3.58	1714.23	27.45	1731.46	17.23	
			1900	47 26 43.07	1727.75	26.61	1744.95	17.20	
992	B. A. C. 7704 . .	7.5	1755	. . . . .	. . . .	+ 23.14	+ 1714.63	. . .	
		7.3	1850	− 6 33 33.6	. . . .	22.22	1736.21	. . .	
993	35 Aquarii	5.5	1755	− 19 42 27.11	+ 1716.93	+ 24.26	+ 1717.66	− 0.73	
		5.9	1850	19 15 5.24	1739.48	23.20	1740.21	0.73	

RIGHT ASCENSIONS.

No.	Star.	Epoch.	Number of observations.	Right ascension.			Centennial variation.	Secular variation.	Struve's precession.	Proper motion.	Sec. var. of proper motion.
				h.	*m.*	*s.*	*s.*	*s.*	*s.*	*s.*	*s.*
994	36 Aquarii	1755	5	21	56	28.567	+ 318.545	− 0.844	+ 318.341	+ 0.204	
		1850	12	22	1	30.809	317.759	0.811	317.551	0.208	
995	B. A. C. 7717 . .	1755	1	21	56	32.693	+ 318.251	− 0.809	+ 317.519	+ 0.732	
		1850	34	22	1	34.671	317.499	0.775	316.768	0.731	
996	e^1 Aquarii	1755	3	21	57	26.278	+ 321.792	− 0.988	+ 321.507	+ 0.285	
		1850	7	22	2	31.540	320.870	0.954	320.586	0.284	
997	B. A. C. 7720 . .	1755	1	21	57	35.452	+ 313.312	− 0.607	+ 313.051	+ 0.261	
		1850	6	22	2	32.828	312.747	0.584	312.487	0.260	
998	e^2 Aquarii	1755	5	21	57	29.899	+ 322.791	− 1.036	+ 322.426	+ 0.365	
		1850	31	22	2	36.089	321.825	0.998	321.467	0.358	
999	B. A. C. 7726 . .	1755	4	21	57	46.738	+ 313.591	− 0.638	+ 313.494	+ 0.097	
		1850	6	22	2	44.366	313.002	0.602	312.910	0.092	
1000	B. A. C. 7740 . .	1755	1	21	59	10.792	+ 322.608	− 0.999	+ 321.625	+ 0.983	
		1850	14	22	4	16.828	321.688	0.938	320.693	0.995	
1001	39 Aquarii	1755	5	21	59	11.373	+ 325.565	− 1.175	+ 325.519	+ 0.046	
		1850	6	22	4	20.134	324.465	1.141	324.418	0.047	
1002	B. A. C. 7744 . .	1755	4	21	59	57.491	+ 313.551	− 0.654	+ 313.929	− 0.378	
		1850	8	22	4	55.074	312.946	0.620	313.328	0.382	
1003	40 Aquarii	1755	4	22	0	18.938	+ 322.326	− 1.042	+ 322.556	− 0.230	
		1850	1	22	5	24.683	321.353	1.007	321.581	0.228	
1004	B. A. C. 7752 . .	1755	1	22	1	4.372	+ 314.282	− 0.642	+ 313.565	+ 0.717	
		1850	9	22	6	2.655	313.689	0.606	312.973	0.716	
1005	42 Aquarii	1755	5	22	3	39.198	+ 323.243	− 1.094	+ 323.258	− 0.015	
		1850	11	22	8	45.791	322.222	1.056	322.242	0.020	
1006	θ Aquarii	1755	5	22	3	53.217	+ 317.967	− 0.809	+ 317.271	+ 0.696	
		1850	409	22	8	54.926	317.216	0.772	316.519	0.697	
		1900	. .	22	11	33.438	316.835	0.752	316.139	0.696	
1007	B. A. C. 7774 . .	1755	. .	22	3	54.815	+ 318.515	− 0.872	+ 318.732	− 0.217	
		1850	3	22	8	57.016	317.704	0.837	317.919	0.215	
1008	44 Aquarii	1755	5	22	4	18.136	+ 314.348	− 0.684	+ 314.483	− 0.135	
		1850	5	22	9	16.463	313.716	0.646	313.853	0.137	
1009	45 Aquarii	1755	5	22	5	50.136	+ 324.037	− 1.116	+ 323.551	+ 0.486	
		1850	19	22	10	57.473	322.995	1.078	322.506	0.489	
1010	ρ Aquarii	1755	5	22	7	17.303	+ 317.110	− 0.806	+ 317.050	+ 0.060	
		1850	46	22	12	18.199	316.362	0.769	316.303	0.059	
1011	B. A. C. 7804 . .	1850	. .	22	15	40.0	. . . .	− 0.725	+ 315.387	. . .	
1012	51 Aquarii	1755	5	22	11	20.294	+ 313.605	− 0.636	+ 313.488	+ 0.117	
		1850	9	22	16	17.938	313.020	0.596	312.903	0.117	

DECLINATIONS.

No.	Star.	Mag.	Epoch.	Declination.	Centennial variation.	Secular variation.	Struve's precession.	Proper motion.	Sec. var. of proper motion.
				° ′ ″	″	″	″	″	″
994	36 Aquarii	7.0	1755	− 9 22 45.88	+ 1726.50	+ 23.13	+ 1722.01	+ 4.49	
		6.3	1850	8 55 15.41	1748.02	22.17	1743.52	4.50	
995	B. A. C. 7717 . .	8.0	1755	. . . . . .	. . . .	+ 23.13	+ 1722.32	. . .	
		6.9	1850	− 8 15 40.1	. . . .	22.18	1743.83	. . .	
996	c^1 Aquarii	6.0	1755	− 12 0 57.10	+ 1730.31	+ 23.20	+ 1726.32	+ 3.99	
		6.8	1850	11 33 22.98	1751.89	22.22	1747.87	4.02	
997	B. A. C. 7720 . .	7.0	1755	− 5 5 2.83	+ 1721.83	+ 22.54	+ 1727.00	− 5.17	
		6.5	1850	4 37 37.06	1742.82	21.64	1747.99	5.17	
998	c^2 Aquarii	6.0	1755	− 12 45 32.70	+ 1726.52	+ 23.34	+ 1726.60	− 0.08	
		5.6	1850	12 18 2.13	1748.22	22.35	1748.24	0.02	
999	B. A. C. 7726 . .	6.5	1755	− 5 27 37.78	+ 1726.47	+ 22.52	+ 1727.85	− 1.38	
		6.3	1850	5 0 7.61	1747.43	21.61	1748.79	1.36	
1000	B. A. C. 7740 . .	7.0	1755	. . . . . .	. . . .	+ 23.01	+ 1734.01	. . .	
		7.0	1850	− 11 48 11.8	. . . .	22.02	1755.38	. . .	
1001	39 Aquarii	7.0	1755	− 15 23 22.78	+ 1730.24	+ 23.15	+ 1734.08	− 3.84	
		6.4	1850	14 55 48.75	1751.75	22.14	1755.59	3.84	
1002	B. A. C. 7744 . .	7.5	1755	− 5 55 7.83	+ 1734.92	+ 22.10	+ 1737.46	− 2.54	
		6.7	1850	5 27 29.82	1755.48	21.19	1758.06	2.58	
1003	40 Aquarii	7.0	1755	− 13 7 36.58	+ 1739.16	+ 22.69	+ 1739.02	+ 0.14	
		7.0	1850	12 39 54.30	1760.25	21.70	1760.14	0.11	
1004	B. A. C. 7752 . .	7.0	1755	. . . . . .	. . . .	+ 22.04	+ 1742.32	. . .	
		6.7	1850	− 5 11 33.3	. . . .	21.12	1762.79	. . .	
1005	42 Aquarii	6.0	1755	− 14 2 33.55	+ 1752.79	+ 22.15	+ 1753.44	− 0.65	
		5.8	1850	13 34 38.55	1773.36	21.16	1774.00	0.64	
1006	θ Aquarii	4.5	1755	− 8 59 35.45	+ 1751.87	+ 21.81	+ 1754.41	− 2.54	+ 0.05
		4.3	1850	8 31 41.48	1772.13	20.85	1774.62	2.49	
		. .	1900	8 16 52.83	1782.43	20.33	1784.90	2.47	
1007	B. A. C. 7774 . .	6.0	1755	− 10 15 2.63	+ 1752.48	+ 21.77	+ 1754.51	− 2.03	
		6.4	1850	9 47 8.10	1772.71	20.81	1774.78	2.07	
1008	44 Aquarii	6.5	1755	− 6 36 4.18	+ 1759.25	+ 21.41	+ 1756.17	+ 3.08	
		6.4	1850	6 8 3.37	1779.16	20.50	1776.08	3.08	
1009	45 Aquarii	6.0	1755	− 14 31 16.55	+ 1761.05	+ 21.86	+ 1762.62	− 1.57	
		6.3	1850	14 3 13.83	1781.34	20.85	1782.89	1.55	
1010	ρ Aquarii	6.0	1755	− 9 2 28.90	+ 1767.88	+ 21.12	+ 1768.57	− 0.69	
		5.6	1850	8 34 20.03	1787.49	20.16	1788.24	0.75	
1011	B. A. C. 7804 . .	6.2	1850	− 7 57 1.1	. . . .	+ 19.58	+ 1801.38	. . .	
1012	51 Aquarii	6.0	1755	− 6 4 1.88	+ 1783.09	+ 20.14	+ 1785.19	− 2.10	
		5.8	1850	5 35 39.01	1801.78	19.22	1803.80	2.02	

RIGHT ASCENSIONS.

No.	Star.	Epoch.	Number of observations.	Right ascension.			Centennial variation.	Secular variation.	Struve's precession.	Proper motion.	Sec. var. of proper motion.
				h.	m.	s.	s.	s.	s.	s.	s.
1013	50 Aquarii	1755	5	22	11	17.974	+ 323.385	— 1.111	+ 323.096	+ 0.289	
		1850	16	22	16	24.697	322.343	1.082	322.050	0.293	
1014	π Aquarii	1755	5	22	12	45.574	+ 306.878	— 0.328	+ 306.845	+ 0.033	
		1850	92	22	17	36.966	306.584	0.292	306.554	0.030	
		1900	. .	22	20	10.223	306.444	0.267	306.416	0.028	
1015	B. A. C. 7818 . .	1755	4	22	13	13.953	+ 328.067	— 1.322	+ 326.494	+ 1.573	
		1850	19	22	18	25.027	326.831	1.279	325.258	1.573	
1016	53 Aquarii	1755	5	22	13	14.501	+ 328.066	— 1.322	+ 326.493	+ 1.573	
		1850	18	22	18	25.575	326.832	1.279	325.259	1.573	
1017	54 Aquarii	1755	5	22	13	39.298	+ 320.527	— 0.979	+ 320.229	+ 0.298	
		1850	3	22	18	43.363	319.617	0.937	319.320	0.297	
1018	B. A. C. 7835 . .	1755	1	22	16	53.535	+ 322.976	— 1.073	+ 321.671	+ 1.305	
		1850	9	22	21	59.884	321.977	1.032	320.674	1.303	
1019	56 Aquarii	1755	5	22	17	7.863	+ 323.573	— 1.168	+ 323.453	+ 0.120	
		1850	14	22	22	14.736	322.480	1.135	322.363	0.117	
1020	σ Aquarii	1755	5	22	17	39.646	+ 319.039	— 0.934	+ 319.194	— 0.155	
		1850	116	22	22	42.319	318.169	0.892	318.326	0.157	
1021	Lal. 43974 . . .	1850	. .	22	23	25.9	. . . .	— 0.564	+ 314.199	. . .	
1022	58 Aquarii	1755	5	22	18	40.658	+ 319.738	— 0.945	+ 319.320	+ 0.418	
		1850	12	22	23	43.989	318.860	0.904	318.441	0.419	
1023	60 Aquarii	1755	5	22	21	24.731	+ 310.000	— 0.438	+ 309.751	+ 0.249	
		1850	6	22	26	19.040	309.604	0.397	309.352	0.252	
1024	η Aquarii	1755	5	22	22	45.649	+ 308.811	— 0.363	+ 308.316	+ 0.495	
		1850	350	22	27	38.862	308.487	0.320	307.990	0.497	
		1900	. .	22	30	13.067	308.333	0.297	307.836	0.497	
1025	226 (B) Cephei . . .	1755	. .	22	27	52.17	+ 112.00	— 2.95	+ 112.17	— 0.14	
		1800	. .	22	28	42.26	110.64	3.09	110.78	0.14	
		1850	. .	22	29	37.18	109.05	3.26	109.19	0.14	
		1900	. .	22	30	31.29	107.38	3.44	107.52	0.14	
1026	κ Aquarii . . .	1755	5	22	25	3.379	+ 311.657	— 0.561	+ 312.167	— 0.510	
		1850	41	22	29	59.207	311.148	0.511	311.654	0.506	
1027	64 Aquarii	1755	4	22	26	21.157	+ 317.292	— 0.873	+ 317.611	— 0.319	
		1850	3	22	31	22.198	316.484	0.827	316.802	0.318	
1028	Lal. 44337 . .	1850	. .	22	33	1.7	. . . .	— 0.475	+ 310.930	. . .	
1029	ζ Pegasi	1755	5	22	29	14.986	+ 298.820	+ 0.168	+ 298.308	+ 0.512	
		1850	722	22	33	58.948	299.002	0.215	298.490	0.512	
		1900	. .	22	36	28.477	299.116	0.242	298,605	0.511	
1030	65 Aquarii	1755	4	22	30	6.543	+ 317.225	— 0.865	+ 317.286	— 0.061	
		1850	6	22	35	7.524	316.425	0.820	316.484	0.059	

DECLINATIONS.

No.	Star.	Mag.	Epoch.	Declination.	Centennial variation.	Secular variation.	Struve's precession.	Proper motion.	Sec. var. of proper motion.
				° ′ ″	″	″	″	″	″
1013	50 Aquarii	6.0	1755	− 14 45 41.18	+ 1785.42	+ 20.81	+ 1784.96	+ 0.46	
		6.1	1850	14 17 15.80	1804.70	19.78	1804.23	0.47	
1014	π Aquarii	5.0	1755	+ 0 8 35.54	+ 1790.32	+ 19.43	+ 1790.79	− 0.47	0.00
		4.9	1850	0 37 4.97	1808.35	18.54	1808.81	0.46	
			1900	0 52 11.45	1817.51	18.08	1817.97	0.46	
1015	B. A. C. 7818 . .	6.5	1755	− 17 58 35.75	+ 1789.96	+ 20.82	+ 1792.64	− 2.68	
		6.7	1850	17 30 6.06	1809.23	19.76	1811.83	2.60	
1016	53 Aquarii	6.5	1755	− 17 58 40.72	+ 1789.99	+ 20.82	+ 1792.68	− 2.69	
		5.8	1850	17 30 11.00	1809.26	19.76	1811.86	2.60	
1017	54 Aquarii	7.5	1755	− 12 27 53.35	+ 1794.13	+ 20.17	+ 1794.29	− 0.16	
		7.0	1850	11 59 19.97	1812.82	19.18	1813.00	0.18	
1018	B. A. C. 7835 . .	6.5	1755	− 14 9 35.53	+ 1805.01	+ 19.77	+ 1806.83	− 1.82	
		6.5	1850	13 40 51.99	1823.30	18.74	1825.05	1.75	
1019	56 Aquarii	6.0	1755	− 15 49 44.83	+ 1803.43	+ 19.71	+ 1807.71	− 4.28	
		6.3	1850	15 21 2.83	1821.68	18.71	1825.94	4.26	
1020	σ Aquarii	5.0	1755	− 11 55 22.76	+ 1806.84	+ 19.32	+ 1809.72	− 2.88	
		5.1	1850	11 26 37.66	1824.76	18.33	1827.61	2.85	
1021	Lal. 43974 . . .	6.2	1850	− 7 18 59.2	. . . .	+ 17.96	+ 1830.22	. . .	
1022	58 Aquarii	6.0	1755	− 12 9 8.21	+ 1809.61	+ 19.20	+ 1813.55	− 3.94	
		6.7	1850	11 40 20.56	1827.38	18.21	1831.30	3.92	
1023	60 Aquarii	6.5	1755	− 2 49 38.08	+ 1819.38	+ 18.09	+ 1823.66	− 4.28	
		6.2	1850	2 20 41.65	1836.13	17.18	1840.39	4.26	
1024	η Aquarii	4.0	1755	− 1 22 20.10	+ 1822.80	+ 17.78	+ 1828.54	− 5.74	+ 0.02
		4.1	1850	0 53 20.55	1839.26	16.87	1844.98	5.72	
			1900	0 37 58.83	1847.58	16.39	1853.29	5.71	
1025	226 (B) Cephei . . .	. .	1755	+ 74 57 57.80	+ 1845.63	+ 5.59	+ 1846.51	− 0.88	
			1800	75 11 48.89	1848.13	5.46	1849.01	0.88	
		5.3	1850	75 27 13.62	1850.80	5.31	1851.69	0.89	
			1900	75 42 39.68	1853.42	5.14	1854.30	0.88	
1026	κ Aquarii	6.0	1755	− 5 29 1.59	+ 1824.55	+ 17.53	+ 1836.71	−12.16	
		5.2	1850	5 0 0.49	1840.76	16.62	1852.93	12.17	
1027	64 Aquarii	6.5	1755	− 11 17 40.62	+ 1840.03	+ 17.58	+ 1841.27	− 1.24	
		6.9	1850	10 48 24.81	1856.26	16.60	1857.53	1.27	
1028	Lal. 44337 . . .	6.3	1850	− 4 19 56.8	. . . .	+ 16.00	+ 1862.95	. . .	
1029	ζ Pegasi	3.0	1755	+ 9 33 34.58	+ 1849.92	+ 16.04	+ 1851.17	− 1.25	+ 0.02
		3.3	1850	10 2 59.11	1864.75	15.19	1866.01	1.26	
			1900	10 18 33.37	1872.23	14.75	1873.51	1.28	
1030	65 Aquarii	7.0	1755	− 11 22 43.05	+ 1855.00	+ 16.89	+ 1854.09	+ 0.91	
		7.0	1850	10 53 13.34	1870.57	15.90	1869.67	0.90	

RIGHT ASCENSIONS.

No.	Star.	Epoch.	Number of observations.	Right ascension.			Centennial variation.	Secular variation.	Struve's precession.	Proper motion.	Sec. var. of proper motion.
				h.	m.	s.	s.	s.	s.	s.	s.
1031	67 Aquarii	1755	2	22	30	25.952	+ 314.209	— 0.693	+ 314.375	— 0.166	
		1850	9	22	35	24.145	313.572	0.647	313.739	0.167	
1032	τ^1 Aquarii	1755	4	22	34	40.859	+ 320.258	— 1.085	+ 320.344	— 0.086	
		1850	6	22	39	44.621	319.251	1.036	319.336	0.085	
1033	B. A. C. 7951 (mean)	1755	5	22	35	11.492	+ 310.160	— 0.550	+ 311.649	— 1.489	
		1850	9	22	40	5.908	309.677	0.467	311.178	1.501	
1034	70 Aquarii	1755	5	22	35	35.215	+ 317.431	— 0.875	+ 317.096	+ 0.335	
		1850	15	22	40	36.389	316.621	0.832	316.285	0.336	
1035	τ^2 Aquarii	1755	5	22	36	35.666	+ 319.532	— 1.054	+ 319.648	— 0.116	
		1850	78	22	41	38.754	318.554	1.004	318.673	0.119	
1036	ι Cephei	1755	5	22	41	1.33	+ 209.31	+ 2.03	+ 210.46	— 1.15	
		1800	. .	22	42	35.72	210.23	2.10	211.39	1.16	
		1850	. .	22	44	21.10	211.31	2.18	212.47	1.16	
		1900	. .	22	46	7.03	212.42	2.27	213.58	1.16	
1037	λ Aquarii	1755	5	22	39	49.055	+ 314.167	— 0.691	+ 314.158	+ 0.009	
		1850	257	22	44	47.209	313.532	0.645	313.523	0.009	
		1900	. .	22	47	23.895	313.216	0.620	313.208	0.008	
1038	Lal. 44734 . . .	1850	. .	22	44	50.5	. . . .	— 0.781	+ 315.435	. . .	
1039	74 Aquarii	1755	5	22	40	33.396	+ 317.516	— 0.919	+ 317.414	+ 0.102	
		1850	9	22	45	34.629	316.668	0.868	316.564	0.104	
1040	75 Aquarii	1755	3	22	41	10.652	+ 317.901	— 0.948	+ 317.812	+ 0.089	
		1850	7	22	46	12.238	317.025	0.897	316.933	0.092	
1041	78 Aquarii	1755	5	22	41	47.826	+ 313.410	— 0.659	+ 313.686	— 0.276	
		1850	13	22	46	45.276	312.809	0.612	313.080	0.271	
1042	1 Piscium . . .	1755	1	22	42	26.944	+ 307.518	— 0.229	+ 307.178	+ 0.340	
		1850	13	22	47	18.992	307.328	0.172	306.980	0.348	
1043	B. A. C. 7986 . .	1850	8	22	47	24.084	+ 311.632	— 0.493	+ 311.397	+ 0.235	
1044	α Piscis Australis .	1755	20	22	44	3.483	+ 335.399	— 2.212	+ 333.055	+ 2.344	—0.009
		1850	. .	22	49	21.123	333.326	2.151	330.992	2.334	
		1900	. .	22	52	7.519	332.262	2.106	329.935	2.327	
1045	Lal. 44872 . . .	1850	. .	22	49	22.4	. . . .	— 0.392	+ 310.045	. . .	
1046	B. A. C. 7993 . .	1755	4	22	44	35.694	+ 311.310	— 0.520	+ 311.625	— 0.315	
		1850	5	22	49	31.211	310.838	0.474	311.153	0.315	
1047	2 Piscium . . .	1755	5	22	46	53.957	+ 307.709	— 0.211	+ 307.244	+ 0.465	
		1850	5	22	51	46.192	307.531	0.164	307.066	0.465	
1048	3 Piscium . . .	1755	5	22	48	4.340	+ 307.501	— 0.249	+ 307.806	— 0.305	
		1850	6	22	52	56.361	307.287	0.201	307.593	0.306	
1049	81 Aquarii	1755	5	22	48	38.884	+ 312.810	— 0.626	+ 313.025	— 0.215	
		1850	16	22	53	35.778	312.239	0.577	312.454	0.215	

DECLINATIONS.

No.	Star.	Mag.	Epoch.	Declination.	Centennial variation.	Secular variation.	Struve's precession.	Proper motion.	Sec. var. of proper motion.
				° ′ ″	″	″	″	″	″
1031	67 Aquarii	6.0	1755	− 8 14 17.37	+ 1855.66	+ 16.67	+ 1855.17	+ 0.49	
		6.4	1850	7 44 47.11	1871.03	15.67	1870.54	0.49	
1032	τ^1 Aquarii	6.0	1755	− 15 20 25.22	+ 1866.87	+ 16.20	+ 1869.01	− 2.14	
		5.8	1850	14 50 44.54	1881.78	15.18	1883.92	2.14	
1033	B. A. C. 7951 (mean)	7.5	1755	− 5 29 38.25	+ 1842.07	+ 15.48	+ 1870.64	−28.57	
		6.7	1850	5 0 21.43	1856.37	14.64	1884.99	28.62	
1034	70 Aquarii	6.0	1755	− 11 50 33.41	+ 1873.63	+ 15.91	+ 1871.89	+ 1.74	
		6.2	1850	11 20 46.43	1888.27	14.93	1886.50	1.77	
1035	τ^2 Aquarii . . .	5.5	1755	− 14 52 42.00	+ 1870.15	+ 15.80	+ 1875.04	− 4.89	
		4.2	1850	14 22 58.39	1884.67	14.78	1889.56	4.89	
1036	ι Cephei	4.0	1755	+ 64 54 58.57	+ 1875.27	+ 9.45	+ 1888.50	−13.23	
		. .	1800	65 9 3.39	1879.49	9.29	1892.75	13.26	
		3.3	1850	65 24 44.28	1884.08	9.12	1897.37	13.29	
			1900	65 40 27.46	1888.60	8.94	1901.91	13.31	
1037	λ Aquarii	4.0	1755	− 8 52 35.71	+ 1888.50	+ 14.88	+ 1884.92	+ 3.58	− 0.01
		3.6	1850	8 22 35.07	1902.17	13.90	1898.60	3.57	
			1900	8 6 42.27	1908.99	13.38	1905.42	3.57	
1038	Lal. 44734 . . .	6.8	1850	− 10 51 17.3	. . . .	+ 14.00	+ 1898.75	. . .	
1039	74 Aquarii	6.0	1755	− 12 54 44.23	+ 1885.70	+ 14.93	+ 1887.11	− 1.41	
		6.0	1850	12 24 46.23	1899.42	13.95	1900.82	1.40	
1040	75 Aquarii	7.5	1755	− 13 29 4.97	+ 1884.75	+ 14.83	+ 1888.95	− 4.20	
		7.0	1850	12 59 7.91	1898.36	13.82	1902.58	4.22	
1041	78 Aquarii	6.0	1755	− 8 30 0.64	+ 1886.42	+ 14.47	+ 1890.78	− 4.36	
		6.4	1850	8 0 2.16	1899.71	13.50	1904.09	4.38	
1042	1 Piscium . . .	6.0	1755	− 0 14 2.76	+ 1891.37	+ 14.10	+ 1892.68	− 1.31	
		6.3	1850	+ 0 16 0.27	1904.33	13.19	1905.62	1.29	
1043	B. A. C. 7986 . .	5.9	1850	− 5 47 8.24	+ 1906.16	+ 13.35	+ 1905.86	+ 0.30	
1044	α Piscis Australis .	1.0	1755	− 30 54 49.81	+ 1880.12	+ 15.19	+ 1897.32	−17.20	+ 0.10
		1.4	1850	30 24 57.02	1893.99	14.02	1911.09	17.10	
			1900	30 9 8.30	1900.85	13.40	1917.91	17.06	
1045	Lal. 44872 . . .	7.0	1850	− 4 2 42.2	. . . .	+ 12.90	+ 1911.15	. . .	
1046	B. A. C. 7993 . .	7.5	1755	− 6 6 48.00	+ 1898.78	+ 13.84	+ 1898.83	− 0.05	
		6.6	1850	5 36 38.06	1911.48	12.89	1911.54	0.06	
1047	2 Piscium . . .	6.5	1755	− 0 20 20.83	+ 1897.26	+ 13.28	+ 1905.26	− 8.00	
		5.4	1850	+ 0 9 47.42	1909.44	12.36	1917.42	7.98	
1048	3 Piscium . . .	6.0	1755	− 1 7 26.05	+ 1910.52	+ 13.07	+ 1908.42	+ 2.10	
		6.4	1850	0 37 5.29	1922.50	12.14	1920.38	2.12	
1049	81 Aquarii	6.0	1755	− 8 22 15.80	+ 1909.81	+ 13.15	+ 1910.00	− 0.19	
		6.6	1850	7 51 55.70	1921.84	12.18	1922.04	0.20	

RIGHT ASCENSIONS.

No.	Star.	Epoch.	Number of observations.	Right ascension.	Centennial variation.	Secular variation.	Struve's precession.	Proper motion.	Sec. var. of proper motion.
				h. m. s.	s.	s.	s.	s.	s.
1050	B. A. C. 8017	1850	. .	22 53 45.7	. . . .	— 0.448	+ 310.858	. . .	
1051	82 Aquarii	1755	5	22 49 48.551	+ 312.544	— 0.592	+ 312.580	— 0.036	
		1850	8	22 54 45.207	312.004	0.544	312.039	0.035	
1052	α Pegasi	1755	10	22 52 34.419	+ 297.777	+ 0.498	+ 297.405	+ 0.372	+0.001
		1850	. .	22 57 17.540	298.276	0.552	297.904	0.372	
		1900	. .	22 59 46.748	298.559	0.581	298.183	0.376	
1053	h^1 Aquarii	1755	5	22 52 22.385	+ 313.935	— 0.639	+ 313.165	+ 0.770	
		1850	18	22 57 20.342	313.349	0.596	312.574	0.775	
1054	h^2 Aquarii	1755	1	22 52 32.744	+ 313.296	— 0.649	+ 313.192	+ 0.104	
		1850	5	22 57 30.090	312.703	0.598	312.598	0.105	
1055	W^2 22^h 1220	1850	. .	22 57 37.3	. . . .	— 0.122	+ 306.863	. . .	
1056	h^3 Aquarii	1755	5	22 53 6.755	+ 313.321	— 0.656	+ 313.266	+ 0.055	
		1850	3	22 58 4.121	312.722	0.605	312.668	0.054	
1057	h^4 Aquarii	1755	3	22 54 26.269	+ 313.307	— 0.635	+ 312.982	+ 0.325	
		1850	6	22 59 23.631	312.727	0.585	312.400	0.327	
1058	A Piscium	1755	5	22 56 8.034	+ 307.357	— 0.118	+ 306.483	+ 0.874	
		1850	16	23 0 59.977	307.269	0.068	306.397	0.872	
1059	B. A. C. 8065	1755	2	22 56 51.730	+ 306.394	— 0.109	+ 306.475	— 0.081	
		1850	4	23 1 42.762	306.314	0.059	306.395	0.081	
1060	φ Aquarii	1755	5	23 1 37.459	+ 311.513	— 0.509	+ 311.355	+ 0.158	
		1850	86	23 6 33.173	311.054	0.458	310.893	0.161	
1061	B. A. C. 8094	1850	. .	23 7 50.8	. . . .	— 0.319	+ 309.451	. . .	
1062	ψ^1 Aquarii	1755	5	23 3 2.400	+ 315.505	— 0.685	+ 313.048	+ 2.457	
		1850	41	23 8 1.829	314.881	0.632	312.426	2.455	
1063	χ Aquarii	1755	5	23 4 8.270	+ 311.908	— 0.596	+ 312.144	— 0.236	
		1850	13	23 9 4.321	311.366	0.544	311.604	0.238	
1064	γ Piscium	1755	5	23 4 28.531	+ 310.799	— 0.011	+ 305.904	+ 4.895	
		1850	286	23 9 23.792	310.813	+ 0.041	305.916	4.897	
1065	ψ^2 Aquarii	1755	5	23 5 9.410	+ 312.885	— 0.673	+ 312.883	+ 0.002	
		1850	33	23 10 6.354	312.271	0.619	312.269	0.002	
1066	ψ^3 Aquarii	1755	5	23 6 12.131	+ 313.236	— 0.692	+ 313.015	+ 0.221	
		1850	56	23 11 9.401	312.604	0.638	312.383	0.221	
1067	96 Aquarii	1755	5	23 6 41.386	+ 311.651	— 0.444	+ 310.489	+ 1.162	
		1850	28	23 11 37.262	311.255	0.391	310.094	1.161	
1068	ο Cephei	1755	5	23 8 40.33	+ 239.12	+ 3.72	+ 237.77	+ 1.35	
		1800	. .	23 10 28.32	240.82	3.85	239.46	1.36	
		1850	. .	23 12 29.22	242.79	4.01	241.41	1.38	
		1900	. .	23 14 31.13	244.84	4.18	243.44	1.40	
1069	κ Piscium	1755	5	23 14 22.520	+ 307.492	— 0.064	+ 307.035	+ 0.457	
		1850	250	23 19 14.616	307.456	— 0.012	306.997	0.459	
		1900	. .	23 21 48.343	307.459	+ 0.016	306.998	0.461	

DECLINATIONS.

No.	Star.	Mag.	Epoch.	Declination.	Centennial variation.	Secular variation.	Struve's precession.	Proper motion.	Sec. var. of proper motion.
				° ′ ″	″	″	″	″	″
1050	B. A. C. 8017 . .	6.1	1850	− 5 31 2.9	. . . .	+ 12.12	+ 1922.46	. . .	
1051	82 Aquarii	6.0	1755	− 7 53 0.61	+ 1908.88	+ 12.92	+ 1913.08	− 4.20	
		6.4	1850	7 22 41.50	1920.70	11.96	1924.91	4.21	
1052	α Pegasi	2.0	1755	+ 13 52 32.51	+ 1915.22	+ 11.79	+ 1920.24	− 5.02	+ 0.01
		2.0	1850	14 23 57.16	1926.01	10.93	1931.02	5.01	
			1900	14 40 1.52	1931.37	10.49	1936.38	5.01	
1053	h^1 Aquarii	6.0	1755	− 9 0 39.11	+ 1921.26	+ 12.53	+ 1919.73	+ 1.53	
		5.4	1850	8 30 8.42	1932.69	11.54	1931.15	1.54	
1054	h^2 Aquarii	7.5	1755	− 9 4 16.84	+ 1920.19	+ 12.44	+ 1920.18	+ 0.01	
		7.4	1850	8 33 47.20	1931.54	11.46	1931.54	0.00	
1055	W^2 22^h 1220 . .	6.6	1850	+ 0 29 59.3	. . . .	+ 11.07	+ 1931.80	. . .	
1056	h^3 Aquarii	7.0	1755	− 9 15 13.64	+ 1921.07	+ 12.32	+ 1921.61	− 0.54	
		7.0	1850	8 44 43.21	1932.32	11.35	1932.86	0.54	
1057	h^4 Aquarii	8.0	1755	− 9 0 38.18	+ 1920.29	+ 12.08	+ 1924.92	− 4.63	
		8.0	1850	8 30 8.61	1931.30	11.10	1935.94	4.64	
1058	A Piscium . . .	6.0	1755	+ 0 47 55.47	+ 1939.65	+ 11.53	+ 1929.05	+10.60	
		5.6	1850	1 18 43.20	1950.17	10.61	1939.55	10.62	
1059	B. A. C. 8065 . .	8.0	1755	+ 0 49 16.89	+ 1929.20	+ 11.32	+ 1930.80	− 1.60	
		8.0	1850	1 19 54.60	1939.52	10.40	1941.13	1.61	
1060	φ Aquarii	5.6	1755	− 7 21 55.24	+ 1922.43	+ 10.65	+ 1941.68	−19.25	
		4.1	1850	6 51 24.28	1932.09	9.68	1951.36	19.27	
1061	B. A. C. 8094 . .	5.4	1850	− 4 18 42.3	. . . .	+ 9.34	+ 1953.94	. . .	
1062	ψ^1 Aquarii	5.5	1755	− 10 25 5.72	+ 1943.58	+ 10.60	+ 1944.79	− 1.21	
		4.1	1850	9 54 14.69	1953.18	9.58	1954.30	1.12	
1063	χ Aquarii	5.5	1755	− 9 3 29.02	+ 1943.02	+ 10.15	+ 1947.14	− 4.12	
		5.3	1850	8 32 38.71	1952.21	9.20	1956.33	4.12	
1064	γ Piscium . . .	4.5	1755	+ 1 56 52.94	+ 1948.62	+ 10.21	+ 1947.84	+ 0.78	
		3.6	1850	2 27 48.59	1957.86	9.25	1956.94	0.92	
1065	ψ^2 Aquarii	5.0	1755	− 10 30 56.62	+ 1946.99	+ 10.02	+ 1949.27	− 2.28	
		4.2	1850	10 0 2.61	1956.03	9.02	1958.30	2.27	
1066	ψ^3 Aquarii	5.0	1755	− 10 56 45.49	+ 1950.76	+ 9.82	+ 1951.43	− 0.67	
		4.8	1850	10 25 48.01	1959.61	8.82	1960.26	0.65	
1067	96 Aquarii	6.0	1755	− 6 27 32.20	+ 1949.99	+ 9.71	+ 1952.41	− 2.42	
		5.6	1850	5 56 35.50	1958.75	8.73	1961.05	2.30	
1068	ο Cephei	7.0	1755	+ 66 46 25.74	+ 1958.02	+ 6.95	+ 1956.35	+ 1.67	
			1800	67 1 7.58	1961.12	6.73	1959.43	1.69	
		5.3	1850	67 17 28.96	1964.41	6.49	1962.70	1.71	
			1900	67 33 51.96	1967.59	6.23	1965.85	1.74	
1069	κ Piscium . . .	5.5	1755	− 0 4 55.39	+ 1955.70	+ 8.07	+ 1966.86	−11.16	
		4.7	1850	+ 0 26 6.03	1962.92	7.13	1974.00	11.08	
			1900	0 42 28.35	1966.35	6.60	1977.43	11.08	

RIGHT ASCENSIONS.

No.	Star.	Epoch.	Number of observations.	Right ascension.			Centennial variation.	Secular variation.	Struve's precession.	Proper motion.	Sec. var. of proper motion.
				h.	m.	s.	s.	s.	s.	s.	s.
1070	9 Piscium . . .	1755	5	23	14	41.776	+ 307.417	− 0.071	+ 307.097	+ 0.316	
		1850	18	23	19	33.798	307.375	0.018	307.054	0.321	
1071	θ Piscium . . .	1755	5	23	15	32.957	+ 303.793	+ 0.199	+ 304.724	− 0.931	
		1850	76	23	20	21.658	304.007	0.251	304.938	0.931	
		1900	. .	23	22	53.694	304.140	0.280	305.072	0.932	
1072	11 Piscium . . .	1755	5	23	16	52.379	+ 308.145	− 0.211	+ 308.367	− 0.222	
		1850	5	23	21	45.029	307.970	0.158	308.192	0.222	
1073	B. A. C. 8184 . .	1755	0	23	16	51.586	+ 310.851	− 0.334	+ 309.567	+ 1.284	
		1850	15	23	21	46.749	310.550	0.300	309.255	1.295	
1074	12 Piscium . . .	1755	5	23	16	56.347	+ 307.940	− 0.171	+ 308.035	− 0.095	
		1850	5	23	21	48.820	307.802	0.119	307.897	0.095	
1075	13 Piscium . . .	1755	5	23	19	23.296	+ 307.944	− 0.162	+ 307.999	− 0.055	
		1850	3	23	24	15.778	307.816	0.109	307.871	0.055	
1076	14 Piscium . . .	1755	5	23	21	33.033	+ 308.764	− 0.159	+ 308.009	+ 0.755	
		1850	7	23	26	26.295	308.639	0.106	307.886	0.753	
1077	15 Piscium . . .	1775	5	23	22	57.601	+ 306.239	− 0.018	+ 307.010	− 0.771	
		1850	5	23	27	48.528	306.248	+ 0.036	307.019	0.771	
1078	16 Piscium . . .	1755	5	23	23	53.457	+ 305.922	+ 0.024	+ 306.725	− 0.803	
		1850	15	23	28	44.102	305.970	0.079	306.774	0.804	
1079	ι Piscium . . .	1755	5	23	27	21.461	+ 308.039	+ 0.240	+ 305.569	+ 2.470	+0.006
		1850	612	23	32	14.214	308.292	0.293	305.816	2.476	
		1900	. .	23	34	48.398	308.445	0.320	305.968	2.477	
1080	γ Cephei	1755	6	23	29	30.20	+ 232.41	+ 6.23	+ 234.38	− 1.97	
		1800	. .	23	31	15.43	235.29	6.59	237.31	2.02	
		1850	. .	23	33	13.92	238.69	7.03	240.77	2.08	
		1900	. .	23	35	14.16	242.32	7.51	244.46	2.14	
1081	λ Piscium . . .	1755	5	23	29	33.033	+ 305.866	+ 0.065	+ 306.863	− 0.997	
		1850	39	23	34	23.640	305.943	0.097	306.928	0.985	
1082	19 Piscium . . .	1755	4	23	33	52.925	+ 306.038	+ 0.153	+ 306.431	− 0.393	
		1850	19	23	38	43.739	306.209	0.207	306.602	0.393	
1083	20 Piscium . . .	1755	5	23	35	20.775	+ 308.601	− 0.166	+ 308.032	+ 0.569	
		1850	48	23	40	13.878	308.470	0.110	307.901	0.569	
1084	B. A. C. 8274 . .	1850	13	23	40	49.942	+ 308.524	− 0.299	+ 308.591	− 0.067	
1085	21 Piscium . . .	1755	5	23	36	55.147	+ 306.888	+ 0.046	+ 307.063	− 0.175	
		1850	33	23	41	46.718	306.956	0.099	307.132	0.176	
1086	22 Piscium . . .	1755	5	23	39	25.849	+ 306.594	+ 0.155	+ 306.667	− 0.073	
		1850	13	23	44	17.191	306.766	0.208	306.842	0.076	
1087	24 Piscium . . .	1755	5	23	40	20.378	+ 308.344	− 0.155	+ 307.902	+ 0.442	
		1850	7	23	45	13.242	308.221	0.103	307.777	0.444	
1088	25 Piscium . . .	1755	5	23	40	32.322	+ 306.873	+ 0.118	+ 306.852	+ 0.021	
		1850	8	23	45	23.913	307.011	0.172	306.998	0.013	

DECLINATIONS.

No.	Star.	Mag.	Epoch.	Declination.	Centennial variation.	Secular variation.	Struve's precession.	Proper motion.	Sec. var. of proper motion.
				° ′ ″	″	″	″	″	″
1070	9 Piscium . . .	6.0	1755	− 0 13 11.06	+ 1962.31	+ 7.97	+ 1967.39	− 5.08	
		6.6	1850	+ 0 17 56.60	1969.43	7.02	1974.50	5.07	
1071	θ Piscium . . .	5.0	1755	+ 5 2 11.90	+ 1963.24	+ 7.67	+ 1968.85	− 5.61	− 0.02
		4.2	1850	5 33 20.29	1970.09	6.76	1975.72	5.63	
			1900	5 49 46.16	1973.35	6.28	1978.99	5.64	
1072	11 Piscium . . .	6.5	1755	− 3 8 11.85	+ 1969.37	+ 7.56	+ 1971.04	− 1.67	
		6.4	1850	2 36 57.68	1976.10	6.61	1977.77	1.67	
1073	B. A. C. 8184 . .	. .	1755	− 5 51 51.24	+ 1948.19	+ 7.62	+ 1971.00	−22.81	
		6.3	1850	5 20 57.15	1954.96	6.65	1977.81	22.85	
1074	12 Piscium . . .	7.0	1755	− 2 22 52.85	+ 1970.14	+ 7.54	+ 1971.15	− 1.01	
		6.8	1850	1 51 37.96	1976.85	6.60	1977.86	1.01	
1075	13 Piscium . . .	7.0	1755	− 2 26 11.05	+ 1977.40	+ 7.07	+ 1975.04	+ 2.36	
		6.4	1850	1 54 49.46	1983.66	6.12	1981.30	2.36	
1076	14 Piscium . . .	6.5	1755	− 2 35 52.53	+ 1976.86	+ 6.69	+ 1978.28	− 1.42	
		5.9	1850	2 4 31.64	1982.76	5.73	1984.17	1.41	
1077	15 Piscium . . .	7.0	1755	− 0 2 13.82	+ 1976.23	+ 6.32	+ 1980.31	− 4.08	
		6.6	1850	+ 0 29 6.31	1981.79	5.38	1985.89	4.10	
1078	16 Piscium . . .	6.0	1755	+ 0 44 42.63	+ 1987.19	+ 6.16	+ 1981.63	+ 5.56	
		5.8	1850	1 16 13.09	1992.59	5.23	1987.00	5.59	
1079	ι Piscium . . .	4.5	1755	+ 4 18 2.35	+ 1941.76	+ 5.59	+ 1986.13	−44.37	+ 0.04
		4.1	1850	4 48 49.39	1946.61	4.63	1990.94	44.33	
			1900	5 5 3.25	1948.79	4.11	1993.10	44.31	
1080	γ Cephei	3.0	1755	+ 76 15 58.67	+ 2003.12	+ 3.60	+ 1988.72	+14.40	
			1800	76 31 0.43	2004.71	3.40	1990.33	14.38	
		3.3	1850	76 47 43.19	2006.34	3.17	1991.98	14.36	
			1900	77 4 26.74	2007.86	2.92	1993.52	14.34	
1081	λ Piscium . . .	5.0	1755	+ 0 26 2.56	+ 1971.54	+ 5.07	+ 1988.76	−17.22	
		4.5	1850	0 57 17.67	1975.89	4.12	1993.14	17.25	
1082	19 Piscium . . .	6.0	1755	+ 2 7 45.09	+ 1990.26	+ 4.21	+ 1993.44	− 3.18	
		4.9	1850	2 39 17.60	1993.82	3.28	1997.01	3.19	
1083	20 Piscium . . .	5.5	1755	− 4 7 19.24	+ 1994.76	+ 3.99	+ 1994.86	− 0.10	
		5.5	1850	3 35 42.56	1998.08	3.02	1998.18	0.10	
1084	B. A. C. 8274 . .	7.0	1850	− 7 12 47.12	+ 1995.44	+ 2.90	+ 1998.63	− 3.19	
1085	21 Piscium . . .	6.0	1755	− 0 16 58.27	+ 1993.54	+ 3.65	+ 1996.29	− 2.75	
		5.8	1850	+ 0 14 37.08	1996.50	2.62	1999.30	2.80	
1086	22 Piscium . . .	6.0	1755	+ 1 34 10.32	+ 1996.37	+ 3.15	+ 1998.38	− 2.01	
		5.0	1850	2 5 48.15	1998.92	2.22	2000.92	2.00	
1087	24 Piscium . . .	6.5	1755	− 4 30 54.52	+ 1995.59	+ 3.01	+ 1999.08	− 3.49	
		6.1	1850	3 59 17.50	1997.99	2.04	2001.47	3.48	
1088	25 Piscium . . .	6.5	1755	+ 0 43 44.35	+ 1997.68	+ 2.94	+ 1999.23	− 1.55	
		6.4	1850	1 15 23.33	2000.02	1.99	2001.57	1.55	

RIGHT ASCENSIONS.

No.	Star.	Epoch.	Number of observations.	Right ascension.			Centennial variation.	Secular variation.	Struve's precession.	Proper motion.	Sec. var. of proper motion.
				h.	m.	s.	s.	s.	s.	s.	s.
1089	26 Piscium . . .	1755	5	23	42	36.818	+ 305.848	+ 0.388	+ 305.983	− 0.135	
		1850	14	23	47	27.557	306.242	0.443	306.379	0.137	
1090	Groombridge 4163	1755	.	23	43	10.18	+ 275.19	+ 7.85	+ 274.80	+ 0.39	
		1800	. .	23	45	14.84	278.81	8.22	278.41	0.40	
		1850	. .	23	47	35.30	283.02	8.66	282.62	0.40	
		1900	. .	23	49	57.91	287.47	9.12	287.06	0.41	
1091	27 Piscium . . .	1755	5	23	46	7.826	+ 307.230	− 0.145	+ 307.693	− 0.463	
		1850	56	23	50	59.637	307.118	0.090	307.577	0.459	
1092	ω Piscium . . .	1755	5	23	46	44.602	+ 307.243	+ 0.403	+ 306.251	+ 0.992	+0.001
		1850	384	23	51	36.673	307.653	0.460	306.660	0.993	
		1900	. .	23	54	10.558	307.890	0.489	306.896	0.994	
1093	Lal. 47041 . . .	1850	. .	23	52	5.8	. . . .	+ 0.087	+ 307.267	. . .	
1094	29 Piscium . . .	1755	5	23	49	16.136	+ 307.488	− 0.101	+ 307.478	+ 0.010	
		1850	33	23	54	8.213	307.419	0.046	307.408	0.011	
1095	30 Piscium . . .	1755	5	23	49	23.523	+ 308.007	− 0.261	+ 307.799	+ 0.208	
		1850	28	23	54	16.019	307.786	0.205	307.579	0.207	
1096	B. A. C. 8351 . .	1755	1	23	49	28.854	+ 307.774	− 0.086	+ 307.440	+ 0.334	
		1850	5	23	54	21.208	307.717	0.035	307.384	0.333	
1097	c^2 Piscium . . .	1755	5	23	49	59.133	+ 305.777	+ 0.504	+ 306.266	− 0.489	
		1850	10	23	54	49.857	306.282	0.560	306.772	0.490	
1098	33 Piscium . . .	1755	5	23	52	47.587	+ 307.336	− 0.224	+ 307.520	− 0.184	
		1850	76	23	57	39.464	307.150	0.168	307.334	0.184	

DECLINATIONS.

No.	Star.	Mag.	Epoch.	Declination.	Centennial variation.	Secular variation.	Struve's precession.	Proper motion.	Sec. var. of proper motion.
				° ′ ″	″	″	″	″	″
1089	26 Piscium . . .	6.0	1755	+ 5 42 33.47	+ 1999.06	+ 2.53	+ 2000.68	− 1.62	
		6.3	1850	6 14 13.57	2001.01	1.59	2002.64	1.63	
1090	Groombridge 4163	. .	1755	+ 73 2 51.58	+ 2000.15	+ 2.09	+ 2001.05	− 0.90	
			1800	73 17 51.84	2001.03	1.76	2001.93	0.90	
		7.0	1850	73 34 32.55	2001.80	1.38	2002.70	0.90	
			1900	73 51 13.60	2002.38	0.98	2003.27	0.89	
1091	27 Piscium . . .	5.0	1755	− 4 54 54.60	+ 1996.68	+ 1.84	+ 2002.77	− 6.09	
		5.1	1850	4 23 17.17	1997.98	0.90	2004.10	6.12	
1092	ω Piscium . . .	4.5	1755	+ 5 30 25.55	+ 1991.63	+ 1.74	+ 2003.10	−11.47	
		4.0	1850	6 1 58.24	1992.84	0.78	2004.30	11.46	
			1900	6 18 34.74	1993.10	0.28	2004.56	11.46	
1093	Lal. 47041 . . .	7.1	1850	− 1 6 48.2	. . . .	+ 0.68	+ 2004.45	. . .	
1094	29 Piscium . .	5.0	1755	− 4 23 28.00	+ 2003.13	+ 1.24	+ 2004.24	− 1.11	
		5.0	1850	3 51 44.61	2003.85	0.29	2004.98	1.13	
1095	30 Piscium . . .	4.5	1755	− 7 22 32.83	+ 2000.46	+ 1.23	+ 2004.33	− 3.87	
		4.4	1850	6 50 51.99	2001.17	0.27	2005.01	3.84	
1096	B. A. C. 8351 . .	8.0	1755	− 4 7 47.72	+ 2002.40	+ 1.20	+ 2004.34	− 1.94	
		8.0	1850	3 36 5.07	2003.09	0.24	2005.03	1.94	
1097	c^2 Piscium . . .	6.0	1755	+ 7 7 26.14	+ 2001.49	+ 1.08	+ 2004.54	− 3.05	
		5.7	1850	7 39 7.90	2002.07	0.14	2005.13	3.06	
1098	33 Piscium . . .	5.0	1755	− 7 4 42.44	+ 2014.97	+ 0.55	+ 2005.50	+ 9.47	
		4.8	1850	6 32 48.12	2015.04	− 0.40	2005.54	9.50	

AUWERS' PERIODIC CORRECTIONS TO BE APPLIED TO THE POSITIONS OF SIRIUS AND PROCYON, ON ACCOUNT OF INEQUALITY OF PROPER MOTION.

Periodic terms to be applied to the position of Sirius.

[P, correction to the right ascension. P′, correction to the declination.]

α Canis Majoris.							α Canis Majoris—Continued.						
Year.	Year.	Year.	P		P′		Year.	Year.	Year.	P		P′	
			s.		″					s.		″	
1750.6	1800.0	1849.4	+.026	−20	+1.41	−6	1775.6	1825.0	1874.4	−.131	+7	−1.16	−7
1751.6	1801.0	1850.4	+.006	−20	+1.35	−7	1776.6	1826.0	1875.4	−.124	+7	−1.23	−6
1752.6	1802.0	1851.4	−.014	−17	+1.28	−9	1777.6	1827.0	1876.4	−.117	+9	−1.29	−5
1753.6	1803.0	1852.4	−.031	−16	+1.19	−10	1778.6	1828.0	1877.4	−.108	+10	−1.34	−4
1754.6	1804.0	1853.4	−.047	−15	+1.09	−11	1779.6	1829.0	1878.4	−.098	+12	−1.38	−4
1755.6	1805.0	1854.4	−.062	−14	+.98	−11	1780.6	1830.0	1879.4	−.086	+12	−1.42	−2
1756.6	1806.0	1855.4	−.076	−12	+.87	−11	1781.6	1831.0	1880.4	−.074	+13	−1.44	−1
1757.6	1807.0	1856.4	−.088	−11	+.76	−12	1782.6	1832.0	1881.4	−.061	+15	−1.45	+1
1758.6	1808.0	1857.4	−.099	−10	+.64	−12	1783.6	1833.0	1882.4	−.046	+16	−1.44	+2
1759.6	1809.0	1858.4	−.109	−9	+.52	−12	1784.6	1834.0	1883.4	−.030	+18	−1.42	+5
1760.6	1810.0	1859.4	−.118	−8	+.40	−12	1785.6	1835.0	1884.4	−.012	+19	−1.37	+7
1761.6	1811.0	1860.4	−.126	−6	+.28	−12	1786.6	1836.0	1885.4	+.007	+20	−1.30	+9
1762.6	1812.0	1861.4	−.132	−6	+.16	−12	1787.6	1837.0	1886.4	+.027	+22	−1.21	+13
1763.6	1813.0	1862.4	−.138	−5	+.04	−11	1788.6	1838.0	1887.4	+.049	+23	−1.08	+18
1764.6	1814.0	1863.4	−.143	−4	−.07	−12	1789.6	1839.0	1888.4	+.072	+24	−.90	+23
1765.6	1815.0	1864.4	−.147	−2	−.19	−11	1790.6	1840.0	1889.4	+.096	+24	−.67	+30
1766.6	1816.0	1865.4	−.149	−2	−.30	−11	1791.6	1841.0	1890.4	+.120	+21	−.37	+39
1767.6	1817.0	1866.4	−.151	−1	−.41	−11	1792.6	1842.0	1891.4	+.141	+11	+.02	+44
1768.6	1818.0	1867.4	−.152	0	−.52	−11	1793.6	1843.0	1892.4	+.152	−5	+.46	+42
1769.6	1819.0	1868.4	−.152	+1	−.63	−10	1794.6	1844.0	1893.4	+.147	−17	+.88	+29
1770.6	1820.0	1869.4	−.151	+2	−.73	−9	1795.6	1845.0	1894.4	+.130	−23	+1.17	+17
1771.6	1821.0	1870.4	−.149	+3	−.82	−10	1796.6	1846.0	1895.4	+.107	−25	+1.34	+9
1772.6	1822.0	1871.4	−.146	+4	−.92	−8	1797.6	1847.0	1896.4	+.082	−24	+1.43	+2
1773.6	1823.0	1872.4	−.142	+5	−1.00	−8	1798.6	1848.0	1897.4	+.058	−23	+1.45	−2
1774.6	1824.0	1873.4	−.137	+6	−1.08	−8	1799.6	1849.0	1898.4	+.035	−21	+1.43	−5
1775.6	1825.0	1874.4	−.131	+7	−1.16	−7	1800.6	1850.0	1899.4	+.014	−20	+1.38	−7
1776.6	1826.0	1875.4	−.124	+7	−1.23	−6	1801.6	1851.0	1900.4	−.006	−18	+1.31	−9
1777.6	1827.0	1876.4	−.117	+9	−1.29	−5	1802.6	1852.0	1901.4	−.024		+1.22	

Periodic terms to be applied to the position of Procyon.

α Canis Minoris.

Year.	Year.	Year.	Year.	P	Diff.	P′	Diff.
				s.		″	
1750.0	1790.0	1830.0	1870.0	−.045	−8	−.80	+11
1751.0	1791.0	1831.0	1871.0	−.053	−7	−.69	+14
1752.0	1792.0	1832.0	1872.0	−.060	−5	−.55	+14
1753.0	1793.0	1833.0	1873.0	−.065	−3	−.41	+16
1754.0	1794.0	1834.0	1874.0	−.068	−2	−.25	+16
1755.0	1795.0	1835.0	1875.0	−.070	0	−.09	+17
1756.0	1796.0	1836.0	1876.0	−.070	+2	+.08	+16
1757.0	1797.0	1837.0	1877.0	−.068	+3	+.24	+16
1758.0	1798.0	1838.0	1878.0	−.065	+5	+.40	+15
1759.0	1799.0	1839.0	1879.0	−.060	+7	+.55	+13
1760.0	1800.0	1840.0	1880.0	−.053	+8	+.68	+12
1761.0	1801.0	1841.0	1881.0	−.045	+9	+.80	+10
1762.0	1802.0	1842.0	1882.0	−.036	+10	+.90	+7
1763.0	1803.0	1843.0	1883.0	−.026	+10	+.97	+5
1764.0	1804.0	1844.0	1884.0	−.016	+10	+1.02	+3
1765.0	1805.0	1845.0	1885.0	−.006	+11	+1.05	0
1766.0	1806.0	1846.0	1886.0	+.005	+10	+1.05	−3
1767.0	1807.0	1847.0	1887.0	+.015	+11	+1.02	−5
1768.0	1808.0	1848.0	1888.0	+.026	+10	+.97	−7
1769.0	1809.0	1849.0	1889.0	+.036	+9	+.90	−10
1770.0	1810.0	1850.0	1890.0	+.045	+8	+.80	−11

α Canis Minoris—Continued.

Year.	Year.	Year.	Year.	P	Diff.	P′	Diff.
				s.		″	
1770.0	1810.0	1850.0	1890.0	+.045	+8	+.80	−11
1771.0	1811.0	1851.0	1891.0	+.053	+7	+.69	−14
1772.0	1812.0	1852.0	1892.0	+.060	+5	+.55	−14
1773.0	1813.0	1853.0	1893.0	+.065	+3	+.41	−16
1774.0	1814.0	1854.0	1894.0	+.068	+2	+.25	−17
1775.0	1815.0	1855.0	1895.0	+.070	0	+.08	−16
1776.0	1816.0	1856.0	1896.0	+.070	−2	−.08	−16
1777.0	1817.0	1857.0	1897.0	+.068	−3	−.24	−16
1778.0	1818.0	1858.0	1898.0	+.065	−5	−.40	−15
1779.0	1819.0	1859.0	1899.0	+.060	−6	−.55	−13
1780.0	1820.0	1860.0	1900.0	+.054	−8	−.68	−12
1781.0	1821.0	1861.0	1901.0	+.046	−9	−.80	−10
1782.0	1822.0	1862.0	1902.0	+.037	−10	−.90	−7
1783.0	1823.0	1863.0	1903.0	+.027	−10	−.97	−5
1784.0	1824.0	1864.0	1904.0	+.017	−11	−1.02	−3
1785.0	1825.0	1865.0	1905.0	+.006	−11	−1.05	0
1786.0	1826.0	1866.0	1906.0	−.005	−10	−1.05	+3
1787.0	1827.0	1867.0	1907.0	−.015	−12	−1.02	+5
1788.0	1828.0	1868.0	1908.0	−.027	−10	−.97	+7
1789.0	1829.0	1869.0	1909.0	.037	−9	−.90	+10
1790.0	1830.0	1870.0	1910.0	−.046		−.80	

RIGHT ASCENSIONS OF TIME STARS FOR 1800

AND

FOR QUINQUENNIAL EPOCHS, 1830–1900.

Right Ascensions of Time Stars for 1800 *and for Quinquennial Epochs,* 1830–1900.

Year.	α Andromedæ. R. A.	Ann. var.	γ Pegasi. R. A.	Ann. var.	12 Ceti. R. A.	Ann. var.	α Cassiopeæ. R. A.	Ann. var.	β Ceti. R. A.	Ann. var.
	23h; 0h		0h		0h		0h		0h	
1800 . .	58 4.620	3.0753	2 57.148	3.0749	19 50.023	3.0605	29 14.615	3.3245	33 32.581	3.0194
1830 . .	59 36.956	3.0806	4 29.440	3.0778	21 21.839	3.0606	30 54.602	3.3410	35 3.137	3.0176
1835 . .	52.361	3.0815	44.830	3.0783	37.143	3.0606	31 11.314	3.3437	18.224	3.0173
1840 . .	0 7.771	3.0824	5 0.223	3.0788	52.446	3.0607	28.039	3.3465	33.310	3.0170
1845 . .	23.185	3.0833	15.618	3.0793	22 7.749	3.0607	44.778	3.3492	48.395	3.0168
1850 . .	38.603	3.0842	31.016	3.0798	23.053	3.0607	32 1.531	3.3519	36 3.478	3.0165
1855 . .	54.026	3.0851	46.416	3.0803	38.357	3.0608	18.298	3.3547	18.560	3.0162
1860 . .	1 9.454	3.0860	6 1.819	3.0808	53.661	3.0608	35.078	3.3574	33.640	3.0159
1865 . .	24.886	3.0869	17.224	3.0813	23 8.965	3.0608	51.872	3.3602	48.719	3.0156
1870 . .	40.323	3.0879	32.632	3.0818	24.269	3.0609	33 8.680	3.3629	37 3.796	3.0154
1875 . .	55.764	3.0888	48.042	3.0823	39.574	3.0609	25.502	3.3657	18.872	3.0151
1880 . .	2 11.210	3.0897	7 3.455	3.0828	54.878	3.0610	42.337	3.3685	33.947	3.0148
1885 . .	26.661	3.0906	18.870	3.0833	24 10.183	3.0610	59.187	3.3713	49.020	3.0145
1890 . .	42.116	3.0915	34.288	3.0838	25.489	3.0611	34 16.051	3.3741	38 4.092	3.0142
1895 . .	57.575	3.0925	49.708	3.0843	40.794	3.0611	32.928	3.3768	19.163	3.0140
1900 . .	3 13.040	3.0934	8 5.131	3.0848	56.100	3.0611	49.820	3.3784	34.232	3.0137

Year.	ε Piscium. R. A.	Ann. var.	β Andromedæ. R. A.	Ann. var.	θ′ Ceti. R. A.	Ann. var.	η Piscium. R. A.	Ann. var.	ο Piscium. R. A.	Ann. var.
	0h		0h; 1h		1h		1h		1h	
1800 . .	52 34.573	3.1013	58 34.585	3.3186	14 1.823	2.9955	20 48.158	3.1900	34 50.929	3.1526
1830 . .	54 7.650	3.1039	0 14.276	3.3272	15 31.697	2.9960	22 23.921	3.1942	36 25.553	3.1558
1835 . .	23.171	3.1043	30.916	3.3287	46.677	2.9961	39.893	3.1949	41.333	3.1563
1840 . .	38.693	3.1047	47.563	3.3301	16 1.658	2.9962	55.869	3.1956	57.116	3.1568
1845 . .	54.218	3.1052	1 4.217	3.3315	16.639	2.9963	23 11.849	3.1963	37 12.902	3.1574
1850 . .	55 9.745	3.1056	20.878	3.3330	31.621	2.9964	27.832	3.1970	28.690	3.1579
1855 . .	25.274	3.1060	37.546	3.3344	46.603	2.9964	43.819	3.1977	44.481	3.1585
1860 . .	40.805	3.1065	54.222	3.3358	17 1.585	2.9965	59.809	3.1984	38 0.275	3.1590
1865 . .	56.339	3.1069	2 10.905	3.3372	16.568	2.9966	24 15.803	3.1991	16.071	3.1596
1870 . .	56 11.874	3.1073	27.594	3.3387	31.552	2.9967	31.800	3.1998	31.871	3.1601
1875 . .	27.412	3.1078	44.291	3.3401	46.535	2.9968	47.801	3.2005	47.673	3.1607
1880 . .	42.952	3.1082	3 0.996	3.3416	18 1.519	2.9969	25 3.805	3.2012	39 3.477	3.1612
1885 . .	58.494	3.1087	17.707	3.3430	16.504	2.9970	19.813	3.2019	19.285	3.1618
1890 . .	57 14.039	3.1091	34.426	3.3444	31.489	2.9971	35.824	3.2026	35.095	3.1624
1895 . .	29.585	3.1095	51.152	3.3459	46.475	2.9971	51.839	3.2033	50.908	3.1629
1900 . .	45.134	3.1100	4 7.885	3.3473	19 1.461	2.9972	26 7.858	3.2041	40 6.724	3.1634

***Right Ascensions of Time Stars for* 1800 *and for Quinquennial Epochs,* 1830–1900—Continued.**

Year.	β Arietis.		α Arietis.		ξ^1 Ceti.		ξ^2 Ceti.		γ Ceti.	
	R. A.	Ann. var.	R. A.	Ann. var.	R. A.	Ann. var.	R. A.	Ann. var.	R. A.	Ann. var.
	1^h		$1^h;\ 2^h$		2^h		2^h		2^h	
1800 . .	43 37.197	3.2872	55 55.765	3.3530	2 24.982	3.1638	17 32.554	3.1735	32 57.102	3.0951
1830 . .	45 15.898	3.2927	57 36.445	3.3590	3 59.947	3.1672	19 7.808	3.1769	34 29.994	3.0978
1835 . .	32.364	3.2936	53.242	3.3600	4 15.784	3.1678	23.694	3.1774	45.484	3.0982
1840 . .	48.834	3.2946	58 10.045	3.3610	31.625	3.1684	39.583	3.1780	35 0.976	3.0987
1845 . .	46 5.309	3.2955	26.852	3.3620	47.468	3.1689	55.475	3.1786	16.471	3.0991
1850 . .	21.789	3.2964	43.665	3.3630	5 3.314	3.1695	[illegible] 11.379	3.1792	31.968	3.0996
1855 . .	38.273	3.2973	59 0.482	3.3640	19.163	3.1701	27.276	3.1797	47.467	3.1001
1860 . .	54.761	3.2982	17.305	3.3650	35.015	3.1706	43.176	3.1803	36 2.969	3.1005
1865 . .	47 11.255	3.2991	34.133	3.3661	50.870	3.1712	59.080	3.1809	18.472	3.1010
1870 . .	27.753	3.3000	50.966	3.3671	6 6.727	3.1718	21 14.986	3.1815	33.979	3.1014
1875 . .	44.255	3.3009	0 7.803	3.3681	22.588	3.1724	30.894	3.1821	49.487	3.1019
1880 . .	48 0.762	3.3018	24.646	3.3691	38.452	3.1730	46.806	3.1826	37 4.998	3.1024
1885 . .	17.273	3.3027	41.494	3.3701	54.318	3.1736	22 2.721	3.1832	20.511	3.1028
1890 . .	33.789	3.3036	58.347	3.3711	7 10.187	3.1741	18.638	3.1838	36.026	3.1033
1895 . .	50.309	3.3046	1 15.206	3.3721	26.059	3.1747	34.559	3.1844	51.544	3.1037
1900 . .	49 6.835	3.3055	32.069	3.3732	41.935	3.1753	50.482	3.1850	38 7.064	3.1042

Year.	α Ceti.		ζ Arietis.		α Persei.		ε Eridani.		δ Persei.	
	R. A.	Ann. var.	R. A.	Ann. var.	R. A.	Ann. var.	R. A.	Ann. var.	R. A.	Ann. var.
	2^h		3^h		3^h		3^h		3^h	
1800 . .	51 50.391	3.1218	3 25.880	3.4235	10 7.015	4.2143	23 30.955	2.8187	28 43.815	4.2127
1830 . .	53 24.087	3.1246	5 8.666	3.4288	12 13.662	4.2288	24 55.540	2.8203	30 50.385	4.2253
1835 . .	39.712	3.1251	25.812	3.4297	34.812	4.2312	25 9.642	2.8206	31 12.517	4.2274
1840 . .	55.338	3.1256	42.963	3.4306	55.974	4.2336	23.746	2.8209	33.659	4.2294
1845 . .	54 10.968	3.1261	6 0.118	3.4315	13 17.149	4.2361	37.851	2.8212	54.811	4.2315
1850 . .	26.599	3.1266	17.278	3.4324	38.335	4.2385	51.958	2.8215	32 15.974	4.2336
1855 . .	42.233	3.1271	34.442	3.4333	59.533	4.2409	26 6.066	2.8217	37.147	4.2357
1860 . .	57.870	3.1275	51.611	3.4341	14 20.744	4.2433	20.175	2.8220	58.331	4.2378
1865 . .	55 13.509	3.1280	7 8.784	3.4350	41.967	4.2457	34.286	2.8223	33 19.525	4.2398
1870 . .	29.150	3.1285	25.961	3.4359	15 3.201	4.2481	48.398	2.8226	40.729	4.2419
1875 . .	44.794	3.1290	43.143	3.4368	24.448	4.2505	27 2.511	2.8228	34 1.944	4.2440
1880 . .	56 0.440	3.1295	8 0.329	3.4377	45.707	4.2530	16.626	2.8231	23.169	4.2461
1885 . .	16.088	3.1300	17.519	3.4386	16 6.978	4.2554	30.743	2.8234	44.405	4.2481
1890 . .	31.739	3.1304	34.714	3.4394	28.261	4.2578	44.860	2.8237	35 5.651	4.2502
1895 . .	47.393	3.1309	51.914	3.4403	49.556	4.2602	58.979	2.8240	26.907	4.2523
1900 . .	57 3.049	3.1314	9 9.117	3.4412	17 10.863	4.2626	28 13.100	2.8242	48.174	4.2543

***Right Ascensions of Time Stars for* 1800 *and for Quinquennial Epochs*, 1830–1900—Continued.**

Year.	η Tauri.		ζ Persei.		γ¹ Eridani.		γ Tauri.		ε Tauri.	
	R. A.	Ann. var.	R. A.	Ann. var.	R. A.	Ann. var.	R. A.	Ann. var.	R. A.	Ann. var.
	3^h		3^h		3^h		4^h		4^h	
1800 . .	35 37.298	3.5411	41 35.521	3.7403	48 42.186	2.7944	8 25.656	3.3987	16 57.301	3.4866
1830 . .	37 23.609	3.5464	43 27.832	3.7470	50 6.040	2.7958	10 7.671	3.4022	18 41.953	3.4903
1835 . .	41.343	3.5473	46.570	3.7481	20.019	2.7960	24.683	3.4028	59.406	3.4909
1840 . .	59.082	3.5481	44 5.313	3.7493	34.000	2.7963	41.699	3.4033	19 16.862	3.4915
1845 . .	38 16.825	3.5490	24.062	3.7504	47.982	2.7965	58.717	3.4039	34.321	3.4921
1850 . .	34.572	3.5499	42.817	3.7515	51 1.965	2.7967	11 15.738	3.4045	51.783	3.4927
1855 . .	52.324	3.5508	45 1.577	3.7526	15.949	2.7969	32.762	3.4051	20 9.248	3.4933
1860 . .	39 10.080	3.5517	20.343	3.7537	29.935	2.7972	49.789	3.4056	26.716	3.4939
1865 . .	27.841	3.5526	39.114	3.7548	43.921	2.7974	12 6.818	3.4062	44.187	3.4945
1870 . .	45.606	3.5535	57.891	3.7559	57.909	2.7976	23.851	3.4068	21 1.662	3.4951
1875 . .	40 3.375	3.5543	46 16.674	3.7570	52 11.897	2.7979	40.886	3.4073	19.139	3.4957
1880 . .	21.149	3.5552	35.462	3.7581	25.887	2.7981	57.924	3.4079	36.619	3.4963
1885 . .	38.927	3.5561	54.255	3.7592	39.878	2.7983	13 14.965	3.4085	54.102	3.4969
1890 . .	56.710	3.5570	47 13.054	3.7604	53.870	2.7985	32.009	3.4091	22 11.588	3.4975
1895 . .	41 14.497	3.5579	31.859	3.7615	53 7.864	2.7988	49.055	3.4096	29.078	3.4981
1900 . .	32.288	3.5587	50.669	3.7626	21.858	2.7990	14 6.105	3.4102	46.570	3.4987

Year.	α Tauri.		ι Aurigæ.		11 Orionis.		α Aurigæ.		β Orionis.	
	R. A.	Ann. var.	R. A.	Ann. var.	R. A.	Ann. var.	R. A.	Ann. var.	R. A.	Ann. var.
	4^h		4^h		4^h		5^h		5^h	
1800 . .	24 27.562	3.4281	43 59.339	3.8875	53 9.103	3.4172	1 56.230	4.4096	4 55.919	2.8777
1830 . .	26 10.452	3.4312	45 56.031	3.8920	54 51.657	3.4197	4 8.597	4.4148	6 22.269	2.8789
1835 . .	27.609	3.4318	46 15.493	3.8927	55 8.756	3.4201	30.673	4.4157	36.665	2.8791
1840 . .	44.769	3.4323	34.959	3.8935	25.857	3.4205	52.754	4.4165	51.061	2.8793
1845 . .	27 1.932	3.4328	54.428	3.8942	42.961	3.4209	5 14.839	4.4174	7 5.458	2.8795
1850 . .	19.097	3.4333	47 13.901	3.8949	56 0.066	3.4212	36.928	4.4182	19.856	2.8797
1855 . .	36.265	3.4339	33.378	3.8957	17.173	3.4216	59.022	4.4190	34.256	2.8799
1860 . .	53.436	3.4344	52.858	3.8964	34.282	3.4220	6 21.119	4.4199	48.656	2.8801
1865 . .	28 10.609	3.4349	48 12.342	3.8971	51.394	3.4224	43.221	4.4207	8 3.057	2.8803
1870 . .	27.785	3.4354	31.829	3.8979	57 8.507	3.4228	7 5.326	4.4215	17.459	2.8805
1875 . .	44.963	3.4359	51.320	3.8986	25.622	3.4232	27.436	4.4223	31.862	2.8807
1880 . .	29 2.144	3.4364	49 10.815	3.8993	42.739	3.4236	49.550	4.4232	46.267	2.8809
1885 . .	19.327	3.4370	30.313	3.9000	59.858	3.4240	8 11.668	4.4240	9 0.672	2.8811
1890 . .	36.514	3.4375	49.815	3.9007	58 16.979	3.4244	33.790	4.4248	15.078	2.8813
1895 . .	53.702	3.4380	50 9.321	3.9014	34.102	3.4248	55.916	4.4256	29.485	2.8815
1900 . .	30 10.894	3.4385	28.830	3.9022	51.226	3.4252	9 18.045	4.4264	43.893	2.8817

Right Ascensions of Time Stars for 1800 *and for Quinquennial Epochs*, 1830–1900—**Continued.**

Year.	β Tauri.		δ Orionis.		α Leporis.		ε Orionis.		α Columbæ.	
	R. A.	Ann. var.	R. A.	Ann. var.	R. A.	Ann. var.	R. A.	Ann. var.	R. A.	Ann. var.
	5^h		5^h		5^h		5^h		5^h	
1800 . .	13 39.585	3.7819	21 47.659	3.0599	23 54.810	2.6421	26 4.225	3.0391	32 24.575	2.1703
1830 .	15 33.081	3.7845	23 19.474	3.0611	25 14.085	2.6430	27 35.415	3.0402	33 29.696	2.1710
1835 . .	52.005	3.7849	34.780	3.0613	27.301	2.6431	50.617	3.0404	40.552	2.1712
1840 . .	16 10.930	3.7853	50.087	3.0615	40.517	2.6433	28 5.819	3.0406	51.409	2.1714
1845 . .	29.858	3.7857	24 5.395	3.0617	53.733	2.6434	21.023	3.0408	34 2.266	2.1715
1850 . .	48.787	3.7861	20.704	3.0619	26 6.951	2.6436	36.227	3.0410	13.124	2.1716
1855 . .	17 7.719	3.7865	36.014	3.0621	20.169	2.6437	51.432	3.0411	23.983	2.1718
1860 . .	26.652	3.7869	51.325	3.0623	33.388	2.6439	29 6.638	3.0413	34.842	2.1719
1865 . .	45.588	3.7873	25 6.636	3.0624	46.608	2.6440	21.845	3.0415	45.702	2.1721
1870 . .	18 4.526	3.7877	21.949	3.0626	59.829	2.6442	37.053	3.0417	56.563	2.1722
1875 . .	23.465	3.7881	37.263	3.0628	27 13.050	2.6443	52.262	3.0418	35 7.424	2.1723
1880 . .	42.407	3.7885	52.577	3.0630	26.272	2.6445	30 7.471	3.0420	18.286	2.1725
1885 . .	19 1.350	3.7889	26 7.893	3.0632	39.494	2.6446	22.682	3.0422	29.149	2.1726
1890 . .	20.296	3.7893	23.209	3.0634	52.718	2.6448	37.893	3.0424	40.012	2.1728
1895 . .	39.243	3.7897	38.527	3.0636	28 5.942	2.6449	53.106	3.0425	50.876	2.1729
1900 . .	58.193	3.7901	53.845	3.0638	19.167	2.6450	31 8.319	3.0427	36 1.741	2.1730

Year.	α Orionis.		ν Orionis.		μ Geminorum.		γ Geminorum.		α Canis Majoris.	
	R. A.	Ann. var.	R. A.	Ann. var.	R. A.	Ann. var.	R. A.	Ann. var.	R. A.	Ann. var.
	5^h		5^h; 6^h		6^h		6^h		6^h	
1800 . .	44 20.865	3.2445	56 9.150	3.4257	10 51.526	3.6317	26 9.326	3.4685	36 20.102	2.6443
1830 . .	45 58.212	3.2454	57 51.932	3.4263	12 40.477	3.6317	27 53.376	3.4682	37 39.427	2.6441
1835 . .	46 14.439	3.2455	58 9.064	3.4264	58.636	3.6317	28 10.717	3.4681	52.647	2.6440
1840 . .	30.667	3.2456	26.196	3.4265	13 16.794	3.6317	28.057	3.4680	38 5.867	2.6440
1845 . .	46.896	3.2458	43.329	3.4266	34.953	3.6317	45.397	3.4680	19.087	2.6440
1850 . .	47 3.125	3.2459	59 00.462	3.4267	53.111	3.6317	29 02.737	3.4679	32.307	2.6439
1855 . .	19.355	3.2461	17.596	3.4268	14 11.269	3.6316	20.076	3.4678	45.527	2.6439
1860 . .	35.586	3.2462	34.730	3.4269	29.427	3.6316	37.415	3.4678	58.746	2.6439
1865 . .	51.817	3.2464	51.864	3.4270	47.586	3.6316	54.754	3.4677	39 11.965	2.6438
1870 . .	48 8.050	3.2465	0 8.999	3.4270	15 5.743	3.6316	30 12.092	3.4676	25.184	2.6438
1875 . .	24.282	3.2466	26.135	3.4271	23.901	3.6316	29.430	3.4675	38.403	2.6437
1880 . .	40.516	3.2468	43.271	3.4272	42.059	3.6315	46.768	3.4675	51.622	2.6437
1885 . .	56.750	3.2469	1 0.407	3.4273	16 0.217	3.6315	31 4.105	3.4674	40 4.840	2.6437
1890 . .	49 12.985	3.2470	17.544	3.4274	18.374	3.6315	21.442	3.4673	18.058	2.6436
1895 . .	29.220	3.2472	34.681	3.4275	36.531	3.6314	38.778	3.4672	31.276	2.6436
1900 . .	45.457	3.2473	51.818	3.4275	54.688	3.6314	56.114	3.4672	44.494	2.6436

***Right Ascensions of Time Stars for* 1800 *and for Quinquennial Epochs,* 1830–1900—Continued.**

Year.	ε Canis Majoris.		δ Canis Majoris.		δ Geminorum.		α^2 Geminorum.		α Canis Minoris.	
	R. A.	Ann. var.	R. A.	Ann. var.	R. A.	Ann. var.	R. A.	Ann. var.	R. A.	Ann. var.
	6^h		7^h		7^h		7^h		7^h	
1800 . .	50 46.049	2.3565	0 15.700	2.4374	8 10.019	3.5942	21 48.945	3.8501	28 49.499	3.1482
1830 . .	51 56.750	2.3569	1 28.828	2.4378	9 57.816	3.5922	23 44.392	3.8463	30 23.920	3.1466
1835 . .	52 8.535	2.3570	41.017	2.4378	10 15.776	3.5919	24 3.622	3.8456	39.653	3.1463
1840 . .	20.320	2.3570	53.206	2.4379	33.735	3.5915	22.849	3.8450	55.384	3.1461
1845 . .	32.105	2.3571	2 5.396	2.4380	51.691	3.5911	42.072	3.8443	31 11.113	3.1458
1850 . .	43.891	2.3572	17.586	2.4380	11 9.646	3.5908	25 1.292	3.8437	26.842	3.1456
1855 . .	55.677	2.3572	29.776	2.4381	27.599	3.5904	20.509	3.8430	42.569	3.1453
1860 . .	53 7.463	2.3573	41.967	2.4381	45.550	3.5901	39.722	3.8423	58.295	3.1450
1865 . .	19.250	2.3574	54.157	2.4382	12 3.500	3.5897	58.932	3.8417	32 14.019	3.1448
1870 . .	31.037	2.3574	3 6.349	2.4382	21.448	3.5894	26 18.139	3.8410	29.742	3.1445
1875 . .	42.825	2.3575	18.540	2.4383	39.393	3.5890	37.342	3.8403	45.464	3.1442
1880 . .	54.612	2.3576	30.732	2.4384	57.337	3.5886	56.542	3.8397	33 1.185	3.1440
1885 . .	54 6.400	2.3577	42.924	2.4384	13 15.280	3.5883	27 15 739	3.8390	16.904	3.1437
1890 . .	18.189	2.3577	55.116	2.4385	33.220	3.5879	34.932	3.8383	32.622	3.1434
1895 . .	29.978	2.3578	4 7.308	2.4385	51.159	3.5875	54.122	3.8376	48.338	3.1432
1900 .	41.767	2.3578	19.501	2.4386	14 9.095	3.5872	28 13.309	3.8370	34 4.053	3.1429

Year.	β Geminorum.		φ Geminorum.		15 Argus.		η Cancri.		ε Hydræ.	
	R. A.	Ann. var.	R. A.	Ann. var.	R. A.	Ann. var.	R. A.	Ann. var.	R. A.	Ann. var.
	7^h		7^h		7^h; 8^h		8^h		8^h	
1800 . .	33 3.462	3.6904	41 14.212	3.6914	59 1.710	2.5536	21 7.326	3.4899	36 10.422	3.1882
1830 . .	34 54.119	3.6867	43 4.899	3.6876	0 18.321	2.5538	22 51.966	3.4860	37 46.035	3.1860
1835 . .	35 12.551	3.6861	23.336	3.6870	31.091	2.5539	23 9.395	3.4854	38 1.964	3.1857
1840 . .	30.980	3.6855	41.769	3.6863	43.860	2.5539	26.820	3.4847	17.891	3.1853
1845 . .	49.405	3.6848	44 0.199	3.6857	56.630	2.5540	44.242	3.4841	33.817	3.1850
1850 . .	36 7.828	3.6842	18.626	3.6850	1 9.400	2.5540	24 1.661	3.4834	38 49.741	3.1846
1855 . .	26.248	3.6836	37.050	3.6844	22.170	2.5541	19.077	3.4828	39 5.663	3.1843
1860 . .	44.664	3.6829	55.470	3.6838	34.941	2.5541	36.489	3.4821	21.584	3.1839
1865 . .	37 3.077	3.6823	45 13.887	3.6831	47.712	2.5542	53.898	3.4815	37.502	3.1835
1870 . .	21.487	3.6817	32.301	3.6824	2 0.483	2.5542	25 11.304	3.4808	53.419	3.1832
1875 . .	39.894	3.6810	50.712	3.6818	13.254	2.5543	28.706	3.4802	40 9.334	3.1828
1880 . .	58.298	3.6804	46 9.119	3.6811	26.026	2.5543	46.105	3.4795	25.247	3.1825
1885 . .	38 16.699	3.6798	27.523	3.6805	38.797	2.5544	26 3.501	3.4789	41.159	3.1821
1890 . .	35.097	3.6791	45.924	3.6798	51.570	2.5544	20.894	3.4782	57.069	3.1818
1895 . .	53.491	3.6785	47 4.321	3.6792	3 4.342	2.5545	38.284	3.4776	41 12.977	3.1814
1900 . .	39 11.882	3.6778	22.715	3.6785	17.114	2.5545	55.670	3.4769	28.883	3.1811

***Right Ascensions of Time Stars for* 1800 *and for Quinquennial Epochs*, 1830–1900—Continued.**

Year.	ι Ursæ Majoris. R. A.	ι Ursæ Majoris. Ann. var.	κ Cancri. R. A.	κ Cancri. Ann. var.	α Hydræ. R. A.	α Hydræ. Ann. var.	θ Ursæ Majoris. R. A.	θ Ursæ Majoris. Ann. var.	ε Leonis. R. A.	ε Leonis. Ann. var.
	8^h		8^h; 9^h		9^h		9^h		9^h	
1800 . .	45 26.691	4.1730	56 53.985	3.2643	17 45.448	2.9504	19 23.885	4.0907	34 28.358	3.4312
1830 . .	47 31.682	4.1597	58 31.872	3.2615	19 13.953	2.9500	21 26.355	4.0740	36 11.211	3.4257
1835 . .	52.475	4.1575	48.178	3.2610	28.703	2.9499	46.718	4.0712	28.338	3.4248
1840 . .	48 13.257	4.1553	59 4.482	3.2605	43.452	2.9498	22 7.067	4.0684	45.460	3.4239
1845 . .	34.028	4.1531	20.783	3.2600	58.201	2.9497	27.403	4.0657	37 2.577	3.4230
1850 . .	54.788	4.1509	37.082	3.2596	20 12.950	2.9497	47.724	4.0629	19.690	3.4221
1855 . .	49 15.537	4.1486	53.379	3.2591	27.698	2.9496	23 8.032	4.0601	36.798	3.4212
1860 . .	36.274	4.1464	0 9.673	3.2586	42.446	2.9495	28.325	4.0573	53.902	3.4203
1865 . .	57.001	4.1442	25.965	3.2582	57.193	2.9495	48.605	4.0546	38 11.002	3.4194
1870 . .	50 17.716	4.1420	42.255	3.2577	21 11.940	2.9494	24 8.871	4.0518	28.097	3.4185
1875 . .	38.420	4.1397	58.542	3.2572	26.687	2.9493	29.123	4.0491	45.187	3.4176
1880 . .	59.114	4.1375	1 14.827	3.2568	41.433	2.9492	49.362	4.0463	39 2.273	3.4168
1885 . .	51 19.796	4.1353	31.110	3.2563	56.179	2.9492	25 9.586	4.0436	19.355	3.4159
1890 . .	40.467	4.1331	47.390	3.2558	22 10.925	2.9491	29.797	4.0408	36.432	3.4150
1895 . .	52 1.126	4.1308	2 3.668	3.2554	25.670	2.9490	49.994	4.0380	53.504	3.4141
1900 . .	21.775	4.1286	19.944	3.2549	40.416	2.9490	26 10.178	4.0353	40 10.572	3.4132

Year.	μ Leonis. R. A.	μ Leonis. Ann. var.	α Leonis. R. A.	α Leonis. Ann. var.	γ^1 Leonis. R. A.	γ^1 Leonis. Ann. var.	ρ Leonis. R. A.	ρ Leonis. Ann. var.	l Leonis. R. A.	l Leonis. Ann. var.
	9^h		9^h; 10^h		10^h		10^h		10^h	
1800 . .	41 21.668	3.4397	57 42.373	3.2096	8 55.528	3.3285	22 16.082	3.1715	38 43.953	3.1659
1830 . .	43 4.769	3.4338	59 18.617	3.2066	10 35.312	3.3239	23 51.190	3.1690	40 18.891	3.1634
1835 . .	21.936	3.4328	34.648	3.2061	51.930	3.3231	24 7.034	3.1686	34.707	3.1629
1840 . .	39.097	3.4318	50.677	3.2056	11 8.543	3.3223	22.876	2.1682	50.521	3.1625
1845 . .	56.253	3.4308	0 6.704	3.2050	25.153	3.3216	38.716	3.1678	41 6.332	3.1621
1850 . .	44 13.405	3.4298	22.728	3.2045	41.759	3.3208	54.554	3.1674	22.142	3.1617
1855 . .	30.552	3.4288	38.749	3.2040	58.361	3.3201	25 10.390	3.1670	37.949	3.1613
1860 . .	47.693	3.4278	54.768	3.2035	12 14.960	3.3193	26.224	3.1666	53.755	3.1609
1865 . .	45 4.830	3.4269	1 10.784	3.2030	31.554	3.3185	42.055	3.1661	42 9.558	3.1605
1870 . .	21.962	3.4259	26.798	3.2025	48.145	3.3178	57.885	3.1657	25.360	3.1601
1875 . .	39.089	3.4249	42.810	3.2020	13 4.732	3.3170	26 13.713	3.1653	41.159	3.1597
1880 . .	56.211	3.4239	58.818	3.2015	21.315	3.3163	29.538	3.1649	56.956	3.1593
1885 . .	46 13.328	3.4229	2 14.825	3.2010	37.895	3.3155	45.362	3.1645	43 12.751	3.1588
1890 . .	30.440	3.4220	30.829	3.2005	54.470	3.3148	27 1.184	3.1641	28.545	3.1584
1895 . .	47.548	3.4210	46.830	3.2000	14 11.042	3.3140	17.004	3.1637	44.336	3.1580
1900 . .	47 4.650	3.4200	3 2.829	3.1995	27.611	3.3133	32.821	3.1633	44 0.125	3.1576

***Right Ascensions of Time Stars for* 1800 *and for Quinquennial Epochs*, 1830–1900—Continued.**

Year.	α Ursæ Majoris. R. A.	α Ursæ Majoris. Ann. var.	δ Leonis. R. A.	δ Leonis. Ann. var.	δ Crateris. R. A.	δ Crateris. Ann. var.	τ Leonis. R. A.	τ Leonis. Ann. var.	ν Leonis. R. A.	ν Leonis. Ann. var.
	10^h		11^h		11^h		11^h		11^h	
1800 . .	51 15.575	3.8217	3 27.123	3.2103	9 21.077	2.9908	17 38.979	3.0881	26 42.597	3.0711
1830 . .	53 9.845	3.7964	5 3.371	3.2062	10 50.827	2.9926	19 11.611	3.0874	28 14.731	3.0711
1835 . .	28.816	3.7922	19.400	3.2055	11 5.791	2.9929	27.048	3.0873	30.087	3.0711
1840 . .	47.767	3.7881	35.426	3.2048	20.756	2.9932	42.484	3.0872	45.443	3.0711
1845 . .	54 6.697	3.7839	51.449	3.2042	35.723	2.9935	57.919	3.0870	29 0.798	3.0711
1850 . .	25.606	3.7798	6 7.468	3.2035	50.691	2.9938	20 13.354	3.0869	16.154	3.0712
1855 . .	44.495	3.7756	23.484	3.2028	12 5.661	2.9941	28.788	3.0868	31.510	3.0712
1860 . .	55 3.362	3.7715	39.496	3.2022	20.632	2.9944	44.222	3.0867	46.866	3.0712
1865 . .	22.210	3.7674	55.505	3.2015	35.605	2.9948	59.655	3.0866	30 2.222	3.0712
1870 . .	41.037	3.7633	7 11.511	3.2008	50.580	2.9951	21 15.088	3.0865	17.578	3.0712
1875 . .	59.843	3.7592	27.514	3.2002	13 5.556	2.9954	30.520	3.0864	32.934	3.0712
1880 . .	56 18.629	3.7552	43.513	3.1995	20.534	2.9957	45.952	3.0863	48.290	3.0712
1885 . .	37.395	3.7511	59.508	3.1988	35.513	2.9960	22 1.383	3.0862	31 3.646	3.0713
1890 . .	56.140	3.7471	8 15.501	3.1982	50.494	2.9963	16.814	3.0861	19.003	3.0713
1895 . .	57 14.865	3.7430	31.490	3.1975	14 5.476	2.9967	32.244	3.0860	34.359	3.0713
1900 . .	33.570	3.7390	47.476	3.1968	20.461	2.9970	47.674	3.0859	49.716	3.0713

Year.	β Leonis. R. A.	β Leonis. Ann. var.	γ Ursæ Majoris. R. A.	γ Ursæ Majoris. Ann. var.	ο Virginius. R. A.	ο Virginius. Ann. var.	γ Corvi. R. A.	γ Corvi. Ann. var.	η Virginius. R. A.	η Virginius. Ann. var.
	11^h		11^h		11^h		12^h		12^h	
1800 . .	38 50.879	3.0706	43 14.545	3.2214	55 1.029	3.0605	5 32.262	3.0695	9 40.627	3.0664
1830 . .	40 22.963	3.0683	44 50.985	3.2079	56 32.829	3.0595	7 04.395	3.0728	11 12.629	3.0671
1835 . .	38.304	3.0680	45 7.019	3.2057	48.126	3.0593	19.760	3.0733	27.965	3.0672
1840 . .	53.642	3.0676	23.042	3.2035	57 3.422	3.0591	35.128	3.0739	43.301	3.0673
1845 . .	41 8.980	3.0672	39.053	3.2012	18.717	3.0590	50.499	3.0745	58.638	3.0675
1850 . .	24.315	3.0669	55.054	3.1990	34.012	3.0588	8 05.873	3.0751	12 13.976	3.0676
1855 . .	39.648	3.0665	46 11.044	3.1968	49.306	3 0587	21.250	3.0756	29.314	3.0677
1860 . .	54.980	3.0661	27.022	3.1946	58 4.599	3.0585	36.629	3.0762	44.653	3.0678
1865 . .	42 10.309	3.0658	42.990	3.1924	19.891	3.0583	52.012	3.0768	59.993	3.0680
1870 . .	25.637	3.0654	58.946	3.1902	35.182	3.0582	9 07.397	3.0773	13 15.333	3.0681
1875 . .	40.963	3.0650	47 14.892	3.1880	50.473	3.0580	22.785	3.0779	30.674	3.0682
1880 . .	56.288	3.0647	30.827	3.1859	59 5.762	3.0579	38.176	3.0785	46.015	3.0684
1885 . .	43 11.610	3.0643	46.751	3.1837	21.051	3.0577	53.570	3.0791	14 1.357	3.0685
1890 . .	26.931	3.0640	48 2.664	3.1815	36.340	3.0576	10 08.967	3.0797	16.700	3.0686
1895 . .	42.250	3.0636	18.566	3.1794	51.627	3.0574	24.366	3.0802	32.044	3.0688
1900 . .	57.567	3.0633	34.457	3.1772	60 6.914	3.0573	39.769	3.0808	47.388	3.0689

***Right Ascensions of Time Stars for* 1800 *and for Quinquennial Epochs*, 1830–1900—Continued.**

Year.	β Corvi.		α Canum Venaticorum.		θ Virginis.		α Virginis.		ζ Virginis.	
	R. A.	Ann. var.	R. A.	Ann. var.	R. A.	Ann. var.	R. A.	Ann. var.	R. A.	Ann. var.
	12^h		12^h		12^h; 13^h		13^h		13^h	
1800 . .	23 54.450	3.1272	46 38.968	2.8291	59 36.462	3.0942	14 40.489	3.1436	24 30.748	3.0475
1830 . .	25 28.337	3.1320	48 3.769	2.8244	1 9.322	3.0965	16 14.846	3.1469	26 2.201	3.0494
1835 . .	43.999	3.1328	17.890	2.8236	24.806	3.0969	30.582	3.1475	17.449	3.0497
1840 . .	59.665	3.1336	32.006	2.8229	40.291	3.0972	46.321	3.1481	32.698	3.0500
1845 . .	26 15.335	3.1344	46.118	2.8221	55.778	3.0976	17 2.063	3.1486	47.949	3.0503
1850 . .	31.069	3.1352	49 0.227	2.8213	2 11.267	3.0980	17.807	3.1492	27 3.201	3.0506
1855 . .	46.687	3.1360	14.332	2.8206	26.758	3.0984	33.555	3.1498	18.455	3.0509
1860 . .	27 2.369	3.1368	28.433	2.8198	42.251	3.0988	49.305	3.1503	33.710	3.0512
1865 . .	18.055	3.1376	42.530	2.8191	57.746	3.0992	18 5.058	3.1509	48.967	3.0515
1870 . .	33.746	3.1385	56.624	2.8183	3 13.243	3.0996	20.814	3.1515	28 4.226	3.0519
1875 . .	49.440	3.1393	50 10.713	2.8176	28.741	3.1000	36.573	3.1520	19.486	3.0522
1880 . .	28 5.139	3.1401	24.800	2.8168	44.242	3.1003	52.334	3.1526	34.747	3.0525
1885 . .	20.841	3.1409	38.882	2.8161	59.745	3.1007	19 8.099	3.1532	50.011	3.0528
1890 . .	36.548	3.1417	52.960	2.8153	4 15.250	3.1011	23.866	3.1538	29 5.275	3.0531
1895 . .	52.259	3.1426	51 7.035	2.8146	30.756	3.1015	39.637	3.1544	20.542	3.0535
1900 . .	29 7.973	3.1434	21.106	2.8139	46.265	3.1019	55.410	3.1549	35.810	3.0538

Year.	η Ursæ Majoris.		η Bootis.		α Bootis.		θ Bootis.		ρ Bootis.	
	R. A.	Ann. var.	R. A.	Ann. var.	R. A.	Ann. var.	R. A.	Ann. var.	R. A.	Ann. var.
	13^h		13^h		14^h		14^h		14^h	
1800 . .	39 38.586	2.3806	45 9.710	2.8572	6 32.601	2.7330	18 23.156	2.0454	23 12.458	2.5894
1830 . .	40 49.955	2.3774	46 35.424	2.8570	7 54.599	2.7336	19 24.511	2.0450	24 30.130	2.5888
1835 . .	41 1.841	2.3769	49.709	2.8570	8 8.267	2.7337	34.735	2.0449	43.074	2.5887
1840 . .	13.724	2.3763	47 3.994	2.8570	21.936	2.7338	44.959	2.0448	56.017	2.5886
1845 . .	25.604	2.3758	18.279	2.8570	35.605	2.7339	55.183	2.0448	25 8.960	2.5885
1850 . .	37.482	2.3753	32.564	2.8569	49.275	2.7340	20 5.407	2.0447	21.902	2.5884
1855 . .	49.357	2.3748	46.849	2.8569	9 2.945	2.7341	15.630	2.0446	34.844	2.5883
1860 . .	42 1.230	2.3743	48 1.133	2.8569	16.616	2.7342	25.853	2.0446	47.785	2.5882
1865 . .	13.100	2.3737	15.417	2.8569	30.288	2.7344	36.076	2.0445	26 0.726	2.5881
1870 . .	24.967	2.3732	29.702	2.8568	43.960	2.7345	46.298	2.0444	13.667	2.5881
1875 . .	36.832	2.3727	43.986	2.8568	57.633	2.7346	56.520	2.0444	26.607	2.5880
1880 . .	48.694	2.3722	58.270	2.8568	10 11.306	2.7347	21 6.742	2.0443	39.546	2.5879
1885 . .	43 0.554	2.3717	49 12.554	2.8568	24.980	2.7348	16.963	2.0443	52.486	2.5878
1890 . .	12.411	2.3712	26.838	2.8568	38.654	2.7350	27.185	2.0442	27 5.425	2.5877
1895 . .	24.266	2.3707	41.121	2.8567	52.329	2.7351	37.405	2.0441	18.363	2.5876
1900 . .	36.118	2.3702	55.405	2.8567	11 6.005	2.7352	47.626	2.0441	31.301	2.5876

***Right Ascensions of Time Stars for* 1800 *and for Quinquennial Epochs*, 1830–1900—Continued.**

Year.	ε Bootis.		α^2 Libræ.		β Bootis.		β Libræ.		μ^1 Bootis.	
	R. A.	Ann. var.	R. A.	Ann. var.	R. A.	Ann. var.	R. A.	Ann. var.	R. A.	Ann. var.
	14^h		14^h		14^h		15^h		15^h	
1800 . .	36 15.120	2.6215	39 50.321	3.2957	54 24.772	2.2601	6 15.763	3.2113	16 56.209	2.2652
1830 . .	37 33.762	2.6214	41 29.263	3.3004	55 32.574	2.2601	7 52.153	3.2148	18 4.167	2.2655
1835 . .	46.869	2.6214	45.766	3.3011	43.874	2.2601	8 8.229	3.2154	15.495	2.2655
1840 . .	59.975	2.6214	42 2.274	3.3019	55.175	2.2601	24.307	3.2160	26.823	2.2656
1845 . .	38 13.082	2.6214	18.785	3.3026	56 6.475	2.2601	40.389	3.2166	38.151	2.2656
1850 . .	26.189	2.6214	35.301	3.3034	17.775	2.2601	56.473	3.2172	49.479	2.2657
1855 . .	39.296	2.6213	51.820	3.3042	29.075	2.2601	9 12.560	3.2177	19 0.808	2.2658
1860 . .	52.402	2.6213	43 8.343	3.3050	40.376	2.2601	28.650	3.2183	12.137	2.2658
1865 . .	39 5.509	2.6213	24.870	3.3058	51.676	2.2601	44.744	3.2189	23.466	2.2659
1870 . .	18.616	2.6213	41.401	3.3065	57 2.976	2.2601	10 0.840	3.2195	34.796	2.2660
1875 . .	31.723	2.6213	57.935	3.3073	14.276	2.2601	16.939	3.2201	46.126	2.2660
1880 . .	44.829	2.6213	44 14.474	3.3081	25.577	2.2601	33.041	3.2207	57.456	2.2661
1885 . .	57.936	2.6213	31.016	3.3089	36.877	2.2601	49.146	3.2213	20 8.787	2.2662
1890 . .	40 11.043	2.6213	47.562	3.3096	48.177	2.2601	11 5.254	3.2219	20.118	2.2662
1895 . .	24.149	2.6214	45 4.113	3.3104	59.478	2.2601	21.365	3.2225	31.449	2.2663
1900 . .	37.256	2.6214	20.667	3.3112	58 10.778	2.2601	37.478	3.2231	42.781	2.2664

Year.	α Coronæ Borealis.		α Serpentis.		ε Serpentis.		ε Coronæ Borealis.		δ Scorpii.	
	R. A.	Ann. var.	R. A.	Ann. var.	R. A.	Ann. var.	R. A.	Ann. var.	R. A.	Ann. var.
	15^h		15^h		15^h		15^h		15^h	
1800 . .	26 13.416	2.5372	34 25.586	2.9462	40 51.391	2.9812	49 18.705	2.4805	48 31.919	3.5244
1830 . .	27 29.541	2.5378	35 53.998	2.9480	42 20.859	2.9832	50 33.135	2.4814	50 17.723	3.5292
1835 . .	42.231	2.5380	36 8.739	2.9483	35.776	2.9835	45.543	2.4816	35.371	3.5300
1840 . .	54.921	2.5381	23.481	2.9486	50.694	2.9839	57.951	2.4817	53.023	3.5308
1845 . .	28 7.612	2.5382	38.225	2.9489	43 5.614	2.9842	51 10.360	2.4819	51 10.679	3.5316
1850 . .	20.303	2.5383	52.970	2.9492	20.536	2.9845	22.770	2.4820	28.339	3.5324
1855 . .	32.995	2.5384	37 7.717	2.9495	35.459	2.9848	35.181	2.4822	46.003	3.5332
1860 . .	45.687	2.5386	22.465	2.9498	50.384	2.9852	47.592	2.4823	52 3.671	3.5340
1865 . .	58.380	2.5387	37.215	2.9501	44 5.311	2.9855	52 0.004	2.4825	21.343	3.5348
1870 . .	29 11.074	2.5388	51.966	2.9504	20.239	2.9858	12.417	2.4826	39.019	3.5356
1875 . .	23.768	2.5389	38 6.719	2.9507	35.169	2.9861	24.830	2.4828	56.699	3.5364
1880 . .	36.463	2.5390	21.473	2.9510	50.101	2.9865	37.245	2.4829	53 14.383	3.5372
1885 . .	49.159	2.5392	36.229	2.9513	45 5.034	2.9868	49.660	2.4831	32.071	3.5380
1890 . .	30 1.855	2.5393	50.987	2.9516	19.969	2.9871	53 2.075	2.4832	49.763	3.5388
1895 . .	14.552	2.5394	39 5.746	2.9520	34.905	2.9875	14.492	2.4834	54 7.459	3.5396
1900 . .	27.249	2.5395	20.506	2.9523	49.844	2.9878	26.909	2.4835	25.159	3.5404

***Right Ascensions of Time Stars for* 1800 *and for Quinquennial Epochs*, 1830–1900—Continued.**

Year.	β^1 Scorpii.		δ Ophiuchi.		τ Herculis.		α Scorpii.		η Draconis.	
	R. A.	Ann. var.	R. A.	Ann. var.	R. A.	Ann. var.	R. A.	Ann. var.	R. A.	Ann. var.
	15^h		16^h		16^h		16^h		16^h	
1800 . .	53 49.770	3.4680	3 52.628	3.1322	13 44.202	1.7965	17 10.056	3.6566	21 18.422	0.7899
1830 . .	55 33.875	3.4723	5 26.631	3.1347	14 38.117	1.7980	18 59.822	3.6612	21 42.199	.7954
1835 . .	51.238	3.4730	42.306	3.1351	47.108	1.7982	19 18.130	3.6619	46.179	.7963
1840 . .	56 8.605	3.4737	57.983	3.1355	56.100	1.7985	36.442	3.6627	50.162	.7972
1845 . .	25.976	3.4744	6 13.661	3.1359	15 5.093	1.7987	54.757	3.6635	54.151	.7982
1850 . .	43.350	3.4752	29.342	3.1364	14.087	1.7990	20 13.076	3.6642	58.144	.7991
1855 . .	57 0.728	3.4759	45.025	3.1368	23.083	1.7993	31.399	3.6650	22 2.142	.8000
1860 . .	18.109	3.4766	7 0.710	3.1372	32.080	1.7995	49.726	3.6657	6.144	.8009
1865 . .	35.494	3.4773	16.397	3.1376	41.078	1.7998	21 8.057	3.6665	10.151	.8019
1870 . .	52.882	3.4780	32.086	3.1380	50.077	1.8000	26.391	3.6673	14.163	.8028
1875 . .	58 10.274	3.4788	47.777	3.1384	59.078	1.8003	44.730	3.6680	18.179	.8037
1880 . .	27.670	3.4795	8 3.470	3.1388	16 8.080	1.8005	22 3.072	3.6688	22.200	.8046
1885 . .	45.069	3.4802	19.165	3.1393	17.084	1.8008	21.417	3.6695	26.225	.8055
1890 . .	59 2.472	3.4809	34.863	3.1397	26.088	1.8011	39.767	3.6703	30.255	.8064
1895 . .	19.878	3.4816	50.562	3.1401	35.094	1.8013	58.120	3.6710	34.290	.8074
1900 . .	37.288	3.4823	9 6.264	3.1405	44.101	1.8016	23 16.477	3.6718	38.329	0.8083

Year.	β Herculis.		ζ Ophiuchi.		η Herculis.		κ Ophiuchi.		d Herculis.	
	R. A.	Ann. var.	R. A.	Ann. var.	R. A.	Ann. var.	R. A.	Ann. var.	R. A.	Ann. var.
	16^h		16^h		16^h		16^h		16^h	
1800 . .	21 37.649	2.5742	26 9.540	3.2911	36 2.778	2.0504	48 12.510	2.8336	54 13.798	2.2084
1830 . .	22 54.892	2.5753	27 48.315	3.2938	37 4.305	2.0515	49 37.535	2.8349	55 20.064	2.2094
1835 . .	23 7.769	2.5755	28 4.785	3.2942	14.563	2.0517	51.710	2.8351	31.112	2.2095
1840 . .	20.646	2.5757	21.258	3.2947	24.822	2.0519	50 5.886	2.8353	42.160	2.2097
1845 . .	33.525	2.5759	37.732	3.2952	35.082	2.0521	20.063	2.8355	53.208	2.2098
1850 . .	46.405	2.5760	54.209	3.2957	45.343	2.0523	34.241	2.8357	56 4.258	2.2100
1855 . .	59.286	2.5762	29 10.688	3.2960	55.605	2.0525	48.420	2.8359	15.308	2.2102
1860 . .	24 12.167	2.5764	27.169	3.2965	38 5.868	2.0526	51 2.600	2.8362	26.360	2.2103
1865 . .	25.050	2.5766	43.653	3.2969	16.131	2.0528	16.782	2.8364	37.412	2.2105
1870 . .	37.933	2.5768	30 0.138	3.2973	26.396	2.0530	30.964	2.8366	48.465	2.2107
1875 . .	50.817	2.5769	16.626	3.2978	36.662	2.0532	45.148	2.8368	59.518	2.2108
1880 . .	25 3.703	2.5771	33.116	3.2982	46.928	2.0534	59.332	2.8370	57 10.573	2.2110
1885 . .	16.589	2.5773	49.608	3.2987	57.196	2.0536	52 13.518	2.8373	21.628	2.2111
1890 . .	29.476	2.5775	31 6.103	3.2991	39 7.464	2.0538	27.705	2.8375	32.684	2.2113
1895 . .	42.364	2.5777	22.599	3.2995	17.733	2.0540	41.893	2.8377	43.741	2.2115
1900 . .	55.252	2.5779	39.098	3.3000	28.004	2.0542	56.082	2.8379	54.799	2.2116

***Right Ascensions of Time Stars for* 1800 *and for Quinquennial Epochs,* 1830–1900—Continued.**

Year.	α^1 Herculis.		*b* Ophiuchi.		β Draconis.		α Ophiuchi.		μ Herculis.	
	R. A.	Ann. var.	R. A.	Ann. var.	R. A.	Ann. var.	R. A.	Ann. var.	R. A.	Ann. var.
	17^h		17^h		17^h		17^h		17^h	
1800 . .	5 32.029	2.7304	14 10.128	3.6522	25 55.274	1.3487	25 39.377	2.7798	38 38.219	2.3429
1830 . .	6 53.956	2.7314	15 59.731	3.6546	26 35.760	1.3503	27 2.788	2.7809	39 48.524	2.3441
1835 . .	7 7.614	2.7316	16 18.005	3.6550	42.512	1.3506	16.692	2.7810	40 0.245	2.3443
1840 . .	21.273	2.7318	36.281	3.6553	49.265	1.3508	30.598	2.7812	11.967	2.3445
1845 . .	34.932	2.7320	54.558	3.6557	56.020	1.3511	44.504	2.7814	23.690	2.3447
1850 . .	48.592	2.7321	17 12.838	3.6561	27 2.776	1.3513	58.411	2.7815	35.414	2.3449
1855 . .	8 2.253	2.7323	31.119	3.6565	9.533	1.3516	28 12.319	2.7817	47.139	2.3451
1860 . .	15.915	2.7325	49.403	3.6568	16.292	1.3519	26.228	2.7819	58.865	2.3452
1865 . .	29.578	2.7327	18 7.688	3.6572	23.052	1.3521	40.138	2.7820	41 10.591	2.3454
1870 . .	43.242	2.7328	25.975	3.6576	29.813	1.3524	54.049	2.7822	22.319	2.3456
1875 . .	56.906	2.7330	44.264	3.6580	36.576	1.3526	29 7.960	2.7824	34.047	2.3458
1880 . .	9 10.572	2.7332	19 2.554	3.6583	43.339	1.3529	21.872	2.7825	45.777	2.3460
1885 . .	24.238	2.7333	20.847	3.6587	50.104	1.3531	35.785	2.7827	57.507	2.3462
1890 . .	37.905	2.7335	39.141	3.6590	56.871	1.3534	49.699	2.7829	42 9.239	2.3464
1895 . .	51.573	2.7337	57.437	3.6594	28 3.638	1.3536	30 3.614	2.7830	20.971	2.3466
1900 . .	10 5.242	2.7339	20 15.735	3.6598	10.407	1.3539	17.530	2.7832	32.704	2.3467

Year.	γ Draconis.		γ^2 Sagittarii.		μ Sagittarii.		η Serpentis.		1 Aquilæ.	
	R. A.	Ann. var.	R. A.	Ann. var.	R. A.	Ann. var.	R. A.	Ann. var.	R. A.	Ann. var.
	17^h		17^h		18^h		18^h		18^h	
1800 . .	51 58.008	1.3886	52 57.951	3.8491	1 48.346	3.5856	10 57.941	3.1005	24 19.468	3.2644
1830 . .	52 39.681	1.3896	54 53.439	3.8500	3 35.919	3.5860	12 30.969	3.1012	25 57.401	3.2645
1835 . .	46.629	1.3897	55 12.690	3.8502	53.849	3.5860	46.475	3.1013	26 13.724	3.2645
1840 .	53.578	1.3899	31.941	3.8503	4 11.779	3.5861	13 1.982	3.1014	30.046	3.2645
1845 . .	53 0.528	1.3901	51.193	3.8504	29.710	3.5862	17.489	3.1015	46.368	3.2645
1850 . .	7.479	1.3902	56 10.445	3.8505	47.641	3.5862	32.997	3.1016	27 2.691	3.2645
1855 . .	14.431	1.3904	29.698	3.8507	5 5.572	3.5863	48.505	3.1017	19.014	3.2645
1860 . .	21.382	1.3906	48.952	3.8508	23.504	3.5863	14 4.014	3.1018	35.336	3.2645
1865 . .	28.336	1.3907	57 8.206	3.8509	41.435	3.5864	19.523	3.1019	51.659	3.2645
1870 . .	35.290	1.3909	27.461	3.8510	59.367	3.5864	35.033	3.1020	28 7.981	3.2645
1875 . .	42.245	1.3910	46.717	3.8512	6 17.300	3.5865	50.543	3.1021	24.304	3.2645
1880 . .	49.200	1.3912	58 5.973	3.8513	35.232	3.5865	15 6.053	3.1022	40.627	3.2645
1885 . .	56.157	1.3914	25.229	3.8514	53.165	3.5866	21.564	3.1022	56.949	3.2645
1890 . .	54 3.114	1.3915	44.487	3.8515	7 11.098	3.5866	37.076	3.1023	29 13.272	3.2645
1895 . .	10.072	1.3917	59 3.744	3.8516	29.031	3.5866	52.588	3.1024	29.595	3.2645
1900 . .	17.031	1.3918	23.003	3.8517	46.964	3.5867	16 8.100	3.1025	45.917	3.2645

***Right Ascensions of Time Stars for* 1800 *and for Quinquennial Epochs,* 1830–1900—Continued.**

Year.	α Lyræ.		β Lyræ.		σ Sagittarii.		ζ Aquilæ.		d Sagittarii.	
	R. A.	Ann. var.	R. A.	Ann. var.	R. A.	Ann. var.	R. A.	Ann. var.	R. A.	Ann. var.
	18^h		18^h		18^h		18^h; 19^h		19^h	
1800 . .	30 10.089	2.0304	42 41.917	2.2129	42 51.515	3.7261	56 13.163	2.7565	5 55.579	3.5176
1830 . .	31 11.005	2.0307	43 48.310	2.2133	44 43.278	3.7247	57 35.859	2.7566	7 41.081	3.5159
1835 . .	21.159	2.0307	59.377	2.2134	45 1.901	3.7245	49.643	2.7567	58.660	3.5156
1840 . .	31.313	2.0308	44 10.444	2.2135	20.522	3.7242	58 3.426	2.7567	8 16.237	3.5153
1845 . .	41.467	2.0308	21.512	2.2136	39.143	3.7240	17.210	2.7567	33.813	3.5150
1850 . .	51.621	2.0309	32.580	2.2136	57.762	3.7237	30.993	2.7567	51.387	3.5147
1855 . .	32 1.776	2.0310	43.648	2.2137	46 16.380	3.7235	44.777	2.7567	9 8.960	3.5144
1860 . .	11.931	2.0310	54.717	2.2138	34.997	3.7232	58.560	2.7568	26.531	3.5141
1865 . .	22.086	2.0311	45 5.786	2.2139	53.612	3.7229	59 12.344	2.7568	44.100	3.5138
1870 . .	32.241	2.0311	16.856	2.2139	47 12.226	3.7227	26.128	2.7568	10 1.669	3.5135
1875 . .	42.397	2.0312	27.925	2.2140	30.839	3.7224	39.912	2.7568	19.235	3.5132
1880 . .	52.553	2.0312	38.996	2.2141	49.450	3.7222	53.696	2.7568	36.800	3.5129
1885 . .	33 2.709	2.0313	50.066	2.2141	48 8.060	3.7219	0 7.481	2.7569	54.364	3.5126
1890 . .	12.865	2.0313	46 1.137	2.2142	26.669	3.7216	21.265	2.7569	11 11.926	3.5123
1895 . .	23.022	2.0314	12.208	2.2143	45.277	3.7214	35.049	2.7569	29.486	3.5119
1900 . .	33.179	2.0314	23.280	2.2144	49 3.883	3.7211	48.834	2.7569	47.045	3.5116

Year.	δ Aquilæ.		κ Aquilæ.		γ Aquilæ.		α Aquilæ.		β Aquilæ.	
	R. A.	Ann. var.	R. A.	Ann. var.	R. A.	Ann. var.	R. A.	Ann. var.	R. A.	Ann. var.
	19^h		19^h		19^h		19^h		19^h	
1800 . .	15 24.783	3.0268	26 7.626	3.2328	36 45.066	2.8532	41 1.425	2.9293	45 29.308	2.9483
1830 . .	16 55.581	3.0263	27 44.591	3.2315	38 10.657	2.8529	42 29.296	2.9287	46 57.752	2.9479
1835 . .	17 10.712	3.0262	28 0.748	3.2313	24.921	2.8528	43.939	2.9286	47 12.492	2.9479
1840 . .	25.843	3.0261	16.904	3.2311	39.185	2.8528	58.582	2.9286	27.231	2.9478
1845 . .	40.974	3.0261	33.059	3.2309	53.448	2.8527	43 13.225	2.9285	41.970	2.9477
1850 . .	56.104	3.0260	49.213	3.2307	39 7.712	2.8527	27.867	2.9284	56.708	2.9476
1855 . .	18 11.234	3.0259	29 5.366	3.2304	21.975	2.8526	42.508	2.9283	48 11.446	2.9476
1860 . .	26.363	3.0258	21.517	3.2302	36.238	2.8526	57.150	2.9282	26.184	2.9475
1865 . .	41.492	3.0257	37.668	3.2300	50.500	2.8525	44 11.790	2.9281	40.921	2.9474
1870 . .	56.620	3.0256	53.817	3.2298	40 4.763	2.8524	26.430	2.9280	55.658	2.9473
1875 . .	19 11.748	3.0255	30 9.966	3.2296	19.025	2.8524	41.070	2.9279	49 10.394	2.9473
1880 . .	26.875	3.0254	26.113	3.2293	33.287	2.8523	55.710	2.9278	25.131	2.9472
1885 . .	42.002	3.0253	42.259	3.2291	47.548	2.8523	45 10.349	2.9277	39.866	2.9471
1890 . .	57.128	3.0252	58.404	3.2289	41 1.810	2.8522	24.987	2.9276	54.602	2.9471
1895 . .	20 12.254	3.0251	31 14.548	3.2287	16.071	2.8522	39.625	2.9275	50 9.337	2.9470
1900 . .	27.380	3.0251	30.691	3.2284	30.331	2.8521	54.262	2.9274	24.072	2.9469

***Right Ascensions of Time Stars for 1800 and for Quinquennial Epochs, 1830–1900*—Continued.**

Year.	τ Aquilæ.			α² Capricorni.			γ Cygni.			π Capricorni.			ε Delphini.		
	R. A.		Ann. var.	R. A.		Ann. var.	R. A.		Ann. var.	R. A.		Ann. var.	R. A.		Ann. var.
	19h			20h			20h			20h			20h		
1800 . .	54	21.975	2.9349	6	56.844	3.3397	15	3.177	2.1520	15	51.493	3.4499	23	39.394	2.8683
1830 . .	55	50.011	2.9343	8	36.999	3.3372	16	7.747	2.1526	17	34.935	3.4464	25	5.438	2.8679
1835 . .	56	4.682	2.9342		53.684	3.3368		18.510	2.1527		52.166	3.4458		19.777	2.8679
1840 . .		19.353	2.9341	9	10.367	3.3364		29.274	2.1528	18	9.393	3.4452		34.116	2.8678
1845 . .		34.023	2.9340		27.048	3.3360		40.038	2.1529		26.618	3.4447		48.455	2.8677
1850 . .		48.692	2.9339		43.726	3.3355		50.803	2.1530		43.840	3.4441	26	2.794	2.8677
1855 . .	57	3.361	2.9338	10	0.403	3.3351	17	1.568	2.1531	19	1.059	3.4435		17.132	2.8676
1860 . .		18.030	2.9337		17.078	3.3347		12.334	2.1532		18.275	3.4429		31.470	2.8675
1865 . .		32.698	2.9336		33.750	3.3343		23.100	2.1533		35.488	3.4424		45.808	2.8675
1870 . .		47.365	2.9335		50.421	3.3339		33.866	2.1534		52.699	3.4418	27	0.145	2.8674
1875 . .	58	2.032	2.9334	11	7.090	3.3334		44.633	2.1535	20	9.906	3.4412		14.482	2.8674
1880 . .		16.699	2.9333		23.756	3.3330		55.401	2.1536		27.111	3.4406		28.819	2.8673
1885 . .		31.365	2.9332		40.420	3.3326	18	6.169	2.1537		44.313	3.4400		43.155	2.8672
1890 . .		46.030	2.9331		57.082	3.3322		16.937	2.1537	21	1.512	3.4395		57.492	2.8672
1895 . .	59	0.695	2.9330	12	13.742	3.3317		27.706	2.1538		18.708	3.4389	28	11.827	2.8671
1900 . .		15.360	2.9329		30.400	3.3313		38.476	2.1539		35.901	3.4383		26.163	2.8671

Year.	α Cygni.			μ Aquarii.			ν Cygni.			61 Cygni (pr).			ζ Cygni.		
	R. A.		Ann. var.	R. A.		Ann. var.	R. A.		Ann. var.	R. A.		Ann. var.	R. A.		Ann. var.
	20h			20h			20h			20h; 21h			21h		
1800 . .	34	37.034	2.0424	41	51.342	3.2472	49	43.410	2.2308	57	56.643	2.6795	4	25.934	2.5461
1830 . .	35	38.316	2.0431	43	28.720	3.2447	50	50.352	2.2319	59	17.046	2.6807	5	42.337	2.5473
1835 . .		48.532	2.0432		44.943	3.2443	51	1.512	2.2321		30.450	2.6809		55.074	2.5475
1840 . .		58.748	2.0433	44	1.163	3.2439		12.673	2.2323		43.855	2.6811	6	7.812	2.5477
1845 . .	36	8.965	2.0434		17.382	3.2435		23.835	2.2325		57.261	2.6813		20.551	2.5479
1850 . .		19.182	2.0435		33.598	3.2431		34.998	2.2327	0	10.668	2.6815		33.291	2.5481
1855 . .		29.399	2.0436		49.812	3.2427		46.162	2.2329		24.076	2.6817		46.032	2.5483
1860 . .		39.618	2.0437	45	6.025	3.2422		57.327	2.2330		37.485	2.6819		58.774	2.5485
1865 . .		49.836	2.0438		22.235	3.2418	52	8.492	2.2332		50.896	2.6822	7	11.517	2.5487
1870 . .	37	0.056	2.0439		38.443	3.2414		19.659	2.2334	1	4.307	2.6824		24.260	2.5489
1875 . .		10.276	2.0440		54.649	3.2410		30.826	2.2336		17.719	2.6826		37.005	2.5491
1880 . .		20.496	2.0441	46	10.853	3.2406		41.995	2.2338		31.133	2.6828		49.751	2.5492
1885 . .		30.717	2.0443		27.055	3.2402		53.164	2.2340		44.547	2.6830	8	2.498	2.5494
1890 . .		40.939	2.0444		43.255	3.2398	53	4.334	2.2342		57.963	2.6832		15.245	2.5496
1895 . .		51.161	2.0445		59.453	3.2394		15.506	2.2343	2	11.379	2.6834		27.994	2.5498
1900 .	38	1.383	2.0446	47	15.649	3.2390		26.678	2.2345		24.796	2.6835		40.744	2.5500

Right Ascensions of Time Stars for 1800 *and for Quinquennial Epochs*, 1830–1900—**Continued.**

Year.	α Cephei.		ι Pegasi.		β Aquarii.		ξ Aquarii.		ε Pegasi.	
	R. A.	Ann. var.	R. A.	Ann. var.	R. A.	Ann. var.	R. A.	Ann. var.	R. A.	Ann. var.
	21^h		21^h		21^h		21^h		21^h	
1800 . .	13 47.703	1.4425	12 50.507	2.7705	21 1.254	3.1681	27 5.647	3.2052	34 21.799	2.9472
1830 . .	14 30.946	1.4405	14 13.633	2.7711	22 36.267	3.1660	28 41.770	3.2028	35 50.213	2.9470
1835 . .	38.147	1.4401	27.489	2.7712	52.096	3.1657	57.783	3.2024	36 4.949	2.9470
1840 . .	45.347	1.4398	41.345	2.7713	23 7.924	3.1653	29 13.794	3.2020	19.683	2.9470
1845 . .	52.545	1.4395	55.202	2.7714	23.749	3.1649	29.803	3.2016	34.418	2.9469
1850 . .	59.742	1.4392	15 9.059	2.7715	39.573	3.1646	45.810	3.2012	49.153	2.9469
1855 . .	15 6.937	1.4388	22.917	2.7716	55.395	3.1642	30 1.815	3.2008	37 3.888	2.9469
1860 . .	14.130	1.4385	36.775	2.7717	24 11.215	3.1639	17.818	3.2004	18.622	2.9469
1865 . .	21.322	1.4382	50.633	2.7717	27.034	3.1635	33.819	3.2000	33.356	2.9468
1870 . .	28.512	1.4379	16 4.492	2.7718	42.851	3.1632	49.817	3.1996	48.090	2.9468
1875 . .	35.701	1.4375	18.351	2.7719	58.666	3.1628	31 5.814	3.1991	38 2.824	2.9468
1880 . .	42.888	1.4372	32.211	2.7720	25 14.479	3.1625	21.809	3.1987	17.558	2.9467
1885 . .	50.073	1.4369	46.072	2.7721	30.290	3.1621	37.801	3.1983	32.292	2.9467
1890 . .	57.256	1.4365	59.932	2.7722	46.100	3.1617	53.792	3.1979	47.025	2.9467
1895 . .	16 4.438	1.4362	17 13.794	2.7723	26 1.908	3.1614	32 9.780	3.1975	39 1.759	2.9467
1900 . .	11.619	1.4359	27.656	2.7724	17.714	3.1610	25.767	3.1971	16.492	2.9466

Year.	μ Capricorni.		α Aquarii.		α Gruis.		θ Aquarii.		π Aquarii.	
	R. A.	Ann. var.	R. A.	Ann. var.	R. A.	Ann. var.	R. A.	Ann. var.	R. A.	Ann. var.
	21^h		21^h; 22^h		21^h; 22^h		22^h		22^h	
1800 . .	42 22.636	3.2861	55 30.433	3.0866	55 33.507	3.8472	6 16.221	3.1760	15 3.638	3.0672
1830 . .	44 1.169	3.2827	57 3.010	3.0852	57 28.717	3.8334	7 51.467	3.1737	16 35.644	3.0664
1835 . .	17.582	3.2822	18.435	3.0850	47.878	3.8311	8 7.335	3.1733	50.976	3.0663
1840 . .	33.991	3.2816	33.860	3.0848	7.027	3.8288	23.201	3.1729	17 6.307	3.0661
1845 . .	50.397	3.2810	49.283	3.0846	26.165	3.8265	39.064	3.1725	21.637	3.0660
1850 . .	45 6.801	3.2805	58 4.706	3.0844	45.292	3.8242	54.926	3.1722	36.967	3.0659
1855 . .	23.202	3.2799	20.127	3.0842	59 4.407	3.8219	9 10.786	3.1718	52.296	3.0657
1860 . .	39.600	3.2793	35.548	3.0840	23.511	3.8196	26.644	3.1714	18 7.624	3.0656
1865 . .	55.995	3.2787	50.967	3.0838	42.603	3.8173	42.500	3.1710	22.951	3.0655
1870 . .	46 12.387	3.2782	59 6.385	3.0835	0 1.683	3.8150	58.354	3.1706	38.278	3.0653
1875 . .	28.777	3.2776	21.802	3.0833	20.752	3.8127	10 14.206	3.1702	53.604	3.0651
1880 . .	45.163	3.2770	37.218	3.0831	39.810	3.8104	30.056	3.1699	19 8.929	3.0650
1885 . .	47 1.547	3.2765	52.633	3.0829	58.856	3.8081	45.905	3.1695	24.254	3.0648
1890 . .	17.928	3.2759	0 8.048	3.0827	1 17.891	3.8058	11 1.751	3.1691	39.578	3.0647
1895 . .	34.306	3.2754	23.461	3.0825	36.914	3.8035	17.596	3.1687	54.901	3.0646
1900 . .	50.681	3.2748	38.873	3.0823	55.926	3.8013	33.438	3.1683	20 10.224	3.0644

***Right Ascensions of Time Stars for* 1800 *and for Quinquennial Epochs*, 1830–1900—Continued.**

Year.	η Aquarii.			ζ Pegasi.			λ Aquarii.			α Piscis Australis.		
	R. A.		Ann. var.	R. A.		Ann. var.	R. A.		Ann. var.	R. A.		Ann. var.
	22ʰ			22ʰ			22ʰ			22ʰ		
1800 . .	25	4.578	3.0865	31	29.473	2.9890	42	10.361	3.1386	46	34.189	3.3441
1830 . .	26	37.158	3.0855	32	59.152	2.9896	43	44.490	3.1366	48	14.414	3.3376
1835 . .		52.585	3.0853	33	14.100	2.9897	44	0.172	3.1363		31.100	3.3365
1840 . .	27	8.012	3.0852		29.049	2.9898		15 852	3.1360		47.779	3.3354
1845 . .		23.437	3.0850		43.998	2.9899		31.532	3.1356	49	4.454	3.3343
1850 . .		38.862	3.0849		58.948	2.9900		47.209	3.1353		21.123	3.3333
1855 . .		54.286	3.0847	34	13.898	2.9901	45	2.885	3.1350		37.786	3.3322
1860 . .	28	9.709	3.0846		28.849	2.9902		18.559	3.1347		54.445	3.3311
1865 . .		25.131	3.0844		43.801	2.9903		34.232	3.1344	50	11.097	3.3300
1870 . .		40.553	3.0842		58.753	2.9905		49.903	3.1340		27.745	3.3290
1875 . .		55.974	3.0841	35	13.705	2.9906	46	5.572	3.1337		44.387	3.3279
1880 . .	29	11.394	3.0839		28.659	2.9907		21.240	3.1334	51	1.024	3.3268
1885 . .		26.813	3.0838		43.612	2.9908		36.906	3.1331		17.656	3.3258
1890 . .		42.232	3.0836		58.567	2.9909		52.571	3.1328		34.282	3.3247
1895 . .		57.649	3.0835	36	13.522	2.9910	47	8.234	3.1325		50.903	3.3236
1900 . .	30	13.067	3.0833		28.477	2.9912		23.895	3.1322	52	7.518	3.3226

Year.	α Pegasi.			θ Piscium.			ι Piscium.			ω Piscium.		
	R. A.		Ann. var.	R. A.		Ann. var.	R. A.		Ann. var.	R. A.		Ann. var.
	22ʰ			23ʰ			23ʰ			23ʰ		
1800 . .	54	48.470	2.9801	17	49.685	3.0389	29	40.104	3.0815	49	2.903	3.0744
1830 . .	56	17.896	2.9817	19	20.862	3.0396	31	12.561	3.0823	50	35.152	3.0756
1835 . .		32.805	2.9819		36.060	3.0397		27.973	3.0825		50.530	3.0758
1840 . .		47.715	2.9822		51.259	3.0398		43.386	3.0826	51	5.910	3.0761
1845 . .	57	2.627	2.9825	20	6.458	3.0399		58.800	3.0828		21.291	3.0763
1850 . .		17.540	2.9828		21.658	3.0401	32	14.214	3.0829		36.673	3.0765
1855 . .		32.455	2.9830		36.859	3.0402		29.629	3.0831		52.056	3.0768
1860 . .		47.370	2.9833		52.060	3.0403		45.045	3.0832	52	7.441	3.0770
1865 . .	58	2.288	2.9836	21	7.262	3.0405	33	0.461	3.0834		22.826	3.0772
1870 . .		17.206	2.9839		22.464	3.0406		15.878	3.0835		38.213	3.0775
1875 . .		32.126	2.9841		37.668	3.0407		31.296	3.0837		53.601	3.0777
1880 . .		47.048	2.9844		52.872	3 0409		46.715	3.0838	53	8.990	3.0779
1885 . .	59	1.971	2.9847	22	8.076	3.0410	34	2.135	3.0840		24.380	3.0782
1890 . .		16.895	2.9850		23.282	3.0411		17.555	3.0841		39.772	3.0784
1895 . .		31.821	2.9853		38.487	3.0413		32.976	3.0843		55.164	3.0787
1900 . .		46.748	2.9856		53.694	3.0414		48.398	3.0845	54	10.558	3.0789

ON

GAUSS'S METHOD

OF COMPUTING

SECULAR PERTURBATIONS,

WITH AN APPLICATION TO THE ACTION OF VENUS ON MERCURY.

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I

ON GAUSS'S METHOD OF COMPUTING SECULAR PERTURBATIONS.

In 1818 Gauss presented to the Royal Society of Sciences at Göttingen a memoir, the full title of which is *Determinatio Attractionis quam in punctum quodvis positionis datæ exerceret planeta si ejus massa per totam orbitam ratione temporis quo singulæ partes describuntur uniformiter esset dispertita.* (*Werke, Band* III, s. 331.)

This memoir is a notable one in the history of elliptic functions, as it contains a new algorithm for the computation of the complete functions of Legendre's first and second species. But we shall at present view it from the side of celestial mechanics. Gauss investigates the expressions for the components of the attraction of a certain species of elliptic ring on a point, which can be advantageously employed in computing the secular perturbations of a planet, at least the parts of them which are of the first order with respect to the disturbing forces. This method merits attention because, with it, we can secure almost absolute accuracy at the cost of a comparatively small outlay of labor. Moreover, it is capable of being applied, with success, to all the asteroids, and even to such refractory cases as the periodic comets. Yet, I can find but two published investigations where it has been employed. The first, a computation of the secular perturbations of the earth by Nicolai, results only being given (*Berliner Astronomische Jahrbuch für* 1820). The second, an application of the method to Tuttle's periodic comet by Dr. Thomas Clausen (*Dorpater Beobachtungen, Band* XVI, *Einleitung*). This, perhaps, is due to the circumstance that the memoir of Gauss does not contain all the formulæ needed in the application. A double integration being necessary, Gauss has considered only that in respect to the eccentric anomaly of the disturbing body, and, having regard to elegance only, has not reduced his equations to the forms giving the utmost brevity of calculation. Hence, I propose to give an exposition of the method with the additional formulæ required.

The following notation will be adopted: For the quantities pertaining to the disturbed planet, let

a denote the semi-axis major,
n " " mean motion in a Julian year,
e " " eccentricity,
φ " " angle of the eccentricity, such that $e = \sin\varphi$,
π " " longitude of the perihelion measured from a fixed equinox,
i " " inclination of the orbit to a fixed ecliptic,
Ω " " longitude of the ascending node of the orbit on the fixed ecliptic,
L " " mean longitude at the epoch,
χ " " longitude of the perihelion measured from a point fixed in the shifting plane of the orbit,

ω denote the angular distance of the perihelion from the ascending node $= \pi - \Omega$,
r " " radius vector,
M, E, v " " mean, eccentric, and true anomalies,
u " " argument of the latitude $= v + \omega$,
m " " mass of the planet, the sun's being taken as the unit,
p " " semi-parameter $= a(1 - e^2)$.

The similar quantities belonging to the disturbing planet will be denoted by the same letters accented. In addition, let R denote the component of the disturbing force in the direction of the radius vector, positive outward from the sun; S the component of the same perpendicular to the radius vector and in the plane of the orbit, positive in the direction of motion; and W the component perpendicular to the plane of the orbit, positive northward.

The differential equations, which give the variations of the elements of the disturbed planet, are

$$\frac{da}{dt} = \frac{2a^3 n \sec\varphi}{1+m}\left[e\sin v.\,\mathrm{R} + \frac{p}{r}\mathrm{S}\right]$$

$$\frac{de}{dt} = \frac{a^2 n \cos\varphi}{1+m}\left[\sin v.\,\mathrm{R} + (\cos v + \cos \mathrm{E})\mathrm{S}\right]$$

$$e\frac{d\chi}{dt} = \frac{a^2 n \cos\varphi}{1+m}\left[-\cos v.\,\mathrm{R} + \left(\frac{r}{p} + 1\right)\sin v.\,\mathrm{S}\right]$$

$$\frac{di}{dt} = \frac{an\sec\varphi}{1+m}\,r\cos u.\,\mathrm{W}$$

$$\sin i\,\frac{d\Omega}{dt} = \frac{an\sec\varphi}{1+m}\,r\sin u.\,\mathrm{W}$$

$$\frac{d\pi}{dt} = \frac{d\chi}{dt} + 2\sin^2\frac{i}{2}\cdot\frac{d\Omega}{dt}$$

$$\frac{d\mathrm{L}}{dt} = -\frac{2an}{1+m}\,r\,\mathrm{R} + 2\sin^2\frac{\varphi}{2}\cdot\frac{d\chi}{dt} + 2\sin^2\frac{i}{2}\cdot\frac{d\Omega}{dt} - \frac{3}{2}\int\frac{n}{a}\frac{da}{dt}dt$$

where R, S, and W involve the factor $m' =$ the mass of the disturbing planet measured with the sun's mass as the unit, but are not multiplied by the factor k^2 (k being usually known as the Gaussian constant).*

Provided the orbits do not intersect, and if we limit the approximation to terms of the first order with respect to the disturbing forces, each of these differential coefficients can be expanded in a periodic series of the form

$$\Sigma.\,\mathrm{A}\,{}^{\sin}_{\cos}\,(j\mathrm{M} + j'\mathrm{M}')$$

j and j' being positive or negative integers, and A being constant. The term, for

* For the proof of these formulæ the reader may consult either of the following sources: Encke, *Berliner Astronomische Jahrbuch für* 1837 *und* 1838, in the treatise *Über die Berechnung der Speciellen Störungen*, which has been reprinted in Encke's *Abhandlungen;* or Oppolzer, *Lehrbuch zur Bahnbestimmung der Cometen und Planeten*, Band II, s. 213 *et seq.;* or Watson, Theoretical Astronomy, pp. 516–523.

which both $j = 0$ and $j' = 0$, constitutes the secular portion of the series. The part of any differential coefficient, as $\frac{de}{dt}$, independent of M′, is given by the definite integral

$$\frac{1}{2\pi}\int_0^{2\pi}\frac{de}{dt}\,d\mathrm{M}'$$

and the secular portion, which is independent of both M and M′, by the definite integral

$$\frac{1}{4\pi^2}\int_0^{2\pi}\int_0^{2\pi}\frac{de}{dt}\,d\mathrm{M}\,d\mathrm{M}'$$

But as we have the equations

$$d\mathrm{M} = \frac{r}{a}\,d\mathrm{E} = \frac{r^2}{a^2\cos^2\varphi}\,dv$$

$$d\mathrm{M}' = \frac{r'}{a'}\,d\mathrm{E}' = \frac{r'^2}{a'^2\cos^2\varphi'}\,dv'$$

and as the variables M, E, and v all take the values 0 and 2π together, it is possible to make the integrations with reference to the eccentric or the true anomalies of the planets. Thus we have choice between four different procedures. That in which both of the integrations are executed with reference to the eccentric anomalies is to be preferred; for the inequalities of distribution of a series of points on an elliptic orbit, corresponding to equal intervals in the value of the eccentric anomaly, are of the order of the square of the eccentricity; while, for the other two anomalies, they are of the order of the first power of this quantity. Hence, to get the secular portion of the variation of any element, as e, we shall employ the double integral

$$\frac{1}{4\pi^2}\int_0^{2\pi}\int_0^{2\pi}\frac{de}{dt}\frac{r}{a}\frac{r'}{a'}\,d\mathrm{E}\,d\mathrm{E}' \, .$$

the value of which we shall denote by $\left[\frac{de}{dt}\right]_{00}$

As, in this method, the integration, with reference to E, will be performed by quadratures, instead of the notation

$$\frac{1}{2\pi}\int_0^{2\pi}\mathrm{X}\,d\mathrm{E}$$

we shall use $\mathrm{M}_{\mathrm{E}}[\mathrm{X}]$, which will denote the average of all the values of X with respect to the variable E. In the application of this method to the eight large planets of the solar system, the taking the average of 12 values, evenly distributed about the circumference with reference to E, will give, in all cases, extremely accurate results; and often 8 values will suffice. It can readily be shown, but, for the sake of brevity, we omit the demonstration, that, if the number of these values be even, the order of the error committed in the determination of the secular portions of the differential coeffi-

cients $\frac{de}{dt}$, $e\frac{d\pi}{dt}$, $\frac{di}{dt}$, and $\sin i\,\frac{d\Omega}{dt}$ will be the same as that of a power of the eccentricities or mutual inclination of orbits, whose exponent is one less than the number of these values, while the error, in the case of $\frac{dL}{dt}$, is of an order one degree higher. From this principle it can be judged, in any particular case, how many values ought to be computed.

It is well known that, not only when the approximation is limited to terms of the first order with respect to the disturbing forces, but even when terms of the second order are included, the secular portion of $\frac{da}{dt}$ vanishes. Hence, we can dispense with computing it.

If we put

$$R_0 = \frac{1}{2\pi}\int_0^{2\pi} \frac{ar}{m'}\, R\,(1 - e'\cos E')\,dE'$$

$$S_0 = \frac{1}{2\pi}\int_0^{2\pi} \frac{ar}{m'}\, S\,(1 - e'\cos E')\,dE'$$

$$W_0 = \frac{1}{2\pi}\int_0^{2\pi} \frac{r^2}{m'}\, W\,(1 - e'\cos E')\,dE'$$

we shall have, for the secular portions of the differential coefficients of the elements of m, the equations

$$\left[\frac{da}{dt}\right]_{00} = 0$$

$$\left[\frac{de}{dt}\right]_{00} = \frac{m'n}{1+m}\cos\varphi.\, M_E\left[\sin v.\, R_0 + (\cos v + \cos E)\, S_0\right]$$

$$e\left[\frac{d\chi}{dt}\right]_{00} = \frac{m'n}{1+m}\cos\varphi.\, M_E\left[-\cos v.\, R_0 + \left(\frac{r}{a\cos^2\varphi} + 1\right)\sin v.\, S_0\right]$$

$$\left[\frac{di}{dt}\right]_{00} = \frac{m'n}{1+m}\sec\varphi.\, M_E\left[\cos u.\, W_0\right]$$

$$\sin i\left[\frac{d\Omega}{dt}\right]_{00} = \frac{m'n}{1+m}\sec\varphi.\, M_E\left[\sin u.\, W_0\right]$$

$$\left[\frac{d\pi}{dt}\right]_{00} = \left[\frac{d\chi}{dt}\right]_{00} + 2\sin^2\frac{i}{2}\cdot\left[\frac{d\Omega}{dt}\right]_{00}$$

$$\left[\frac{dL}{dt}\right]_{00} = \frac{m'n}{1+m} M_E\left[-2\frac{r}{a}R_0\right] + 2\sin^2\frac{\varphi}{2}\cdot\left[\frac{d\chi}{dt}\right]_{00} + 2\sin^2\frac{i}{2}\cdot\left[\frac{d\Omega}{dt}\right]_{00}$$

In the case of the earth, as the ecliptic is usually assumed as the plane of reference, at the epoch i vanishes and Ω is indeterminate. But this inconvenience is avoided by

substituting for i and Ω two variables p and q (where the reader is asked not to confound this p with the p which denotes the semi-parameter), such that

$$p = \sin i \sin \Omega \qquad\qquad q = \sin i \cos \Omega$$

When we shall have

$$\left[\frac{dp}{dt}\right]_{00} = \frac{m'n}{1+m} \sec\varphi . \mathrm{M_E}\left[\sin(v+\pi).\mathrm{W_0}\right]$$

$$\left[\frac{dq}{dt}\right]_{00} = \frac{m'n}{1+m} \sec\varphi . \mathrm{M_E}\left[\cos(v+\pi).\mathrm{W_0}\right]$$

The parts of R, S, and W, which arise from the action of the disturbing planet on the sun, have, in their periodic developments, no terms independent of M′. For

$$\int \frac{x'}{r'^3} d\mathrm{M}' = -\frac{n'}{1+m'} \int \frac{d^2x'}{dt^2} dt = -\frac{n'}{1+m'} \frac{dx'}{dt}$$

which, as it has the same value for $\mathrm{M}' = 0$ and $\mathrm{M}' = 2\pi$, leads to

$$\int_0^{2\pi} \frac{x'}{r'^3} d\mathrm{M}' = 0$$

In like manner

$$\int_0^{2\pi} \frac{y'}{r'^3} d\mathrm{M}' = 0 \qquad\qquad \int_0^{2\pi} \frac{z'}{r'^3} d\mathrm{M}' = 0$$

Hence, for our present purpose, it will suffice to consider only the mutual action of the two planets. Then, assuming a system of rectangular co-ordinates, two of whose axes, x and y, lie in the plane of the orbit of the disturbed planet, so that $z = 0$, R, S, and W are determined by the equations

$$\frac{r}{m'}\mathrm{R} = \frac{xx' + yy' - r^2}{\Delta^3}$$

$$\frac{r}{m'}\mathrm{S} = \frac{xy' - x'y}{\Delta^3}$$

$$\frac{1}{m'}\mathrm{W} = \frac{z'}{\Delta^3}$$

and the distance Δ of the two planets by the equation

$$\Delta^2 = r^2 - 2(xx' + yy') + r'^2$$

In order to accomplish the integrations which $\mathrm{R_0}$, $\mathrm{S_0}$, and $\mathrm{W_0}$ involve, it will be necessary to express R, S, and W explicitly in terms of the variable E′. If I denotes the mutual inclination of the orbits, and Π and Π' severally the angular distances of

the perihelia from the ascending node of the orbit of the disturbing planet on the orbit of the disturbed, these quantities are determined by the equations

$$\begin{aligned}
\sin I \cos(\Pi - \omega) &= -\sin i \cos i' + \cos i \sin i' \cos(\Omega' - \Omega) \\
\sin I \sin(\Pi - \omega) &= -\sin i' \sin(\Omega' - \Omega) \\
\sin I \cos(\Pi' - \omega') &= \cos i \sin i' - \sin i \cos i' \cos(\Omega' - \Omega) \\
\sin I \sin(\Pi' - \omega') &= -\sin i \sin(\Omega' - \Omega)
\end{aligned}$$

We shall then have

$$\begin{aligned}
xx' + yy' &= rr'\left[\cos(v + \Pi)\cos(v' + \Pi') + \cos I \sin(v + \Pi)\sin(v' + \Pi')\right] \\
xy' - x'y &= rr'\left[-\sin(v + \Pi)\cos(v' + \Pi') + \cos I \cos(v + \Pi)\sin(v' + \Pi')\right] \\
z' &= r' \sin I \sin(v' + \Pi')
\end{aligned}$$

But if four auxiliary constants, k, K, k', and K$'$, are so taken that

$$\begin{aligned}
k\cos(\mathrm{K} - \Pi) &= \cos \Pi' & k'\cos(\mathrm{K}' - \Pi) &= \cos I \cos \Pi' \\
k\sin(\mathrm{K} - \Pi) &= -\cos I \sin \Pi' & k'\sin(\mathrm{K}' - \Pi) &= -\sin \Pi'
\end{aligned}$$

the first two equations take the forms

$$\begin{aligned}
xx' + yy' &= kr\cos(v + \mathrm{K}).\, r'\cos v' + k'r\sin(v + \mathrm{K}').\, r'\sin v' \\
xy' - x'y &= -kr\sin(v + \mathrm{K}).\, r'\cos v' + k'r\cos(v + \mathrm{K}').\, r'\sin v'
\end{aligned}$$

By the substitution of the values

$$r'\cos v' = a'(\cos \mathrm{E}' - e) \qquad r'\sin v' = a'\cos\varphi'\sin \mathrm{E}'$$

we have

$$\begin{aligned}
xx' + yy' &= ka'r\cos(v + \mathrm{K})(\cos \mathrm{E}' - e') + k'a'\cos\varphi'.\, r\sin(v + \mathrm{K}')\sin \mathrm{E}' \\
xy' - x'y &= -ka'r\sin(v + \mathrm{K})(\cos \mathrm{E}' - e') + k'a'\cos\varphi'.\, r\cos(v + \mathrm{K}')\sin \mathrm{E}' \\
z' &= a'\sin I \sin \Pi'(\cos \mathrm{E}' - e') + a'\sin I \cos \Pi' \cos\varphi' \sin \mathrm{E}'
\end{aligned}$$

Moreover,

$$r' = a'(1 - e'\cos \mathrm{E}')$$

in consequence, if we put

$$\begin{aligned}
\mathrm{A} &= r^2 + 2ka'e'r\cos(v + \mathrm{K}) + a'^2 \\
\mathrm{B}\cos\varepsilon &= ka'r\cos(v + \mathrm{K}) + a'^2e' \\
\mathrm{B}\sin\varepsilon &= k'a'\cos\varphi'.\, r\sin(v + \mathrm{K}') \\
\mathrm{C} &= a'^2e'^2
\end{aligned}$$

we shall have

$$\triangle^2 = \mathrm{A} - 2\mathrm{B}\cos(\mathrm{E}' - \varepsilon) + \mathrm{C}\cos^2 \mathrm{E}'$$

R, S, and W are now expressed explicitly in terms of E$'$. Gauss's method of

effecting the integrations, which give R_0, S_0, and W_0, consists in taking a new variable T, such that

$$\cos E' = \frac{\alpha + \alpha' \sin T + \alpha'' \cos T}{\gamma + \gamma' \sin T + \gamma'' \cos T}$$

$$\sin E' = \frac{\beta + \beta' \sin T + \beta'' \cos T}{\gamma + \gamma' \sin T + \gamma'' \cos T}$$

where α, β, γ, &c, satisfy certain conditions, and, moreover, are so taken that the coefficients of $\sin T$, $\cos T$ and $\sin T \cos T$ vanish in the expression

$$\triangle^2 [\gamma + \gamma' \sin T + \gamma'' \cos T]^2$$

which, in consequence, takes the form

$$G - G' \sin^2 T + G'' \cos^2 T$$

As the equation

$$[\alpha + \alpha' \sin T + \alpha'' \cos T]^2 + [\beta + \beta' \sin T + \beta'' \cos T]^2 - [\gamma + \gamma' \sin T + \gamma'' \cos T]^2 = 0$$

ought to hold true independently of the value of T, the left member must have the form

$$k (\sin^2 T + \cos^2 T - 1)$$

but as it is plain that the values of α, α', &c., can be multiplied by a common factor without any change resulting in $\sin E'$ and $\cos E'$, we may assume $k = 1$. We then have the six equations of condition

$$\begin{array}{ll}
\alpha^2 + \beta^2 - \gamma^2 = -1 & \alpha\alpha' + \beta\beta' - \gamma\gamma' = 0 \\
\alpha'^2 + \beta'^2 - \gamma'^2 = 1 & \alpha\alpha'' + \beta\beta'' - \gamma\gamma'' = 0 \\
\alpha''^2 + \beta''^2 - \gamma''^2 = 1 & \alpha'\alpha'' + \beta'\beta'' - \gamma'\gamma'' = 0
\end{array}$$

From the values of $\sin E'$ and $\cos E'$ in terms of T, by having regard to the equations of condition just written, we obtain

$$\alpha \cos E' + \beta \sin E' - \gamma = \frac{-1}{\gamma + \gamma' \sin T + \gamma'' \cos T}$$

$$\alpha' \cos E' + \beta' \sin E' - \gamma' = \frac{\sin T}{\gamma + \gamma' \sin T + \gamma'' \cos T}$$

$$\alpha'' \cos E' + \beta'' \sin E' - \gamma'' = \frac{\cos T}{\gamma + \gamma' \sin T + \gamma'' \cos T}$$

Hence, as the equation

$$[\alpha \cos E' + \beta \sin E' - \gamma]^2 - [\alpha' \cos E' + \beta' \sin E' - \gamma']^2 - [\alpha'' \cos E' + \beta'' \sin E' - \gamma'']^2 = 0$$

ought to be satisfied independently of the value assigned to E', the left member must have the form

$$k [\sin^2 E' + \cos^2 E' - 1]$$

Consequently,

$$\alpha^2 - \alpha'^2 - \alpha''^2 = k \qquad \alpha\beta - \alpha'\beta' - \alpha''\beta'' = 0$$
$$\beta^2 - \beta'^2 - \beta''^2 = k \qquad \alpha\gamma - \alpha'\gamma' - \alpha''\gamma'' = 0$$
$$\gamma^2 - \gamma'^2 - \gamma''^2 = -k \qquad \beta\gamma - \beta'\gamma' - \beta''\gamma'' = 0$$

But by comparing the three of these equations which involve squares of the quantities α, α', &c., with the similar three of the equations of condition previously obtained, we get $3k = -3$, or $k = -1$.

The six equations of condition first obtained may be so written as to form three groups of linear equations, thus:

$$\alpha.\ \alpha + \beta.\ \beta - \gamma.\ \gamma = -1 \qquad \alpha.\ \alpha' + \beta.\ \beta' - \gamma.\ \gamma' = 0 \qquad \alpha.\ \alpha'' + \beta.\ \beta'' - \gamma.\ \gamma'' = 0$$
$$\alpha'.\ \alpha + \beta'.\ \beta - \gamma'.\ \gamma = 0 \qquad \alpha'.\ \alpha' + \beta'.\ \beta' - \gamma'.\ \gamma' = 1 \qquad \alpha'.\ \alpha'' + \beta'.\ \beta'' - \gamma'.\ \gamma'' = 0$$
$$\alpha''.\alpha + \beta''.\beta - \gamma''.\gamma = 0 \qquad \alpha''.\alpha' + \beta''.\beta' - \gamma''.\gamma' = 0 \qquad \alpha''.\alpha'' + \beta''.\beta'' - \gamma''.\gamma'' = 1$$

If we put

$$D = \alpha\beta'\gamma'' - \alpha'\beta\gamma'' + \alpha'\beta''\gamma - \alpha''\beta'\gamma + \alpha''\beta\gamma' - \alpha\beta''\gamma'$$

we shall have

$$D\alpha = -\frac{dD}{d\alpha} = \beta''\gamma' - \beta'\gamma'' \qquad D\alpha' = \frac{dD}{d\alpha'} = \beta''\gamma - \beta\gamma''$$

$$D\beta = -\frac{dD}{d\beta} = \alpha'\gamma'' - \alpha''\gamma' \qquad D\beta' = \frac{dD}{d\beta'} = \alpha\gamma'' - \alpha''\gamma$$

$$D\gamma = \frac{dD}{d\gamma} = \alpha'\beta'' - \alpha''\beta' \qquad D\gamma' = -\frac{dD}{d\gamma'} = \alpha\beta'' - \alpha''\beta$$

$$D\alpha'' = \frac{dD}{d\alpha''} = \beta\gamma' - \beta'\gamma$$

$$D\beta'' = \frac{dD}{d\beta''} = \alpha'\gamma - \alpha\gamma'$$

$$D\gamma'' = -\frac{dD}{d\gamma''} = \alpha'\beta - \alpha\beta'$$

The value of D may be found by taking any one of the twelve preceding equations of condition between α, α', &c., and substituting in it the values of α, α', &c., from the preceding nine equations. Thus, if we take the equation

$$\alpha^2 - \alpha'^2 - \alpha''^2 = -1$$

we shall have

$$\begin{aligned}
D^2(-\alpha^2 + \alpha'^2 + \alpha''^2) = D^2 &= (\beta\gamma' - \beta'\gamma)^2 + (\beta''\gamma - \beta\gamma'')^2 - (\beta'\gamma'' - \beta''\gamma')^2 \\
&= \beta^2\gamma'^2 + \beta'^2\gamma^2 + \beta''^2\gamma^2 + \beta^2\gamma''^2 - \beta'^2\gamma''^2 - \beta''^2\gamma'^2 \\
&\quad - 2\,\beta\gamma\beta'\gamma' - 2\,\beta\gamma\beta''\gamma'' + 2\,\beta'\gamma'\beta''\gamma'' \\
&= \beta^2(\gamma^2 - 1) + \beta'^2(\gamma'^2 + 1) + \beta''^2(\gamma''^2 + 1) \\
&\quad - 2\,\beta\gamma\beta'\gamma' - 2\,\beta\gamma\beta''\gamma'' + 2\,\beta'\gamma'\beta''\gamma'' \\
&= -\beta^2 + \beta'^2 + \beta''^2 + (\beta\gamma - \beta'\gamma' - \beta''\gamma'')^2 \\
&= 1
\end{aligned}$$

Hence, $D = \pm 1$. It is evident we may adopt either sign, consequently we take the positive one.

The foregoing equations between the quantities α, α', &c., are all that are necessary for our purposes, but in order to obtain the values of these quantities and also of the three G, G′, and G″ we must have recourse to the equations furnished by the transformation of the expression for Δ^2. This transformation evidently comes to the same thing as the changing of the expression

$$Az^2 - 2B\cos\varepsilon.\ xz - 2B\sin\varepsilon.\ yz + Cx^2$$

into

$$Gu^2 - G'u'^2 + G''u''^2$$

by the employment of the formulæ

$$\begin{aligned} x &= \alpha u + \alpha' u' + \alpha'' u'' \\ y &= \beta u + \beta' u' + \beta'' u'' \\ z &= \gamma u + \gamma' u' + \gamma'' u'' \end{aligned}$$

But, having regard to the equations which the quantities α, α', &c., satisfy, we readily deduce from the last-given equations

$$\begin{aligned} u &= -\alpha x - \beta y + \gamma z \\ u' &= \alpha' x + \beta' y - \gamma' z \\ u'' &= \alpha'' x + \beta'' y - \gamma'' z \end{aligned}$$

By substitution of these values in the expression $Gu^2 - G'u'^2 + G''u''^2$ and comparison of the resulting coefficients with

$$Az^2 - 2B\cos\varepsilon.\ xz - 2B\sin\varepsilon.\ yz + Cx^2$$

we get the following equations:

$$\begin{aligned} G\alpha^2 - G'\alpha'^2 + G''\alpha''^2 &= C & \qquad G\alpha\beta - G'\alpha'\beta' + G''\alpha''\beta'' &= 0 \\ G\beta^2 - G'\beta'^2 + G''\beta''^2 &= 0 & \qquad G\alpha\gamma - G'\alpha'\gamma' + G''\alpha''\gamma'' &= B\cos\varepsilon \\ G\gamma^2 - G'\gamma'^2 + G''\gamma''^2 &= A & \qquad G\beta\gamma - G'\beta'\gamma' + G''\beta''\gamma'' &= B\sin\varepsilon \end{aligned}$$

which, in conjunction with the six independent equations between α, α', &c., previously obtained, suffice to determine the twelve unknowns, α, α', α'', β, β', β'', γ, γ', γ'', G, G′, and G″.

These six equations can be written in three groups of three equations each, the first group being as follows:

$$\begin{aligned} \alpha.\ G\alpha - \alpha'.\ G'\alpha' + \alpha''.\ G''\alpha'' &= C \\ \alpha.\ G\beta - \alpha'.\ G'\beta' + \alpha''.\ G''\beta'' &= 0 \\ \alpha.\ G\gamma - \alpha'.\ G'\gamma' + \alpha''.\ G''\gamma'' &= B\cos\varepsilon \end{aligned}$$

The second and third groups are obtained from this by writing in succession β and γ for α in the first factors of the terms of the left members of the equations, and

making the second members, in the first case, severally o, o, and B sin ε, and in the second, B cos ε, B sin ε, and A. By having regard to the six equations of conditions between α, α', &c., which were first obtained, we get from these three groups severally the following three groups of equations:

$$\left\{\begin{aligned} G\alpha &= -C\alpha + B\cos\varepsilon.\,\gamma \\ G\beta &= B\sin\varepsilon.\,\gamma \\ G\gamma &= -B\cos\varepsilon.\,\alpha - B\sin\varepsilon.\,\beta + A\gamma \end{aligned}\right.$$

$$\left\{\begin{aligned} G'\alpha' &= -C\alpha' + B\cos\varepsilon.\,\gamma' \\ G'\beta' &= B\sin\varepsilon.\,\gamma' \\ G'\gamma' &= -B\cos\varepsilon.\,\alpha' - B\sin\varepsilon.\,\beta' + A\gamma' \end{aligned}\right.$$

$$\left\{\begin{aligned} -G''\alpha'' &= -C\alpha'' + B\cos\varepsilon.\,\gamma'' \\ -G''\beta'' &= B\sin\varepsilon.\,\gamma'' \\ -G''\gamma'' &= -B\cos\varepsilon.\,\alpha'' - B\sin\varepsilon.\,\beta'' + A\gamma'' \end{aligned}\right.$$

From the first two equations of each of these three groups is obtained

$$\alpha = \frac{B\cos\varepsilon}{G+C}\gamma \qquad \alpha' = \frac{B\cos\varepsilon}{G'+C}\gamma' \qquad \alpha'' = \frac{B\cos\varepsilon}{C-G''}\gamma''$$

$$\beta = \frac{B\sin\varepsilon}{G}\gamma \qquad \beta' = \frac{B\sin\varepsilon}{G'}\gamma' \qquad \beta'' = -\frac{B\sin\varepsilon}{G''}\gamma''$$

By substituting these values of α, β, &c., in the last equation of each group we obtain

$$G - A + \frac{B^2\cos^2\varepsilon}{G+C} + \frac{B^2\sin^2\varepsilon}{G} = 0$$

$$G' - A + \frac{B^2\cos^2\varepsilon}{G'+C} + \frac{B^2\sin^2\varepsilon}{G'} = 0$$

$$-G'' - A + \frac{B^2\cos^2\varepsilon}{-G''+C} + \frac{B^2\sin^2\varepsilon}{-G''} = 0$$

It is evident, now, that G, G′, and $-$G″ are the roots of the cubic equation

$$x - A + \frac{B^2\cos^2\varepsilon}{x+C} + \frac{B^2\sin^2\varepsilon}{x} = 0$$

or of

$$x\,[(x-A)\,(x+C) + B^2] + B^2\sin^2\varepsilon = 0$$

The roots of this equation are all real, as can be shown in the following manner: If, for the moment, we adopt Gauss's system of rectangular co-ordinates, that is, put the origin at the center of the ellipse described by the disturbing planet, and make the axes of x and y coincide severally with the major and minor axes of this ellipse, and suppose that the co-ordinates of the disturbed planet, with reference to this system of

axes are denoted by A, B, and C, the expression for $\triangle^2$, which, in our notation, is

$$\triangle^2 = \mathrm{A} - 2\,\mathrm{B} \cos(\mathrm{E}' - \varepsilon) + \mathrm{C} \cos^2 \mathrm{E}'$$

will become

$$\begin{aligned}\triangle^2 &= (A - a' \cos \mathrm{E}')^2 + (B - a' \cos \varphi' \sin \mathrm{E}') + C^2 \\ &= A^2 + B^2 + C^2 + a'^2 \cos^2 \varphi' - 2\,(A a' \cos \mathrm{E}' + B a' \cos \varphi' \sin \mathrm{E}') + a'^2 \sin^2 \varphi' \cos^2 \mathrm{E}'\end{aligned}$$

By comparison of these two expressions for $\triangle^2$, we find that, expressed in terms of the second system of co-ordinates, the equation in x becomes

$$\begin{aligned}x\,[x - (A^2 + B^2 + C^2 + a'^2 \cos^2 \varphi')]\,(x + a'^2 \sin^2 \varphi') + (A^2 a'^2 + B^2 a'^2 \cos^2 \varphi')\,x \\ + B^2 a'^4 \sin^2 \varphi' \cos^2 \varphi' = 0\end{aligned}$$

We substitute for x in this equation the four values $-\mathrm{C}$, 0, $a'^2 \cos^2 \varphi'$, and A, and obtain the results

$x = -a'^2 \sin^2 \varphi' = -\mathrm{C}$	result, $-A^2 a'^4 \sin^2 \varphi'$
$x = 0$	" $+B^2 a'^4 \sin^2 \varphi' \cos^2 \varphi'$
$x = a'^2 \cos^2 \varphi'$	" $-C^2 a'^4 \cos^2 \varphi'$
$x = \mathrm{A}$	" $+\mathrm{B}^2 (\mathrm{A} + \mathrm{C} \sin^2 \varepsilon)$

From this it is apparent that the roots are all real, one being negative and numerically less than C, one positive and less than $a'^2 \cos^2 \varphi'$, and another positive and lying between $a'^2 \cos^2 \varphi'$ and A.

The assignment of these roots as the values of G, G′, and $-$G″ is not indifferent; as we wish both $\triangle$ and the transformation to be real, we put G equal to the larger of the positive roots, G′ equal to the smaller, and $-$G″ equal to the negative root. Consequently, G, G′, and G″ are always positive quantities.

The readiest method of obtaining them from the equation of the third degree, which determines them, appears to be by trial. If we put

$$\begin{aligned}g &= \mathrm{B}^2 \mathrm{C} \sin^2 \varepsilon \\ h &= \tfrac{1}{2}\,[\mathrm{A} - \mathrm{C} + \sqrt{(\mathrm{A} + \mathrm{C})^2 - 4\mathrm{B}^2}] \\ l &= \tfrac{1}{2}\,[\mathrm{A} - \mathrm{C} - \sqrt{(\mathrm{A} + \mathrm{C})^2 - 4\mathrm{B}^2}]\end{aligned}$$

the equation takes the form

$$x\,(x - h)\,(x - l) + g = 0$$

As g is usually a small quantity, having the factor e'^2, the approximate values of the roots are 0, l, and h. G, G′, and G″ can then be obtained, by successive approximations, from the equation put in the forms

$$\mathrm{G} = h - \frac{g}{\mathrm{G}\,(\mathrm{G} - l)}$$

$$\mathrm{G}' = l + \frac{g}{\mathrm{G}'\,(h - \mathrm{G}')}$$

$$\mathrm{G}'' = \frac{g}{(h + \mathrm{G}'')\,(l + \mathrm{G}'')}$$

quite approximate values being

$$G = h - \frac{g}{h\,(h-l)} \qquad G' = l + \frac{g}{l\,(h-l)} \qquad G'' = \frac{g}{\left(h+\frac{g}{hl}\right)\left(l+\frac{g}{hl}\right)}$$

For verification we may employ either or both of the equations

$$G + G' - G'' = A - C$$
$$GG'G'' = B^2C\sin^2\varepsilon$$

It will be seen that, in order to make our desired transformation from the variable E′ to the variable T, we do not need the values of the nine quantities α, α', &c., but only the values of the following ten squares and products of them, viz, α'^2, γ'^2, $\alpha'\beta'$, $\alpha'\gamma'$, $\beta'\gamma'$, α''^2, γ''^2, $\alpha''\beta''$, $\alpha''\gamma''$, and $\beta''\gamma''$; hence, we will limit ourselves to the determination of these.

The values of α' and β', in terms of γ', and of α'' and β'', in terms of γ'', have already been given. If we substitute them in the equations

$$\alpha'^2 + \beta'^2 - \gamma'^2 = 1 \qquad \alpha''^2 + \beta''^2 - \gamma''^2 = 1$$

we obtain

$$\left[\frac{B^2\cos^2\varepsilon}{(G'+C)^2} + \frac{B^2\sin^2\varepsilon}{G'^2} - 1\right]\gamma' = 1$$

$$\left[\frac{B^2\cos^2\varepsilon}{(C-G'')^2} + \frac{B^2\sin^2\varepsilon}{G''^2} - 1\right]\gamma'' = 1$$

Whence

$$\gamma'^2 = \frac{(G'+C)\,G'}{\frac{B^2\cos^2\varepsilon}{G'+C}G' + \frac{B^2\sin^2\varepsilon}{G'}(G'+C) - (G'+C)\,G'}$$

or having regard to the equation which determines G′,

$$\gamma'^2 = \frac{(G'+C)\,G'}{(A-G')\,G' + \frac{B^2C\sin^2\varepsilon}{G'} - (G'+C)\,G'}$$

$$= \frac{(G'+C)\,G'}{(A-C-2G')\,G' + GG''}$$

$$= \frac{(G'+C)\,G'}{(G'+G'')\,(G-G')}$$

And in like manner,

$$\gamma''^2 = \frac{(C - G'')\,G''}{\dfrac{B^2\cos^2\varepsilon}{C - G''}\,G'' + \dfrac{B^2\sin^2\varepsilon}{G''}\,(C - G'') - (C - G'')\,G''}$$

$$= \frac{(C - G'')\,G''}{(A + G'')\,G'' + GG' - (C - G'')\,G''}$$

$$= \frac{(C - G'')\,G''}{(G + G'')\,(G' + G'')}$$

We have

$$\frac{B^2\cos^2\varepsilon}{G' + C} = A - G' - \frac{B^2\sin^2\varepsilon}{G'}$$

consequently,

$$\alpha'^2 = \frac{(A - G')\,G' - B^2\sin^2\varepsilon}{(G' + G'')\,(G - G'')}$$

Also,

$$\frac{B^2\cos^2\varepsilon}{C - G''} = A + G'' + \frac{B^2\sin^2\varepsilon}{G''}$$

consequently,

$$\alpha''^2 = \frac{(A + G'')\,G'' + B^2\sin^2\varepsilon}{(G + G'')\,(G' + G'')}$$

And the values of the six products needed are

$$\alpha'\beta' = \frac{B^2\sin\varepsilon\cos\varepsilon}{(G' + G'')\,(G - G')} \qquad \alpha''\beta'' = -\frac{B^2\sin\varepsilon\cos\varepsilon}{(G + G'')\,(G' + G'')}$$

$$\alpha'\gamma' = \frac{B\cos\varepsilon.\ G'}{(G' + G'')\,(G - G')} \qquad \alpha''\gamma'' = \frac{B\cos\varepsilon.\ G''}{(G + G'')\,(G' + G'')}$$

$$\beta'\gamma' = \frac{B\sin\varepsilon.\ (C + G')}{(G' + G'')\,(G - G')} \qquad \beta''\gamma'' = -\frac{B\sin\varepsilon.\ (C - G'')}{(G + G'')\,(G' + G'')}$$

We have next to ascertain the value of the differential dE' in terms of the differential dT. From the equations

$$H\cos E' = \alpha + \alpha'\sin T + \alpha''\cos T$$
$$H\sin E' = \beta + \beta'\sin T + \beta''\cos T$$

where H stands for $\gamma + \gamma'\sin E' + \gamma''\cos E'$, it follows that

$$H\,dE' = [\cos E'\,(\beta'\cos T - \beta''\sin T) - \sin E'\,(\alpha'\cos T - \alpha''\sin T)]\,dT$$

or

$$H^2\,dE' = [(\alpha''\beta' - \alpha'\beta'') + (\alpha''\beta - \alpha\beta'')\sin T + (\alpha\beta' - \alpha'\beta)\cos T]\,dT$$
$$= -\,[\gamma + \gamma'\sin T + \gamma''\cos T]\,dT$$

Whence

$$H\, dE' = -\, dT$$

The quantity H is always of the same sign, otherwise sin E′ and cos E′ might become infinite in the passage of H through zero. If this consideration is not deemed conclusive, the point can be established as follows:

Since we have

$$(\gamma' \sin T + \gamma'' \cos T)^2 + (\gamma'' \sin T - \gamma' \cos T)^2 = \gamma'^2 + \gamma''^2 = \gamma^2 - 1$$

without regard to signs, $\gamma' \sin T + \gamma'' \cos T$ will always be less than γ. Hence, if γ be negative, T will always increase when E′ increases; but if γ be positive, T will always diminish when E′ increases.

If we put $\sqrt{\gamma^2 - 1} = \delta$, so that $\delta^2 = \alpha^2 + \beta^2 = \gamma'^2 + \gamma''^2$, we shall have

$$\begin{aligned} H\,(\delta + \alpha \cos E' + \beta \sin E') &= \gamma\delta + \alpha^2 + \beta^2 + (\gamma'\delta + \alpha\alpha' + \beta\beta') \sin T \\ &\quad + (\gamma''\delta + \alpha\alpha'' + \beta\beta'') \cos T \\ &= (\gamma + \delta)\,(\delta + \gamma' \sin T + \gamma'' \cos T) \end{aligned}$$

Also,

$$\begin{aligned} H\,(\alpha \sin E' - \beta \cos E') &= (\alpha\beta' - \alpha'\beta) \sin T + (\alpha\beta'' - \alpha''\beta) \cos T \\ &= \gamma'' \sin T - \gamma' \cos T \end{aligned}$$

By putting

$$\frac{\alpha}{\delta} = \cos L \qquad \frac{\beta}{\delta} = \sin L \qquad \frac{\gamma''}{\delta} = \cos M \qquad \frac{\gamma'}{\delta} = \sin M$$

these two equations become

$$\begin{aligned} H\,[1 + \cos (E' - L)] &= (\gamma + \delta)\,[1 + \cos (T - M)] \\ H \sin (E' - L) &= \sin (T - M) \end{aligned}$$

By division we get

$$\tan \tfrac{1}{2}(T - M) = (\gamma + \delta) \tan \tfrac{1}{2}(E' - L)$$

From this equation it is evident that, when E′ augments by a circumference, T augments or diminishes by the same quantity according as γ is negative or positive.

The expressions we have to integrate with respect to E′ are of the form $\frac{\Theta}{\Delta^3}$; hence, whether γ be positive or negative, we shall always have

$$\int_0^{2\pi} \frac{\Theta}{\Delta^3}\, dE' = \int_0^{2\pi} \frac{H^2\, \Theta}{(H^2\, \Delta^2)^{\frac{3}{2}}}\, dT$$

provided that we understand that the radical in the denominator is to have the positive sign.

The general form of Θ is

$$\begin{aligned} \Theta &= [f + g\,(\cos E' - e') + h \sin E']\,(1 - e' \cos E') \\ &= f - ge' + [g\,(1 + e'^2) - fe'] \cos E' + h \sin E' - he' \sin E' \cos E' - ge' \cos^2 E' \end{aligned}$$

If in this expression, multiplied by H^2, are substituted the values of H^2, $H \cos E'$, and $H \sin E'$ in terms of T, and the terms multiplied by $\sin T$, $\cos T$, and $\sin T \cos T$ omitted, as, when integrated between the limits 0 and 2π they contribute nothing to the value of the integral, we get

$$\begin{aligned} H^2 \Theta = & (f - ge') (\gamma^2 + \gamma'^2 \sin^2 T + \gamma''^2 \cos^2 T) \\ & + [g(1 + e'^2) - fe'] (\alpha\gamma + \alpha'\gamma' \sin^2 T + \alpha''\gamma'' \cos^2 T) \\ & + h (\beta\gamma + \beta'\gamma' \sin^2 T + \beta''\gamma'' \cos^2 T) \\ & - he' (\alpha\beta + \alpha'\beta' \sin^2 T + \alpha''\beta'' \cos^2 T) \\ & - ge' (\alpha^2 + \alpha'^2 \sin^2 T + \alpha''^2 \cos^2 T) \end{aligned}$$

But we have the equations

$$\begin{aligned} \alpha^2 &= -1 + \alpha'^2 + \alpha''^2 \\ \gamma^2 &= 1 + \gamma'^2 + \gamma''^2 \\ \alpha\beta &= \alpha'\beta' + \alpha''\beta'' \\ \alpha\gamma &= \alpha'\gamma' + \alpha''\gamma'' \\ \beta\gamma &= \beta'\gamma' + \beta''\gamma'' \end{aligned}$$

Hence, if we put

$$\begin{aligned} \Gamma' &= (f - ge') \gamma'^2 + [g(1 + e'^2) - fe'] \alpha'\gamma' + h\beta'\gamma' - he'\alpha'\beta' - ge'\alpha'^2 \\ \Gamma'' &= (f - ge') \gamma''^2 + [g(1 + e'^2) - fe'] \alpha''\gamma'' + h\beta''\gamma'' - he'\alpha''\beta'' - ge'\alpha''^2 \end{aligned}$$

we shall have

$$H^2 \Theta = [2\Gamma' + \Gamma'' + f] \sin^2 T + [\Gamma' + 2\Gamma'' + f] \cos^2 T$$

If we substitute, in the expressions for Γ' and Γ'', for γ'^2, $\alpha'\gamma'$, &c., the values we have previously obtained for these squares and products, and, moreover, put

$$\begin{aligned} F &= [ge' B \sin \varepsilon - he' B \cos \varepsilon + hC] B \sin \varepsilon \\ J &= - ge' A + (f - ge') C + [g(1 + e'^2) - fe'] B \cos \varepsilon + hB \sin \varepsilon \end{aligned}$$

we shall obtain

$$\Gamma' = \frac{F + JG' + fG'^2}{(G' + G'')(G - G')} \qquad \Gamma'' = \frac{-F + JG'' - fG''^2}{(G + G'')(G' + G'')}$$

Substituting in the values of F and J the values of A, $B \cos \varepsilon$, $B \sin \varepsilon$, and C, we get

$$\begin{aligned} F &= a'e'r B \sin \varepsilon [gk' \cos \varphi' \sin (v + K') - hk \cos (v + K)] \\ J &= - fa'e'kr \cos (v + K) + g [ka' \cos^2 \varphi' . r \cos (v + K) - e'r^2] \\ & \qquad + hk'a' \cos \varphi' . r \sin (v + K') \end{aligned}$$

To apply these formulæ to the three special cases of the computation of R_0, S_0, and W_0. In the case of R_0 we have

$$f = - ar^2 \qquad g = kaa'r \cos (v + K) \qquad h = k'aa' \cos \varphi' . r \sin (v + K')$$

Consequently, here

$$F = 0$$
$$J = aa'^2 \cos^2 \varphi'. \, r^2 \, [k^2 \cos^2 (v + K) + k'^2 \sin^2 (v + K')]$$
$$= aa'^2 \cos^2 \varphi'. \, r^2 \, [1 - \sin^2 I \sin^2 (v + \Pi)]$$

In the case of S_0 we have

$$f = 0 \qquad g = - kaa'r \sin (v + K) \qquad h = k'aa' \cos \varphi'. \, r \cos (v + K')$$

Consequently, here

$$F = - aa'^2 \, kk' \cos (K' - K) \sin \varphi' \cos \varphi'. \, r^2 \, B \sin \varepsilon$$
$$= - aa'^2 \sin \varphi' \cos \varphi' \cos I. \, r^2 \, B \sin \varepsilon$$
$$J = kaa'e'r^3 \sin (v + K) + \tfrac{1}{2} aa'^2 \cos^2 \varphi'. \, r^2 \, [k'^2 \sin 2 (v + K') - k^2 \sin 2 (v + K)]$$
$$= kaa'e'r^3 \sin (v + K) - \tfrac{1}{2} aa'^2 \cos^2 \varphi' \sin^2 I. \, r^2 \sin 2 (v + \Pi)$$

In the case of W_0 we have

$$f = 0 \qquad g = a' \sin I \sin \Pi'. \, r^2 \qquad h = a' \sin I \cos \Pi' \cos \varphi'. \, r^2$$

Consequently, here

$$F = a'^2 \sin \varphi' \cos \varphi' \sin I. \, r^3 \, B \sin \varepsilon \, [k' \sin \Pi' \sin (v + K') - k \cos \Pi' \cos (v + K)]$$
$$= - a'^2 \sin \varphi' \cos \varphi' \sin I. \, r^3 \cos (v + \Pi). \, B \sin \varepsilon$$
$$J = a'^2 \cos^2 \varphi' \sin I. \, r^3 \, [k \sin \Pi' \cos (v + K) + k' \cos \Pi' \sin (v + K')]$$
$$- a' \sin \varphi' \sin I \sin \Pi'. \, r^4$$
$$= a'^2 \cos^2 \varphi' \sin I \cos I. \, r^3 \sin (v + \Pi) - a'e' \sin I \sin \Pi'. \, r^4$$

The values of R_0, S_0, and W_0 are given by the definite integral

$$\frac{1}{2\pi} \int_0^{2\pi} \frac{[2\Gamma' + \Gamma'' + f] \sin^2 T + [\Gamma' + 2\Gamma'' + f] \cos^2 T}{[G + G'']^{\frac{3}{2}} \, [1 - c^2 \sin^2 T]^{\frac{3}{2}}} \, dT$$

provided we attribute to F, J, and f the values they have in each case. In this expression we have put

$$\frac{G' + G''}{G + G''} = c^2$$

c is then the modulus of the elliptic integrals involved in the expression. Let b denote the complementary modulus $= \sqrt{1 - c^2}$. In the notation of Legendre

$$\int_0^{\frac{\pi}{2}} \frac{dT}{[1 - c^2 \sin^2 T]^{\frac{1}{2}}} = F^1 (c) \qquad \int_0^{\frac{\pi}{2}} [1 - c^2 \sin^2 T]^{\frac{1}{2}} \, dT = E^1 (c)$$

We have the equation

$$\frac{d}{dT} \frac{\sin T \cos T}{[1 - c^2 \sin^2 T]^{\frac{1}{2}}} = \frac{1 - 2 \sin^2 T + c^2 \sin^4 T}{[1 - c^2 \sin^2 T]^{\frac{3}{2}}}$$

whence

$$\int_0^{\frac{\pi}{2}} \frac{1 - 2\sin^2 T + c^2 \sin^4 T}{[1 - c^2 \sin^2 T]^{\frac{3}{2}}} = 0$$

In consequence, we have the equations

$$\int_0^{\frac{\pi}{2}} \frac{(1 - c^2)\, dT}{[1 - c^2 \sin^2 T]^{\frac{3}{2}}} = E^1(c)$$

$$\int_0^{\frac{\pi}{2}} \frac{\sin^2 T\, dT}{[1 - c^2 \sin^2 T]^{\frac{3}{2}}} = \frac{1}{c^2}\left[\frac{1}{b^2} E^1(c) - F^1(c)\right]$$

$$\int_0^{\frac{\pi}{2}} \frac{\cos^2 T\, dT}{[1 - c^2 \sin^2 T]^{\frac{3}{2}}} = \frac{1}{c^2}\left[F^1(c) - E^1(c)\right]$$

Legendre, moreover, has put

$$F^1(c) = \frac{\pi}{2} K \qquad\qquad E^1(c) = \frac{\pi}{2} KL$$

Hence,

$$R_0,\ S_0, \text{ or } W_0 = \frac{K}{c^2 (G + G'')^{\frac{3}{2}}}\left[(\Gamma' + 2\Gamma'' + f)(1 - L) + (2\Gamma' + \Gamma'' + f)\left(\frac{L}{b^2} - 1\right)\right]$$

$$= \frac{KL}{b^2 (G + G'')^{\frac{3}{2}}} f + \frac{K}{(G + G'')^{\frac{3}{2}}}\left[\frac{L}{b^2} + \frac{L - b^2}{b^2 c^2}\right]\Gamma' + \frac{K}{(G + G'')^{\frac{3}{2}}}\left[2\frac{L}{b^2} - \frac{L - b^2}{b^2 c^2}\right]\Gamma''$$

We will now put

$$\mathfrak{K} = \frac{KL}{b^2} \qquad\qquad \mathfrak{L} = \frac{L - b^2}{c^2 L}$$

In consequence, the general expression for R_0, S_0, or W_0 will take the form

$$\frac{\mathfrak{K}}{(G + G'')^{\frac{3}{2}}}\left[f + (1 + \mathfrak{L})\,\Gamma' + (2 - \mathfrak{L})\,\Gamma''\right]$$

If we put

$$N = \frac{ar^2 \mathfrak{K}}{(G + G'')^{\frac{3}{2}}} \qquad\qquad N' = \frac{N(1 + \mathfrak{L})}{b^2 c^2 (G + G'')^2} \qquad\qquad N'' = \frac{N(2 - \mathfrak{L})}{c^2 (G + G'')^2}$$

and substitute for Γ' and Γ'' their values, this expression becomes

$$(N' - N'')\frac{F}{ar^2} + (N'G' + N''G'')\frac{J}{ar^2} + (N + N'G'^2 - N''G''^2)\frac{f}{ar^2}$$

This can be rendered more suitable for computation by putting

$$P = N' - N'' = \frac{N\,[-2b^2 + 1 + (1 + b^2)\,\mathfrak{T}]}{b^2c^2\,(G + G'')^2}$$

$$Q = N'\,(G' + G'') = \frac{N\,(1 + \mathfrak{T})}{b^2(G + G'')}$$

$$V = Q - PG''$$

Then the expression takes the form

$$P\frac{F}{ar^2} + V\frac{J}{ar^2} + (N + QG' - VG'')\frac{f}{ar^2}$$

If we call $\frac{F}{ar^2}$, $\frac{J}{ar^2}$, and $\frac{f}{ar^2}$ severally in the cases of R_0, S_0, and W_0 by F_1, J_1, f_1, F_2, J_2, f_2, F_3, J_3, f_3, remembering that $F_1 = 0$, $f_1 = -1$, $f_2 = 0$, and $f_3 = 0$, we shall have

$$\begin{aligned} R_0 &= -(N + QG' - VG'') + VJ_1 \\ S_0 &= PF_2 + VJ_2 \\ W_0 &= PF_3 + VJ_3 \end{aligned}$$

It now only remains to show how the elliptic integrals K and L may be computed. If we adopt a new variable, T^0, such that

$$\sin\,(2T - T^0) = c^0 \sin T^0$$

where $c^0 = \frac{1 - b}{1 + b}$, we shall have the following equations.

$$\cos\,(2T - T^0) = \sqrt{(1 - c^{02}\sin^2 T^0)} = \triangle$$

$$\cos 2T = \triangle \cos T^0 - c^0 \sin^2 T^0$$

$$\begin{aligned} \sin 2T &= \triangle \sin T^0 + c^0 \sin T^0 \cos T^0 \\ &= \sin T^0\,(c^0 \cos T^0 + \triangle) \end{aligned}$$

$$2\,dT = \frac{dT^0}{\triangle}\,(c^0 \cos T^0 + \triangle)$$

$$\sqrt{(1 - c^2 \sin^2 T)} = \frac{c^0 \cos T^0 + \triangle}{1 + c^0}$$

$$\frac{dT}{\sqrt{(1 - c^2 \sin^2 T)}} = \frac{1 + c^0}{2}\,\frac{dT^0}{\triangle}$$

which constitute the well-known transformation of Landen. It is plain, from the values of $\sin\,(2T - T^0)$ and $\cos\,(2T - T^0)$ that, when T passes from the value 0 to

the value $\frac{\pi}{2}$, T^0 passes from 0 to π. Hence,

$$\int_0^{\frac{\pi}{2}} \frac{dT}{\sqrt{(1 - c^2 \sin^2 T)}} = (1 + c^0) \int_0^{\frac{\pi}{2}} \frac{dT^0}{\sqrt{(1 - c^{02} \sin^2 T^0)}}$$

or

$$F^1(c) = (1 + c^0)\, F^1(c^0)$$

If we take c^{00} the same function of c^0 that c^0 is of c, and, again, in like manner, derive c^{000}, and so on, the quantities c, c^0, c^{00}, &c., diminish, and, as $F^1(0) = \frac{\pi}{2}$, we shall have

$$F^1(c) = \frac{\pi}{2}(1 + c^0)(1 + c^{00})(1 + c^{000}) \ldots$$

If the *moduli* complementary to c^0, c^{00}, &c., are denoted by b^0, b^{00}, &c., we shall have $b^0 = \sqrt{1 - c^{02}}$ and $b = \frac{1 - c^0}{1 + c^0}$. Consequently,

$$(1 + c^0) = \frac{b^0}{\sqrt{b}}$$

Hence,

$$K = \sqrt{\frac{b^0\, b^{00}\, b^{000} \ldots}{b}}$$

From the equations

$$\frac{dT}{\sqrt{(1 - c^2 \sin^2 T)}} = \frac{1 + c^0}{2}\, \frac{dT^0}{\Delta} \qquad \sin^2 T = \frac{1}{2}(1 + c^0 \sin^2 T^0 - \Delta \cos T^0)$$

we obtain

$$\int_0^{\frac{\pi}{2}} \frac{A + B \sin^2 T}{\sqrt{(1 - c^2 \sin^2 T)}}\, dT = (1 + c^0) \int_0^{\frac{\pi}{2}} \frac{A + \frac{B}{2} + B\, \frac{c^0}{2} \sin^2 T^0}{\Delta}\, dT^0$$

If this process of transformation is continued as in the case of the former integral we find that

$$\int_0^{\frac{\pi}{2}} \frac{A + B \sin^2 T}{\sqrt{(1 - c^2 \sin^2 T)}}\, dT = \frac{\pi}{2} K \left[A + \frac{B}{2}\left(1 + \frac{c^0}{2} + \frac{c^0 c^{00}}{4} + \frac{c^0 c^{00} c^{000}}{8} + \ldots\right)\right]$$

In the case of $E^1(c)$ we have $A = 1$ and $B = -c^2$; hence,

$$L = 1 - \frac{c^2}{2} - \frac{c^2 c^0}{4} - \frac{c^2 c^0 c^{00}}{8} - \ldots$$

As we have

$$1 - \frac{c^2}{2} - \frac{c^2 c^0}{4} = \frac{c^2}{4c^0} = \frac{b}{b^{0 2}}$$

and as we may, for our purpose, cut off the series at the term which contains c^{000}, and with sufficient approximation put

$$1 + \frac{1}{2} c^{000} = \sqrt{1 + c^{000}} = \sqrt{\frac{2\sqrt{c^{000}}}{c^{00}}} = \frac{\sqrt{b^{000}}}{\sqrt[4]{b^{00}}}$$

we may put

$$L = \frac{b}{b^{0 2}} \left[1 - \frac{1}{2} c^{0 2} c^{00} \frac{\sqrt{b^{000}}}{\sqrt[4]{b^{00}}} \right]$$

In like manner

$$\frac{L - b^2}{c^2} = \frac{1}{2} \left[1 - \frac{c^0}{2} - \frac{c^0 c^{00}}{4} \frac{\sqrt{b^{000}}}{\sqrt[4]{b^{00}}} \right]$$

$$\mathfrak{K} = \sqrt{\frac{b^{00} b^{000}}{b^3 b^{0 3}}} \left[1 - \frac{1}{2} c^{0 2} c^{00} \frac{\sqrt{b^{000}}}{\sqrt[4]{b^{00}}} \right]$$

$$\frac{(1 + b^2)\mathfrak{L} - 2b^2 + 1}{b^2 c^2} = \mathfrak{L}' = \frac{2 - c^2 - \frac{(1 - c^2 + c^4)\, b^{0 2}}{8} \left(1 + \frac{1}{2} c^{00} \frac{\sqrt{b^{000}}}{\sqrt[4]{b^{00}}} \right)}{\frac{b^3}{b^{0 2}} \left[1 - \frac{1}{2} c^{0 2} c^{00} \frac{\sqrt{b^{000}}}{\sqrt[4]{b^{00}}} \right]}$$

$$\frac{1 + \mathfrak{L}}{b^2} = \mathfrak{N} = \frac{\frac{3}{2} - \frac{1}{2} c^2 - \frac{1 + c^2}{2} \left[\frac{c^0}{2} + \frac{c^0 c^{00}}{4} \frac{\sqrt{b^{000}}}{\sqrt[4]{b^{00}}} \right]}{\frac{b^3}{b^{0 2}} \left[1 - \frac{1}{2} c^{0 2} c^{00} \frac{\sqrt{b^{000}}}{\sqrt[4]{b^{00}}} \right]}$$

The common logarithms of the last three functions are tabulated at the end of this memoir. In order to make the data of Legendre's Tables in the second volume of his *Théorie des Fonctions Elliptiques* available, c has been put $= \sin \theta$, and θ adopted as the argument. The quantities are given to eight places of decimals, having been computed with ten. They are tabulated at intervals of a tenth of a degree, and are given from $\theta = 0$ up to $\theta = 50°$. Beyond the latter limit they will scarcely be needed and the interpolation of the tables becomes difficult. Should values, beyond the limit of the table, be wanted, it will be easier to compute them directly from the formulæ than to derive them by interpolation from values tabulated at intervals of $0°.1$ in the value of θ.

Recapitulation of the formulæ needed for the application of this method.

For the benefit of those who wish to make a numerical application of this method, I have here gathered together and arranged, in proper order, all the formulæ necessary to be used. For the signification of the symbols, the preceding discussion must be consulted.

Compute the constants I, Π, Π', k, K, k', K$'$, and C, which are functions of the elements of the two orbits, by means of the equations

$$
\begin{aligned}
\sin I \cos(\Pi - \omega) &= -\sin i \cos i' + \cos i \sin i' \cos(\Omega' - \Omega) \\
\sin I \sin(\Pi - \omega) &= \quad - \quad \sin i' \sin(\Omega' - \Omega) \\
\sin I \cos(\Pi' - \omega') &= \cos i \sin i' - \sin i \cos i' \cos(\Omega' - \Omega) \\
\sin I \sin(\Pi' - \omega') &= \quad - \quad \sin i \sin(\Omega' - \Omega) \\
k \cos(\mathrm{K} - \Pi) &= \cos \Pi' \\
k \sin(\mathrm{K} - \Pi) &= -\cos I \sin \Pi' \\
k' \cos(\mathrm{K}' - \Pi) &= \cos I \cos \Pi' \\
k' \sin(\mathrm{K}' - \Pi) &= -\sin \Pi' \\
\mathrm{C} &= a'^2 e'^2
\end{aligned}
$$

The circumference, with reference to the variable E, will now be divided into a certain number of equal parts, which number ought to be a multiple of 4, and should be large or small as the perturbations are more or less irregular through the variation of the distance of the two planets. For each of these values of E, the values of the varying quantities in the left members of the following equations must be calculated. Here a useful check against large errors may be had by adding the first, third, fifth, &c., numerical values of any one of these quantities, and again the second, fourth, sixth, &c. The difference of the two sums should be very small, except in case of certain angles, where one sum may exceed the other by nearly 180°. The same test may be applied to the logarithms of a quantity, provided it does not change sign and does not approach zero very closely.

$$
\begin{aligned}
r \cos v &= a(\cos \mathrm{E} - e) \\
r \sin v &= a \cos \varphi \sin \mathrm{E} \\
\mathrm{A} &= r^2 + 2ka'e'r \cos(v + \mathrm{K}) + a'^2 \\
\mathrm{B} \cos \varepsilon &= ka'r \cos(v + \mathrm{K}) + a'^2 e' \\
\mathrm{B} \sin \varepsilon &= k'a' \cos \varphi' . \, r \sin(v + \mathrm{K}') \\
g &= \mathrm{B}^2 \mathrm{C} \sin^2 \varepsilon \\
h &= \tfrac{1}{2}\left[\mathrm{A} - \mathrm{C} + \sqrt{(\mathrm{A}+\mathrm{C})^2 - 4\mathrm{B}^2}\right] \\
l &= \tfrac{1}{2}\left[\mathrm{A} - \mathrm{C} - \sqrt{(\mathrm{A}+\mathrm{C})^2 - 4\mathrm{B}^2}\right]
\end{aligned}
$$

Find G, G′, and G″ by trial from the equations

$$G = h - \frac{g}{G(G - l)}$$

$$G' = l + \frac{g}{G'(h - G')}$$

$$G'' = \frac{g}{(h + G'')(l + G'')}$$

Approximate values are

$$G = h - \frac{g}{h(h - l)}$$

$$G' = l + \frac{g}{l(h - l)}$$

$$G'' = \frac{g}{\left(h + \frac{g}{hl}\right)\left(l + \frac{g}{hl}\right)}$$

$$\sin^2\theta = \frac{G' + G''}{G + G''}$$

From the tables at the end of this memoir, with the argument θ, take out the values of log $\mathfrak{K}$, log $\mathfrak{L}'$, and log $\mathfrak{N}$.

$$N = \frac{ar^2\,\mathfrak{K}}{(G + G'')^{\frac{3}{2}}}$$

$$P = \frac{N\mathfrak{L}'}{(G + G'')^2}$$

$$Q = \frac{N\mathfrak{N}}{G + G''}$$

$$V = Q - PG''$$

$$J_1 = a'^2 \cos^2\varphi' \,[1 - \sin^2 I \sin^2 (v + \Pi)] + G''$$

$$J_2 = ka'e'r \sin (v + K) - \tfrac{1}{2}\, a'^2 \cos^2\varphi' \sin^2 I \sin 2\,(v + \Pi)$$

This should read Π

$$J_3 = \frac{a'^2}{a} \cos^2\varphi' \sin I \cos I.\ r \sin (v + \Pi) - \frac{a'}{a} e' \sin I \sin \Pi.\ r^2$$

$$F_2 = -\, a'^2 \sin\varphi' \cos\varphi' \cos I.\ B \sin\varepsilon$$

$$F_3 = -\frac{a'^2}{a} \sin\varphi' \cos\varphi' \sin I.\ r \cos (v + \Pi).\ B \sin\varepsilon$$

$$R_0 = -N - QG' + VJ_1$$
$$S_0 = PF_2 + VJ_2$$
$$W_0 = PF_3 + VJ_3$$

The secular variations of the elements will be given by the following equations:

$$\left[\frac{de}{dt}\right]_{00} = \frac{m'n}{1+m}\cos\varphi.\, M_E\left[\sin v.\, R_0 + (\cos v + \cos E)\, S_0\right]$$

$$e\left[\frac{d\chi}{dt}\right]_{00} = \frac{m'n}{1+m}\cos\varphi.\, M_E\left[-\cos v.\, R_0 + \left(\frac{r}{a\cos^2\varphi} + 1\right)\sin v.\, S_0\right]$$

$$\left[\frac{di}{dt}\right]_{00} = \frac{m'n}{1+m}\sec\varphi.\, M_E\left[\cos u.\, W_0\right]$$

$$\sin i\left[\frac{d\Omega}{dt}\right]_{00} = \frac{m'n}{1+m}\sec\varphi.\, M_E\left[\sin u.\, W_0\right]$$

$$\left[\frac{d\pi}{dt}\right]_{00} = \left[\frac{d\chi}{dt}\right]_{00} + 2\sin^2\frac{i}{2}\cdot\left[\frac{d\Omega}{dt}\right]_{00}$$

$$\left[\frac{dL}{dt}\right]_{00} = \frac{m'n}{1+m} M_E\left[-2\frac{r}{a}R_0\right] + 2\sin^2\frac{\varphi}{2}\cdot\left[\frac{d\chi}{dt}\right]_{00} + 2\sin^2\frac{i}{2}\cdot\left[\frac{d\Omega}{dt}\right]_{00}.$$

EXAMPLE.

Computation of the Secular Perturbations of Mercury produced by the Action of Venus.

The elements of the two planets, adopted for the epoch 1850.0, are

Mercury.	Venus.
$n = 5381016''.26$	$n' = 2106641''.357$
$e = 0.20560476$	$e' = 0.00684311$
$\pi = 75^\circ\ 7'\ 13''.62$	$\pi' = 129^\circ\ 27'\ 42''.83$
$i = 7^\circ\ 0'\ 7''.71$	$i' = 3^\circ\ 23'\ 35''.01$
$\Omega = 46^\circ\ 33'\ 8''.63$	$\Omega' = 75^\circ\ 19'\ 53''.08$
$\log a = 9.5878217$	$\log a' = 9.8593378$
$m = \frac{1}{5000000}$	

From these are deduced

$I = 4^\circ\ 20'\ 42''.98$	$K = 305^\circ\ 43'\ 2''.46$	$\log k' = 9.9999176$
$\Pi = 230^\circ\ 39'\ 31''.39$	$K' = 305^\circ\ 47'\ 57''.54$	$\log C = 5.3891826$
$\Pi' = 284^\circ\ 54'\ 1''.18$	$\log k = 9.9988328$	$C = 0.00002450$

The circumference is now divided into twelve parts with respect to E, the eccentric anomaly of Mercury. The values of the various quantities employed in the computation, computed for each of the points of division, are tabulated below. The result of the application of the test, mentioned above, is given at the foot of the column, opposite

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to the symbols S and S′, whenever it is supposed to be useful. The numbers given are affected with asterisks when the additions have been made on the numbers which correspond to the logarithms in the column of values.

E	log. r	v			A	log. B	ε			log. g
°		°	′	″			°	′	″	
0	9.4878584	0	0	0.00	0.61954395	9.3505444	306	25	17.64	3.90151
30	9.5026623	36	32	7.50	0.62743501	9.3671640	342	33	14.83	3.07719
60	9.5407098	70	50	41.41	0.64711632	9.4050438	16	26	41.01	3.10312
90	9.5878217	101	51	53.65	0.67563289	9.4506321	47	9	9.28	4.02085
120	9.6303194	129	46	44.60	0.70650301	9.4909308	74	53	39.98	4.34050
150	9.6589887	155	27	29.02	0.73029576	9.5171866	100	32	23.25	4.40878
180	9.6690267	180	0	0.00	0.73831733	9.5249278	125	10	50.07	4.26384
210	9.6589887	204	32	30.98	0.72725905	9.5130385	149	56	52.18	3.81457
240	9.6303194	230	13	15.40	0.70124328	9.4833852	175	57	47.29	2.05108
270	9.5878217	258	8	6.35	0.66955948	9.4412922	204	16	31.00	3 49971
300	9.5407098	289	9	18.59	0.64185659	9.3963533	235	38	26.28	4.01534
330	9.5026623	323	27	52.50	0.62439830	9.3618721	270	4	31.93	4.11293
S . .	. . . .	. . .		. . .	4.05458048	6.6511853	934	32	42.27	. . . .
S′ . .	. . . .	. . .		. . .	4.05458049	6.6511855	1114	32	42.47	. . . .

E	h	l	G	G′	G″	θ			log. [illegible]
°						°	′	″	
0	0.52358611	0.09593335	0.52358255	0.09595277	0.00001587	25	20	53.91	0.0667815
30	0.52390824	0.10350226	0.52390770	0.10350501	0.00000220	26	23	25.40	0.0726785
60	0.52384405	0.12324776	0.52384345	0.12325033	0.00000196	29	0	59.16	0.0888373
90	0.52344857	0.15215982	0.52344317	0.15217839	0.00001317	32	37	46.67	0.1142938
120	0.52319735	0.18328117	0.52318503	0.18331632	0.00002284	36	17	45.75	0.1442958
150	0.52358284	0.20668842	0.52356739	0.20672755	0.00002368	38	55	52.65	0.1687224
180	0.52446108	0.21383175	0.52444981	0.21385939	0.00001617	39	41	12.28	0.1762011
210	0.52500793	0.20222662	0.52500408	0.20223662	0.00000615	38	21	51.22	0.1632515
240	0.52470763	0.17651115	0.52470757	0.17651133	0.00000012	35	27	1.86	0.1369807
270	0.52391066	0.14562431	0.52390907	0.14563005	0.00000414	31	49	7.12	0.1082397
300	0.52329644	0.11853565	0.52329155	0.11855724	0.00001670	28	25	30.44	0.0850327
330	0.52323371	0.11014009	0.52322784	0.10117046	0.00002450	26	5	20.74	0.0709442
S . .	3.14309266	0.91134083	3.14305996	0.91144738	0.00007386	194	13	23.40	0.6981291
S′ . .	3.14309195	0.91134152	3.14305925	0.91144808	0.00007384	194	13	23.80	0.6981301

E	log. $\mathfrak{M}'$	log. $\mathfrak{N}$	log. N	log. P	log. Q	log. V	log. J_1
0°	0.3610703	0.2748567	9.0518226	9.9748963	9.6076810	9.6076649	9.7171747
30	0.3687562	0.2834450	9.0869399	0.0171829	9.6511283	9.6511261	9.7161627
60	0.3897436	0.3068691	9.1792740	0.1306114	9.7669400	9.7669380	9.7168407
90	0.4225948	0.3434556	9.2994382	0.2842720	9.9240133	9.9240002	9.7181351
120	0.4609870	0.3860837	9.4147450	0.4383836	0.0821545	0.0821320	9.7186740
150	0.4919942	0.4204077	9.4960332	0.5500430	0.1974487	0.1974255	9.7181915
180	0.5014421	0.4308492	9.5225000	0.5845071	0.2336317	0.2336158	9.7171751
210	0.4850679	0.4127493	9.4887989	0.5335312	0.1813804	0.1813744	9.7163236
240	0.4516579	0.3757381	9.4055651	0.4173882	0.0613858	0.0613857	9.7162443
270	0.4148054	0.3347895	9.2928157	0.2691023	9.9083458	9.9083417	9.7171416
300	0.3848118	0.3013686	9.1761378	0.1234346	9.7587489	9.7587321	9.7183721
330	0.3664971	0.2809213	9.0860239	0.0150988	9.6482341	9.6482093	9.7185270
S . .	2.5497127	2.0757654	5.7500445	1.6692212	9.5105419	9.5104685	8.3044809
S′ . .	2.5497156	2.0757684	5.7500498	1.6692302	9.5105506	9.5104772	8.3044815

E	log. J_2	log. J_3	log. F_2	log. F_3	log. R_0	log. S_0	log. W_0
0°	*n* 7.4321671	*n* 8.3837285	6.8088312	*n* 5.3916432	8.7760911	*n* 6.6886872	*n* 7.9924224
30	*n* 6.7963083	*n* 8.5099324	6.3966713	*n* 3.8820117	8.8092004	*n* 5.3190515	*n* 8.1610823
60	7.2616976	*n* 8.4788955	*n* 6.4096375	*n* 4.9613828	8.9109724	6.8580694	*n* 8.2461381
90	7.4216280	*n* 8.2575909	*n* 6.8685040	*n* 5.6972542	9.0478487	6.9002047	*n* 8.1843223
120	7.3047658	6.7021384	*n* 7.0283282	*n* 5.9515414	9.1783301	*n* 6.6917105	6.5600086
150	7.0091948	8.3158080	*n* 7.0624655	*n* 5.9675881	9.2656128	*n* 7.3958789	8.5088240
180	6.5998867	8.5688794	*n* 6.9899995	*n* 5.7539798	9.2869000	*n* 7.4874988	8.8010010
210	6.5740806	8.6552729	*n* 6.7653615	*n* 5.1245368	9.2427427	*n* 7.1525123	8.8363593
240	6.8487789	8.6332314	*n* 5.8836161	4.0827215	9.1508864	6.7875671	8.6916448
270	6.8412620	8.4906201	6.6079307	*n* 5.2855935	9.0333867	7.1190054	8.3983398
300	*n* 6.6329728	8.0916691	6.8657465	*n* 5.6718326	8.9135270	6.8627036	7.8465591
330	*n* 7.3581667	*n* 7.8939066	6.9145405	*n* 5.6967794	8.8179243	*n* 6.2157912	*n* 7.5484891
S . .	. . . . .	. . . . .	. . . . .	. . . . .	4.2167070	−0.001989228*	+0.09268013*
S′ . .	. . . . .	. . . . .	. . . . .	. . . . .	4.2167156	−0.001984156*	+0.09258717*

E	$R_0 \sin v + S_0 (\cos v + \cos E)$	$-R_0 \cos v + S_0 \left(\frac{r}{a \cos^2 \phi} + 1\right) \sin v$	$W_0 \cos u$	$W_0 \sin u$	$-2 \frac{r}{a} R_0$
0°	− 0.00097660	− 0.0597161	− 0.00863059	− 0.00469931	− 0.0948763
30	+ 0.03833155	− 0.0518053	− 0.00610021	− 0.01314386	− 0.1059427
60	+ 0.07755206	− 0.0254115	+ 0.00288259	− 0.01738805	− 0.1461808
90	+ 0.10909871	+ 0.0245450	+ 0.00991450	− 0.01163594	− 0.2232948
120	+ 0.11643388	+ 0.0956574	− 0.00033746	+ 0.00013397	− 0.3325506
150	+ 0.08098430	+ 0.1653789	− 0.03219222	− 0.00226584	− 0.4343200
180	+ 0.00614510	+ 0.1935976	− 0.05554168	− 0.03024215	− 0.4668045
210	− 0.07011566	+ 0.1603978	− 0.04118259	− 0.05487000	− 0.4120403
240	− 0.10947664	+ 0.0895491	− 0.00962480	− 0.04855987	− 0.3121865
270	− 0.10595401	+ 0.0195723	+ 0.00719195	− 0.02396722	− 0.2159816
300	− 0.07680512	− 0.0282224	+ 0.00519678	− 0.00472486	− 0.1470433
330	− 0.03941924	− 0.0526511	− 0.00350168	+ 0.00049010	− 0.1080923
S . .	+ 0.01287268	+ 0.2654541	− 0.06605516	− 0.10548027	− 1.4996420
S′ . .	+ 0.01292565	+ 0.2654376	− 0.06587025	− 0.10539276	− 1.4996717
	+ 0.02579833	+ 0.5308917	− 0.13192541	− 0.21087303	− 2.9993137

Dividing the numbers at the foot of the last five columns by 12, we have the average values of the several functions written at the top. And, leaving the mass of Venus indefinite, we have

		log. coeff.
$\left[\frac{de}{dt}\right]_{00} = +$	$11321''.28\ m'$	4.0538954
$\left[\frac{d\chi}{dt}\right]_{00} = +$	$1133122''\ m'$	6.0542766
$\left[\frac{di}{dt}\right]_{00} = -$	$60449''.22\ m'$	n 4.7813907
$\left[\frac{d\Omega}{dt}\right]_{00} = -$	$792604''.4\ m'$	n 5.8990565
$\left[\frac{d\pi}{dt}\right]_{00} = +$	$1127210''\ m'$	6.0520049
$\left[\frac{dL}{dt}\right]_{00} = -$	$1326648''.7\ m'$	n 6.1227559

The eccentricity e is supposed to be expressed in seconds of arc; if the variation in parts of the radius is wanted, the result given above must be multiplied by the factor whose logarithm is 94 6855749. It is scarcely necessary to add that the unit of time is the Julian year, and that m' must be expressed in parts of the sun's mass.

If we adopt Leverrier's value of m', viz, $m' = \frac{1}{401847}$, we have the values of the secular variations given below. Alongside, for the sake of comparison, I put Leverrier's values, deduced from the series expanded in powers of the eccentricities and mutual inclination of the plane of the orbits. (*Annales de l'Observatoire de Paris. Mémoires.* Tome V, pp. 6–7–21.)

		Leverrier's values.
$\left[\frac{de}{dt}\right]_{00}$	$= + 0''.0281731$	$+ 0''.02823$
$\left[\frac{d\pi}{dt}\right]_{00}$	$= + 2''.805073$	$+ 2''.8064$
$\left[\frac{di}{dt}\right]_{00}$	$= - 0''.1504284$	$- 0''.15044$
$\left[\frac{d\Omega}{dt}\right]_{00}$	$= - 1''.972403$	$- 1''.9702$
$\left[\frac{dL}{dt}\right]_{00}$	$= - 3''.301377$	$- 3''.3282$

Table of the Values of Three Elliptic Integrals employed in this Memoir.

θ	Log. $\mathfrak{K}$			Log. $\mathfrak{L}'$			Log. $\mathfrak{N}$		
0.0	0.00000000	+ 99		0.27300127	+ 132		0.17609126	+ 149	
0.1	00000099	298	+199	27300259	397	+265	17609275	446	+297
0.2	00000397	496	198	27300656	662	265	17609721	744	298
0.3	00000893	695	199	27301318	926	264	17610465	1042	298
0.4	00001588	893	198	27302244	1191	265	17611507	1340	298
0.5	0.00002481	+1091	+198	0.27303435	+ 1455	+264	0.17612847	+ 1637	+297
0.6	00003572	1290	199	27304890	1720	265	17614484	1935	298
0.7	00004862	1488	198	27306610	1984	264	17616419	2232	297
0.8	00006350	1687	199	27308594	2249	265	17618651	2531	299
0.9	00008037	1886	199	27310843	2514	265	17621182	2828	297
1.0	0.00009923	+2084	+198	0.27313357	+ 2779	+265	0.17624010	+ 3125	+297
1.1	00012007	2282	198	27316136	3043	264	17627135	3424	299
1.2	00014289	2481	199	27319179	3308	265	17630559	3721	297
1.3	00016770	2680	199	27322487	3572	264	17634280	4019	298
1.4	00019450	2878	198	27326059	3838	266	17638299	4317	298
1.5	0.00022328	+3077	+199	0.27329897	+ 4102	+264	0.17642616	+ 4615	+298
1.6	00025405	3275	198	27333999	4367	265	17647231	4913	298
1.7	00028680	3475	200	27338366	4632	265	17652144	5211	298
1.8	00032155	3673	198	27342998	4896	264	17657355	5508	297
1.9	00035828	3871	198	27347894	5162	266	17662863	5807	299
2.0	0.00039699	+4071	+200	0.27353056	+ 5426	+264	0.17668670	+ 6104	+297
2.1	00043770	4269	198	27358482	5692	266	17674774	6403	299
2.2	00048039	4468	199	27364174	5956	264	17681177	6700	297
2.3	00052507	4667	199	27370130	6222	266	17687877	6999	299
2.4	00057174	4866	199	27376352	6486	264	17694876	7297	298
2.5	0.00062040	+5065	+199	0.27382838	+ 6752	+266	0.17702173	+ 7595	+298
2.6	00067105	5263	198	27389590	7017	265	17709768	7894	299
2.7	00072368	5463	200	27396607	7282	265	17717662	8191	297
2.8	00077831	5662	199	27403889	7547	265	17725853	8490	299
2.9	00083493	5861	199	27411436	7812	265	17734343	8789	299
3.0	0.00089354	+6061	+200	0.27419248	+ 8078	+266	0.17743132	+ 9087	+298
3.1	00095415	6259	198	27427326	8344	266	17752219	9385	298
3.2	00101674	6459	200	27435670	8608	264	17761604	9684	299
3.3	00108133	6658	199	27444278	8875	267	17771288	9983	299
3.4	00114791	6858	200	27453153	9140	265	17781271	10281	298
3.5	0.00121649	+7057	+199	0.27462293	+ 9405	+265	0.17791552	+10580	+299
3.6	00128706	7256	199	27471698	9671	266	17802132	10879	299
3.7	00135962	7457	201	27481369	9937	266	17813011	11177	298
3.8	00143419	7655	198	27491306	10203	266	17824188	11477	300
3.9	00151074	7856	201	27501509	10468	265	17835665	11775	298
4.0	0.00158930	+8055	+199	0.27511977	+10735	+267	0.17847440	+12075	+300
4.1	00166985	8255	200	27522712	11000	265	17859515	12373	298
4.2	00175240	8455	200	27533712	11267	267	17871888	12673	300
4.3	00183695	8655	200	27544979	11533	266	17884561	12972	299
4.4	00192350	+8856	201	27556512	+11799	266	17897533	+13272	300
4.5	0.00201206		+199	0.27568311		+266	0.17910805		+299

Table of the Values of Three Elliptic Integrals, &c.—Continued.

θ	Log. K			Log. L′			Log. N		
4°.5	0.00201206	+9055	+199	0.27568311	+12065	+266	0.17910805	+13571	+299
4.6	00210261	9255	200	27580376	12332	267	17924376	13870	299
4.7	00219516	9456	201	27592708	12598	266	17938246	14170	300
4.8	00228972	9656	200	27605306	12864	266	17952416	14470	300
4.9	00238628	9856	200	27618170	13132	268	17966886	14769	299
5.0	0.00248484	+10058	+202	0.27631302	+13398	+266	0.17981655	+15070	+301
5.1	00258542	10257	199	27644700	13665	267	17996725	15369	299
5.2	00268799	10459	202	27658365	13931	266	18012094	15669	300
5.3	00279258	10659	200	27672296	14199	268	18027763	15969	300
5.4	00289917	10860	201	27686495	14466	267	18043732	16270	301
5.5	0.00300777	+11061	+201	0.27700961	+14732	+266	0.18060002	+16570	+300
5.6	00311838	11262	201	27715693	15001	269	18076572	16870	300
5.7	00323100	11463	201	27730694	15267	266	18093442	17171	301
5.8	00334563	11665	202	27745961	15535	268	18110613	17471	300
5.9	00346228	11865	200	27761496	15802	267	18128084	17772	301
6.0	0.00358093	+12068	+203	0.27777298	+16070	+268	0.18145856	+18073	+301
6.1	00370161	12269	201	27793368	16338	268	18163929	18374	301
6.2	00382430	12470	201	27809706	16606	268	18182303	18675	301
6.3	00394900	12672	202	27826312	16874	268	18200978	18976	301
6.4	00407572	12874	202	27843186	17141	267	18219954	19277	301
6.5	0.00420446	+13077	+203	0.27860327	+17410	+269	0.18239231	+19578	+301
6.6	00433523	13278	201	27877737	17679	269	18258809	19880	302
6.7	00446801	13480	202	27895416	17946	267	18278689	20182	302
6.8	00460281	13683	203	27913362	18215	269	18298871	20483	301
6.9	00473964	13885	202	27931577	18484	269	18319354	20786	303
7.0	0.00487849	+14088	+203	0.27950061	+18753	+269	0.18340140	+21087	+301
7.1	00501937	14291	203	27968814	19021	268	18361227	21389	302
7.2	00516228	14493	202	27987835	19291	270	18382616	21691	302
7.3	00530721	14696	203	28007126	19559	268	18404307	21994	303
7.4	00545417	14899	203	28026685	19829	270	18426301	22296	302
7.5	0.00560316	+15103	+204	0.28046514	+20098	+269	0.18448597	+22599	+303
7.6	00575419	15305	202	28066612	20368	270	18471196	22902	303
7.7	00590724	15509	204	28086980	20638	270	18494098	23204	302
7.8	00606233	15713	204	28107618	20907	269	18517302	23507	303
7.9	00621946	15916	203	28128525	21177	270	18540809	23811	304
8.0	0.00637862	+16121	+205	0.28149702	+21447	+270	0.18564620	+24114	+303
8.1	00653983	16324	203	28171149	21717	270	18588734	24417	303
8.2	00670307	16528	204	28192866	21988	271	18613151	24721	304
8.3	00686835	16733	205	28214854	22258	270	18637872	25025	304
8.4	00703568	16937	204	28237112	22529	271	18662897	25328	303
8.5	0.00720505	+17141	+204	0.28259641	+22800	+271	0.18688225	+25633	+305
8.6	00737646	17346	205	28282441	23070	270	18713858	25936	303
8.7	00754992	17551	205	28305511	23342	272	18739794	26241	305
8.8	00772543	17756	205	28328853	23613	271	18766035	26546	305
8.9	00790299	+17962	206	28352466	+23884	271	18792581	+26850	304
9.0	0.00808261		+204	0.28376350		+272	0.18819431		+305

Table of the Values of Three Elliptic Integrals, &c.—Continued.

θ	Log. $\mathfrak{K}$			Log. $\mathfrak{L}'$			Log. $\mathfrak{N}$		
9.0°	0.00808261	+18166	+204	0.28376350	+24156	+272	0.18819431	+27155	+305
9.1	00826427	18372	206	28400506	24427	271	18846586	27460	305
9.2	00844799	18577	205	28424933	24700	273	18874046	27765	305
9.3	00863376	18784	207	28449633	24971	271	18901811	28070	305
9.4	00882160	18989	205	28474604	25243	272	18929881	28376	306
9.5	0.00901149	+19195	+206	0.28499847	+25516	+273	0.18958257	+28681	+305
9.6	00920344	19402	207	28525363	25789	273	18986938	28987	306
9.7	00939746	19608	206	28551152	26061	272	19015925	29293	306
9.8	00959354	19814	206	28577213	26333	272	19045218	29600	307
9.9	00979168	20022	208	28603546	26607	274	19074818	29906	306
10.0	0.00999190	+20228	+206	0.28630153	+26881	+274	0.19104724	+30212	+306
10.1	01019418	20435	207	28657034	27153	272	19134936	30519	307
10.2	01039853	20643	208	28684187	27427	274	19165455	30825	306
10.3	01060496	20850	207	28711614	27701	274	19196280	31133	308
10.4	01081346	21058	208	28739315	27975	274	19227413	31440	307
10.5	0.01102404	+21265	+207	0.28767290	+28248	+273	0.19258853	+31748	+308
10.6	01123669	21474	209	28795538	28524	276	19290601	32055	307
10.7	01145143	21682	208	28824062	28797	273	19322656	32363	308
10.8	01166825	21890	208	28852859	29072	275	19355019	32671	308
10.9	01188715	22099	209	28881931	29348	276	19387690	32979	308
11.0	0.01210814	+22307	+208	0.28911279	+29622	+274	0.19420669	+33287	+308
11.1	01233121	22517	210	28940901	29897	275	19453956	33596	309
11.2	01255638	22725	208	28970798	30173	276	19487552	33905	309
11.3	01278363	22935	210	29000971	30449	276	19521457	34214	309
11.4	01301298	23145	210	29031420	30724	275	19555671	34524	310
11.5	0.01324443	+23354	+209	0.29062144	+31000	+276	0.19590195	+34832	+308
11.6	01347797	23564	210	29093144	31277	277	19625027	35143	311
11.7	01371361	23775	211	29124421	31553	276	19660170	35452	309
11.8	01395136	23985	210	29155974	31830	277	19695622	35762	310
11.9	01419121	24195	210	29187804	32107	277	19731384	36073	311
12.0	0.01443316	+24406	+211	0.29219911	+32384	+277	0.19767457	+36383	+310
12.1	01467722	24617	211	29252295	32661	277	19803840	36693	310
12.2	01492339	24829	212	29284956	32939	278	19840533	37005	312
12.3	01517168	25040	211	29317895	33216	277	19877538	37316	311
12.4	01542208	25251	211	29351111	33494	278	19914854	37627	311
12.5	0.01567459	+25463	+212	0.29384605	+33773	+279	0.19952481	+37939	+312
12.6	01592922	25676	213	29418378	34051	278	19990420	38251	312
12.7	01618598	25888	212	29452429	34330	279	20028671	38563	312
12.8	01644486	26100	212	29486759	34609	279	20067234	38875	312
12.9	01670586	26313	213	29521368	34887	278	20106109	39188	313
13.0	0.01696899	+26526	+213	0.29556255	+35167	+280	0.20145297	+39500	+312
13.1	01723425	26740	214	29591422	35447	280	20184797	39814	314
13.2	01750165	26953	213	29626869	35727	280	20224611	40127	313
13.3	01777118	27166	213	29662596	36006	279	20264738	40440	313
13.4	01804284	+27381	215	29698602	+36287	281	20305178	+40754	314
13.5	0.01831665		+214	0.29734889		+281	0.20345932		+315

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Table of the Values of Three Elliptic Integrals, &c.—Continued.

θ	Log. K			Log. L′			Log. N		
13°.5	0.01831665	+27595	+214	0.29734889	+36568	+281	0.20345932	+41069	+315
13.6	01859260	27809	214	29771457	36848	280	20387001	41382	313
13.7	01887069	28024	215	29808305	37129	281	20428383	41697	315
13.8	01915093	28239	215	29845434	37411	282	20470080	42012	315
13.9	01943332	28454	215	29882845	37692	281	20512092	42327	315
14.0	0.01971786	+28670	+216	0.29920537	+37974	+282	0.20554419	+42642	+315
14.1	02000456	28885	215	29958511	38256	282	20597061	42958	316
14.2	02029341	29101	216	29996767	38539	283	20640019	43274	316
14.3	02058442	29317	216	30035306	38821	282	20683293	43590	316
14.4	02087759	29534	217	30074127	39104	283	20726883	43906	316
14.5	0.02117293	+29751	+217	0.30113231	+39387	+283	0.20770789	+44222	+316
14.6	02147044	29968	217	30152618	39670	283	20815011	44540	318
14.7	02177012	30184	216	30192288	39954	284	20859551	44857	317
14.8	02207196	30403	219	30232242	40238	284	20904408	45174	317
14.9	02237599	30620	217	30272480	40522	284	20949582	45493	319
15.0	0.02268219	+30839	+219	0.30313002	+40807	+285	0.20995075	+45810	+317
15.1	02299058	31056	217	30353809	41091	284	21040885	46128	318
15.2	02330114	31276	220	30394900	41377	286	21087013	46447	319
15.3	02361390	31494	218	30436277	41661	284	21133460	46766	319
15.4	02392884	31714	220	30477938	41948	287	21180226	47086	320
15.5	0.02424598	+31933	+219	0.30519886	+42233	+285	0.21227312	+47404	+318
15.6	02456531	32152	219	30562119	42520	287	21274716	47725	321
15.7	02488683	32373	221	30604639	42806	286	21322441	48044	319
15.8	02521056	32593	220	30647445	43092	286	21370485	48365	321
15.9	02553649	32814	221	30690537	43380	288	21418850	48685	320
16.0	0.02586463	+33035	+221	0.30733917	+43667	+287	0.21467535	+49007	+322
16.1	02619498	33256	221	30777584	43955	288	21516542	49328	321
16.2	02652754	33478	222	30821539	44243	288	21565870	49649	321
16.3	02686232	33699	221	30865782	44531	288	21615519	49972	323
16.4	02719931	33922	223	30910313	44820	289	21665491	50293	321
16.5	0.02753853	+34144	+222	0.30955133	+45109	+289	0.21715784	+50616	+323
16.6	02787997	34367	223	31000242	45398	289	21766400	50939	323
16.7	02822364	34590	223	31045640	45687	289	21817339	51262	323
16.8	02856954	34814	224	31091327	45977	290	21868601	51586	324
16.9	02891768	35037	223	31137304	46268	291	21920187	51909	323
17.0	0.02926805	+35261	+224	0.31183572	+46557	+289	0.21972096	+52234	+325
17.1	02962066	35485	224	31230129	46849	292	22024330	52558	324
17.2	02997551	35711	226	31276978	47140	291	22076888	52883	325
17.3	03033262	35935	224	31324118	47431	291	22129771	53208	325
17.4	03069197	36161	226	31371549	47724	293	22182979	53533	325
17.5	0.03105358	+36386	+225	0.31419273	+48015	+291	0.22236512	+53859	+326
17.6	03141744	36612	226	31467288	48308	293	22290371	54186	327
17.7	03178356	36839	227	31515596	48600	292	22344557	54512	326
17.8	03215195	37065	226	31564196	48894	294	22399069	54839	327
17.9	03252260	+37292	227	31613090	+49187	293	22453908	+55166	327
18.0	0.03289552		+228	0.31662277		+294	0.22509074		+327

DISCUSSION AND RESULTS OF OBSERVATIONS

ON

TRANSITS OF MERCURY,

FROM 1677 TO 1881,

BY

SIMON NEWCOMB,

PROFESSOR, U. S. N., SUPERINTENDENT AMERICAN EPHEMERIS.

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PART I.—DISCUSSION OF OBSERVATIONS.

PART II.—COMPUTATION OF TABULAR ELEMENTS AND COMPARISON OF THEORY WITH OBSERVATIONS.

PART III.—DISCUSSIONS OF RESULTS.

TRANSITS OF MERCURY, 1677 TO 1881.

PART I.

DISCUSSION OF THE OBSERVATIONS.

§ I.

Introductory Remarks.

The series of transits of Mercury discussed by LEVERRIER* terminated with that of 1848. The work of LEVERRIER on this subject is memorable from the fact that in it was first pointed out the discrepancy between the motion of the perihelion of Mercury as derived from observation and as derived from theory. The existence of this discrepancy, at least when the mass of Venus determined in other ways is employed, has been placed beyond doubt by observations of four transits since the publication of LEVERRIER's work. Notwithstanding the thoroughness with which the great astronomer treated the subject, there are a number of circumstances which render a reinvestigation desirable. The mere fact that a third of a century of observations, far more accurate than any made in previous times, are now available, is alone a reason for ascertaining whether any modification of LEVERRIER's results is now possible.

Again, several new questions have arisen which observed transits of Mercury will help to decide. Among these is the question of the uniformity of the earth's rotation. The discrepancies between theory and observation in the moon's secular acceleration, and the inequalities of long period in its longitude, give rise to the question whether the rotation of the earth itself may not be variable. That a slow secular retardation in this rotation exists seems almost certain on theoretical grounds; and it is not impossible that causes may act, capable of producing changes of long period. The question whether the apparent discrepancies in the moon's motion are to be accounted for in this way can best be settled by observation on other rapidly moving bodies.

There are many questions respecting the phenomena of contact, data for the solution of which will be found in the mass of recorded observations of transits of Mercury. Valuable hints may thus be derived in respect to the interpretation and discussion of observed transits of Venus.

Yet another reason for a new discussion is the desirableness of having the results of the theory and of observation so worked out, collated, and compared that the astronomer of the present or future may be able to see how they are to be reconciled

*Annales de l'observatoire de Paris, Tome V.

without the necessity of going over the entire discussion from the beginning. The author hopes that the present paper will make this possible.

It is a part of the general policy which the writer has adopted in carrying on this work to secure the co-operation of other astronomers, and he desires to express his thanks to those who have aided him in the collection of materials for the present discussion. Prominently among these must be mentioned Dr. J. A. C. OUDEMANNS, of Utrecht, who kindly furnished a complete translation of a number of Dutch Memoirs containing observations not found elsewhere. Messrs. R. L. J. ELLERY, JAMES TEBBUT, and H. C. RUSSELL, of Australia, kindly furnished, in advance of publication, the valuable series of Australian observations of the transit of 1881, made under their direction. Mr. J. E. HILGARD, Superintendent of the Coast and Geodetic Survey, also supplied valuable data from the records of that office. Among them are a great number of American observations on the transit of 1845, which had never been published. Dr. OTTO STRUVE also furnished several unpublished observations made in Russia as well as extracts from the Montpelier Memoirs made by Dr. WAGNER.

Most of the heavy work of reducing the observations has been performed by Lieutenant CHAUNCEY THOMAS, U. S. N., and Mr. R. W. PRENTISS

§ 2.

Authorities for Observations.

Completeness in an investigation like the present requires the use of all data which can add to the precision of the result. In the case of contact observations the phenomena are of such a character that precision can be obtained only by combining as many observations as possible, made by different observers and under different circumstances. For these reasons the available published sources of observations were examined with some care. It was soon found, however, that a limit would have to be set to the number used. In the older volumes of the *Berliner Astronomisches Jahrbuch*, *Monatliche Correspondenz*, and *Allgemeine Geographische Ephemeriden* are great numbers of observations which it is hardly possible to use without suspicion. The range from the most valueless to the best is so gradual that it is very difficult to draw a line of distinction. The embarrassment is increased by the circumstance that the longitudes of the stations are in many cases not known with precision. It was desirable to reduce to a minimum the number of observations likely to be rejected merely on account of their discordance from others, and this again was a reason for leaving out of consideration some observations which might possibly have proved accordant, but which, had they been discordant, would not have been considered entitled to any weight. The rule finally adopted was to retain all observations of known observers and all made at well known observatories, or stations, even when the observer was not known as an experienced astronomer; but, as a general rule, to leave out of consideration those of unknown observers at stations whose longitude would be difficult to determine. Except in the case of the Coast Survey records, and a few Russian observations already alluded to, reliance has been placed entirely upon published sources. The author did, indeed, some years ago make a collection of unpublished observations among the registers of the Paris observatory, but these were unfortunately lost through some accident. A critical examination of them had, however, shown that there

was very little material of value not already published, so that the investigation has probably not suffered much from their loss. The principal published sources are indicated at the commencement of the tabular exhibit of the observations, so that it is unnecessary to repeat them here. Thoroughness has not, however, been aimed at in preparing the bibliographical references, it being deemed sufficient to furnish the reader with the data which would enable him to trace any observations quoted.

§ 3.

Distinction of Phases and Methods of Deducing the Time of Contact from the Observations.

It is well known that different definitions of what shall be regarded as the time of interior contact of a planet with the sun may be formulated. Although, in reality, there can be but one phase of true contact, namely, that of interior tangency of the limbs, yet in practice, owing to the effect of irradiation, the limbs are not seen in their true geometrical outline. It has, however, been very generally considered that when allowance is made for irradiation we shall find two distinct phases of contact, namely:

I. A phase in which the apparent limb of the sun, as extended by irradiation, and the apparent limb of the planet, as contracted from the same cause, shall be in geocentric contact.

II. A phase when the light of the sun's limb is first or last seen to completely encircle the planet.

At ingress the phases follow each other in the order here described, while at egress their order is reversed. The first phase is sometimes denominated "apparent contact" or "tangency of limbs"; while the second is commonly known as "true contact" or "formation of the thread of light.". The terms *apparent* and *true* contact are founded on a theory of the phenomenon which presupposes that the breaking or completion of the thread of light marks the moment of true internal contact, while the apparent tangency of limbs occurs at that interval preceding it during which the planet moves over a space equal to double the enlargement produced by irradiation. The geometry of this theory is quite simple, and, assuming the hypothesis to correspond to the facts of the case, it is quite correct. The theory assumes that, owing to the extreme brilliancy of the sun's limb, the completion of the thread of light can be noted the moment that the planet completely enters upon the solar disc.

In accordance with this view it was generally the custom, previous to the transit of Venus in 1874, to divide phases of contact into the two classes just defined, and to assume that the observer noted one or the other of the two phases. Thus, in Encke's classic work on the transits of 1761 and 1769 we find the observations of internal contact classified under the two heads of "*Umkreis in Berührung*" and "*Lichtfaden.*" A similar course was followed by Mr. Stone, in his discussion of the transit of 1769, though he put some observations into one class which his predecessors had put into the other.

Now, assuming that observers always did note one or the other of these two distinct phases; assuming also that it could be inferred, either from their statements or from the times given, which phase each observer noted, it would be necessary, in com-

paring observations, to classify the phase and to avoid comparing one phase with another, except after applying the proper reduction.

But all experience concurs in showing that no such distinct classification of the two phases is possible in actual observation. It is true that observers sometimes describe the phase of tangency of limbs and formation of the thread of light as occurring at different specified times. It is also true that, as a general rule, it will be found that these observers who describe the thread of light as fully formed will be found to have noted their times somewhat later [at ingress] than those who describe their observation as that of tangency of limbs.

But this is by no means universal. As a matter of fact, we find the noted times of contact bridging over both phases in such a way that no clear distinction can be drawn. It is also to be remarked that a majority of observers do not define the phase at all; so that it is impossible to infer from their description which phase they observed. The following is perhaps the most conclusive way of considering the subject.

If there are two distinct and separable phases of contact, and if some observers note one phase and some another, then, by examining the times in a great number of transits observed by a great number of observers, we should find in them a tendency to group themselves near two distinct moments, corresponding to the two phases. If, for example, apparent contact at ingress occurs at 0^s and true contaet at 20^s then we should find one set of observers giving a result near to 0^s and another near to 20^s, while observations at 10^s would be fewer in number. To show whether there is any such tendency we have, in a subsequent section, taken all the good observations given in the following pages between the limits 1769 and 1878 and grouped them in the way supposed. It is thus found that there is no tendency to a grouping about any distinct phase, and that the observations all group themselves around a single general mean, according to the usual law of error.

It appears, therefore, that no distinct line can be drawn between the two phases as they have been actually observed by the hundreds of observers in past times.

Associated with the error thus pointed out is another one, which has not been without influence in the discussion of observations. It is that the phenomena commonly known as "breaking of the black drop" at ingress, and which is frequently described by observers as occurring suddenly, is a real and well defined phase. Now, were this so, there would be at least some approximation to agreement among observers as to the time of this phase. But, as a matter of fact which has been pointed out too frequently to need a full explanation at present, there is no such agreement. On the contrary, the divergence of times between the observers of the supposed phase is as wide as between observers of geometrical tangency. Moreover, optical considerations will show that this is nothing more than we should expect. Whether the black drop is or is not present, the moment of the phase thus described is not that at which the thread of light commences, but that at which it became sufficiently strong to be sensible to the eye of the observer. Now, this moment depends very largely upon the state of the atmosphere, the defining power of the telescope, and the contrast between the sun and the sky. Since irradiation implies a diffusion of the solar light, we may expect the phase in question to occur later the greater the amount of this disturbing

Table of the Values of Three Elliptic Integrals, &c.—Continued.

θ	Log. K			Log. L′			Log. N		
18°.0	0. 03289552	+37520	+228	0. 31662277	+49481	+294	0. 22509074	+55493	+327
18. 1	03327072	37748	228	31711758	49775	294	22564567	55822	329
18. 2	03364820	37975	227	31761533	50070	295	22620389	56149	327
18. 3	03402795	38204	229	31811603	50365	295	22676538	56479	330
18. 4	03440999	38433	229	31861968	50660	295	22733017	56807	328
18. 5	0. 03479432	+38662	+229	0. 31912628	+50956	+296	0. 22789824	+57137	+330
18. 6	03518094	38891	229	31963584	51251	295	22846961	57466	329
18. 7	03556985	39121	230	32014835	51548	297	22904427	57796	330
18. 8	03596106	39351	230	32066383	51845	297	22962223	58127	331
18. 9	03635457	39582	231	32118228	52141	296	23020350	58458	331
19. 0	0. 03675039	+39813	+231	0. 32170369	+52440	+299	0. 23078808	+58789	+331
19. 1	03714852	40045	232	32222809	52737	297	23137597	59120	331
19. 2	03754897	40275	230	32275546	53035	298	23196717	59453	333
19. 3	03795172	40508	233	32328581	53333	298	23256170	59785	332
19. 4	03835680	40741	233	32381914	53633	300	23315955	60117	332
19. 5	0. 03876421	+40973	+232	0. 32435547	+53932	+299	0. 23376072	+60451	+334
19. 6	03917394	41207	234	32489479	54232	300	23436523	60784	333
19. 7	03958601	41440	233	32543711	54532	300	23497307	61118	334
19. 8	04000041	41674	234	32598243	54832	300	23558425	61453	335
19. 9	04041715	41908	234	32653075	55134	302	23619878	61787	334
20. 0	0. 04083623	+42144	+236	0. 32708209	+55434	+300	0. 23681665	+62122	+335
20. 1	04125767	42378	234	32763643	55737	303	23743787	62457	335
20. 2	04168145	42614	236	32819380	56038	301	23806244	62794	337
20. 3	04210759	42850	236	32875418	56341	303	23869038	63130	336
20. 4	04253609	43086	236	32931759	56644	303	23932168	63467	337
20. 5	0. 04296695	+43323	+237	0. 32988403	+56947	+303	0. 23995635	+63804	+337
20. 6	04340018	43561	238	33045350	57251	304	24059439	64141	337
20. 7	04383579	43798	237	33102601	57556	305	24123580	64479	338
20. 8	04427377	44036	238	33160157	57859	303	24188059	64818	339
20. 9	04471413	44274	238	33218016	58165	306	24252877	65157	339
21. 0	0. 04515687	+44514	+240	0. 33276181	+58470	+305	0. 24318034	+65496	+339
21. 1	04560201	44752	238	33334651	58776	306	24383530	65835	339
21. 2	04604953	44993	241	33393427	59083	307	24449365	66176	341
21. 3	04649946	45232	239	33452510	59389	306	24515541	66516	340
21. 4	04695178	45474	242	33511899	59696	307	24582057	66858	342
21. 5	0. 04740652	+45714	+240	0. 33571595	+60004	+308	0. 24648915	+67198	+340
21. 6	04786366	45955	241	33631599	60312	308	24716113	67541	343
21. 7	04832321	46198	243	33691911	60620	308	24783654	67883	342
21. 8	04878519	46440	242	33752531	60929	309	24851537	68226	343
21. 9	04924959	+46683	243	33813460	+61239	310	24919763	+68569	343
22. 0	0. 04971642		+243	0. 33874699		+309	0. 24988332		+344

Table of the Values of Three Elliptic Integrals, &c.—Continued.

θ	Log. K			Log. L′			Log. N		
22.0°	0.04971642	+46926	+243	0.33874699	+61548	+309	0.24988332	+68913	+344
22.1	05018568	47170	244	33936247	61859	311	25057245	69256	343
22.2	05065738	47414	244	33998106	62170	311	25126501	69602	346
22.3	05113152	47658	244	34060276	62481	311	25196103	69947	345
22.4	05160810	47904	246	34122757	62792	311	25266050	70292	345
22.5	0.05208714	+48149	+245	0.34185549	+63105	+313	0.25336342	+70638	+346
22.6	05256863	48396	247	34248654	63417	312	25406980	70985	347
22.7	05305259	48641	245	34312071	63730	313	25477965	71332	347
22.8	05353900	48889	248	34375801	64044	314	25549297	71679	347
22.9	05402789	49136	247	34439845	64358	314	25620976	72027	348
23.0	0.05451925	+49385	+249	0.34504203	+64673	+315	0.25693003	+72375	+348
23.1	05501310	49632	247	34568876	64987	314	25765378	72725	350
23.2	05550942	49882	250	34633863	65303	316	25838103	73074	349
23.3	05600824	50131	249	34699166	65619	316	25911177	73424	350
23.4	05650955	50381	250	34764785	65936	317	25984601	73774	350
23.5	0.05701336	+50631	+250	0.34830721	+66252	+316	0.26058375	+74125	+351
23.6	05751967	50882	251	34896973	66570	318	26132500	74476	351
23.7	05802849	51134	252	34963543	66888	318	26206976	74829	353
23.8	05853983	51385	251	35030431	67207	319	26281805	75181	352
23.9	05905368	51638	253	35097638	67525	318	26356986	75534	353
24.0	0.05957006	+51891	+253	0.35165163	+67845	+320	0.26432520	+75887	+353
24.1	06008897	52144	253	35233008	68166	321	26508407	76241	354
24.2	06061041	52399	255	35301174	68485	319	26584648	76596	355
24.3	06113440	52653	254	35369659	68807	322	26661244	76951	355
24.4	06166093	52907	254	35438466	69129	322	26738195	77306	355
24.5	0.06219000	+53164	+257	0.35507595	+69450	+321	0.26815501	+77662	+356
24.6	06272164	53419	255	35577045	69774	324	26893163	78019	357
24.7	06325583	53676	257	35646819	70096	322	26971182	78377	358
24.8	06379259	53933	257	35716915	70421	325	27049559	78733	356
24.9	06433192	54191	258	35787336	70744	323	27128292	79093	360
25.0	0.06487383	+54449	+258	0.35858080	+71070	+326	0.27207385	+79450	+357
25.1	06541832	54708	259	35929150	71395	325	27286835	79811	361
25.2	06596540	54968	260	36000545	71721	326	27366646	80170	359
25.3	06651508	55227	259	36072266	72048	327	27446816	80531	361
25.4	06706735	55488	261	36144314	72374	326	27527347	80891	360
25.5	0.06762223	+55748	+260	0.36216688	+72702	+328	0.27608238	+81254	+363
25.6	06817971	56011	263	36289390	73031	329	27689492	81615	361
25.7	06873982	56272	261	36362421	73359	328	27771107	81978	363
25.8	06930254	56536	264	36435780	73689	330	27853085	82341	363
25.9	06986790	+56798	262	36509469	+74019	330	27935426	+82706	365
26.0	0.07043588		+265	0.36583488		+330	0.28018132		+364

Table of the Values of Three Elliptic Integrals, &c.—Continued.

θ	Log. K			Log. L′			Log. N		
°									
26.0	0.07043588	+57063	+265	0.36583488	+74349	+330	0.28018132	+83070	+364
26.1	07100651	57327	264	36657837	74680	331	28101202	83434	364
26.2	07157978	57592	265	36732517	75012	332	28184636	83801	367
26.3	07215570	57858	266	36807529	75344	332	28268437	84166	365
26.4	07273428	58124	266	36882873	75678	334	28352603	84534	368
26.5	0.07331552	+58391	+267	0.36958551	+76010	+332	0.28437137	+84901	+367
26.6	07389943	58659	268	37034561	76345	335	28522038	85269	368
26.7	07448602	58926	267	37110906	76680	335	28607307	85637	368
26.8	07507528	59196	270	37187586	77015	335	28692944	86007	370
26.9	07566724	59464	268	37264601	77350	335	28778951	86377	370
27.0	0.07626188	+59735	+271	0.37341951	+77688	+338	0.28865328	+86747	+370
27.1	07685923	60005	270	37419639	78024	336	28952075	87118	371
27.2	07745928	60276	271	37497663	78363	339	29039193	87490	372
27.3	07806204	60548	272	37576026	78701	338	29126683	87863	373
27.4	07866752	60821	273	37654727	79040	339	29214546	88235	372
27.5	0.07927573	+61094	+273	0.37733767	+79379	+339	0.29302781	+88609	+374
27.6	07988667	61367	273	37813146	79720	341	29391390	88983	374
27.7	08050034	61642	275	37892866	80061	341	29480373	89358	375
27.8	08111676	61917	275	37972927	80403	342	29569731	89734	376
27.9	08173593	62193	276	38053330	80745	342	29659465	90110	376
28.0	0.08235786	+62469	+276	0.38134075	+81088	+343	0.29749575	+90486	+376
28.1	08298255	62745	276	38215163	81432	344	29840061	90864	378
28.2	08361000	63024	279	38296595	81775	343	29930925	91243	379
28.3	08424024	63302	278	38378370	82121	346	30022168	91621	378
28.4	08487326	63581	279	38460491	82467	346	30113789	92001	380
28.5	0.08550907	+63861	+280	0.38542958	+82813	+346	0.30205790	+92380	+379
28.6	08614768	64141	280	38625771	83159	346	30298170	92762	382
28.7	08678909	64422	281	38708930	83508	349	30390932	93143	381
28.8	08743331	64704	282	38792438	83856	348	30484075	93526	383
28.9	08808035	64987	283	38876294	84205	349	30577601	93908	382
29.0	0.08873022	+65269	+282	0.38960499	+84555	+350	0.30671509	+94292	+384
29.1	08938291	65554	285	39045054	84905	350	30765801	94677	385
29.2	09003845	65838	284	39129959	85256	351	30860478	95061	384
29.3	09069683	66123	285	39215215	85608	352	30955539	95447	386
29.4	09135806	66410	287	39300823	85961	353	31050986	95833	386
29.5	0.09202216	+66696	+286	0.39386784	+86315	+354	0.31146819	+96221	+388
29.6	09268912	66983	287	39473099	86668	353	31243040	96608	387
29.7	09335895	67272	289	39559767	87023	355	31339648	96998	390
29.8	09403167	67561	289	39646790	87379	356	31436646	97386	388
29.9	09470728	+67851	290	39734169	+87734	355	31534032	+97776	390
30.0	0.09538579		+289	0.39821903		+358	0.31631808		+391

Table of the Values of Three Elliptic Integrals, &c.—Continued.

θ	Log. K			Log. L′			Log. N		
30.0	0.09538579	+68140	+289	0.39821903	+88092	+358	0.31631808	+ 98167	+391
30.1	09606719	68432	292	39909995	88450	358	31729975	98559	392
30.2	09675151	68724	292	39998445	88808	358	31828534	98951	392
30.3	09743875	69017	293	40087253	89167	359	31927485	99344	393
30.4	09812892	69310	293	40176420	89528	361	32026829	99738	394
30.5	0.09882202	+69605	+295	0.40265948	+89888	+360	0.32126567	+100132	+394
30.6	09951807	69899	294	40355836	90250	362	32226699	100527	395
30.7	10021706	70195	296	40446086	90612	362	32327226	100924	397
30.8	10091901	70492	297	40536698	90975	363	32428150	101320	396
30.9	10162393	70789	297	40627673	91339	364	32529470	101718	398
31.0	0.10233182	+71087	+298	0.40719012	+91704	+365	0.32631188	+102117	+399
31.1	10304269	71386	299	40810716	92069	365	32733305	102515	398
31.2	10375655	71686	300	40902785	92435	366	32835820	102915	400
31.3	10447341	71986	300	40995220	92803	368	32938735	103317	402
31.4	10519327	72288	302	41088023	93170	367	33042052	103717	400
31.5	0.10591615	+72589	+301	0.41181193	+93540	+370	0.33145769	+104120	+403
31.6	10664204	72893	304	41274733	93908	368	33249889	104523	403
31.7	10737097	73197	304	41368641	94279	371	33354412	104927	404
31.8	10810294	73501	304	41462920	94650	371	33459339	105332	405
31.9	10883795	73807	306	41557570	95022	372	33564671	105738	406
32.0	0.10957602	+74113	+306	0.41652592	+95395	+373	0.33670409	+106144	+406
32.1	11031715	74421	308	41747987	95769	374	33776553	106551	407
32.2	11106136	74728	307	41843756	96143	374	33883104	106959	408
32.3	11180864	75037	309	41939899	96519	376	33990063	107368	409
32.4	11255901	75347	310	42036418	96894	375	34097431	107778	410
32.5	0.11331248	+75658	+311	0.42133312	+ 97272	+378	0.34205209	+108188	+410
32.6	11406906	75969	311	42230584	97650	378	34313397	108600	412
32.7	11482875	76282	313	42328234	98029	379	34421997	109013	413
32.8	11559157	76594	312	42426263	98408	379	34531010	109425	412
32.9	11635751	76909	315	42524671	98789	381	34640435	109840	415
33.0	0.11712660	+77224	+315	0.42623460	+ 99171	+382	0.34750275	+110254	+414
33.1	11789884	77540	316	42722631	99552	381	34860529	110671	417
33.2	11867424	77857	317	42822183	99937	385	34971200	111087	416
33.3	11945281	78175	318	42922120	100320	383	35082287	111504	417
33.4	12023456	78493	318	43022440	100705	385	35193791	111923	419
33.5	0.12101949	+78813	+320	0.43123145	+101092	+387	0.35305714	+112343	+420
33.6	12180762	79133	320	43224237	101478	386	35418057	112763	420
33.7	12259895	79455	322	43325715	101867	389	35530820	113184	421
33.8	12339350	79777	322	43427582	102255	388	35644004	113606	422
33.9	12419127	+80101	324	43529837	+102645	390	35757610	+114029	423
34.0	0.12499228		+324	0.43632482		+390	0.35871639		+425

Table of the Values of Three Elliptic Integrals, &c.—Continued.

θ	Log. K			Log. L′			Log. N		
34.0	0.12499228	+80425	+324	0.43632482	+103035	+390	0.35871639	+114454	+425
34.1	12579653	80751	326	43735517	103427	392	35986093	114878	424
34.2	12660404	81076	325	43838944	103820	393	36100971	115304	426
34.3	12741480	81404	328	43942764	104214	394	36216275	115731	427
34.4	12822884	81732	328	44046978	104608	394	36332006	116158	427
34.5	0.12904616	+82061	+329	0.44151586	+105003	+395	0.36448164	+116588	+430
34.6	12986677	82392	331	44256589	105400	397	36564752	117017	429
34.7	13069069	82722	330	44361989	105797	397	36681769	117447	430
34.8	13151791	83055	333	44467786	106196	399	36799216	117880	433
34.9	13234846	83388	333	44573982	106595	399	36917096	118312	432
35.0	0.13318234	+83722	+334	0.44680577	+106996	+401	0.37035408	+118745	+433
35.1	13401956	84057	335	44787573	107398	402	37154153	119180	435
35.2	13486013	84394	337	44894971	107799	401	37273333	119616	436
35.3	13570407	84731	337	45002770	108204	405	37392949	120052	436
35.4	13655138	85069	338	45110974	108608	404	37513001	120489	437
35.5	0.13740207	+85408	+339	0.45219582	+109013	+405	0.37633490	+120929	+440
35.6	13825615	85749	341	45328595	109420	407	37754419	121368	439
35.7	13911364	86091	342	45438015	109828	408	37875787	121808	440
35.8	13997455	86433	342	45547843	110237	409	37997595	122250	442
35.9	14083888	86776	343	45658080	110646	409	38119845	122693	443
36.0	0.14170664	+87122	+346	0.45768726	+111057	+411	0.38242538	+123137	+444
36.1	14257786	87467	345	45879783	111469	412	38365675	123582	445
36.2	14345253	87814	347	45991252	111883	414	38489257	124027	445
36.3	14433067	88161	347	46103135	112296	413	38613284	124474	447
36.4	14521228	88512	351	46215431	112711	415	38737758	124922	448
36.5	0.14609740	+88861	+349	0.46328142	+113128	+417	0.38862680	+125372	+450
36.6	14698601	89212	351	46441270	113546	418	38988052	125821	449
36.7	14787813	89565	353	46554816	113963	417	39113873	126273	452
36.8	14877378	89919	354	46668779	114384	421	39240146	126725	452
36.9	14967297	90273	354	46783163	114804	420	39366871	127179	454
37.0	0.15057570	+90630	+357	0.46897967	+115226	+422	0.39494050	+127633	+454
37.1	15148200	90986	356	47013193	115650	424	39621683	128088	455
37.2	15239186	91344	358	47128843	116073	423	39749771	128546	458
37.3	15330530	91704	360	47244916	116499	426	39878317	129003	457
37.4	15422234	92065	361	47361415	116925	426	40007320	129463	460
37.5	0.15514299	+ 92426	+361	0.47478340	+117354	+429	0.40136783	+129923	+460
37.6	15606725	92789	363	47595694	117782	428	40266706	130384	461
37.7	15699514	93153	364	47713476	118212	430	40397090	130846	462
37.8	15792667	93519	366	47831688	118643	431	40527936	131310	464
37.9	15886186	+93885	366	47950331	+119076	433	40659246	+131776	466
38.0	0.15980071		+368	0.48069407		+434	0.40791022		+465

Table of the Values of Three Elliptic Integrals, &c.—Continued.

θ	Log. 𝔎			Log. 𝔏′			Log. 𝔑		
38.°0	0.15980071	+ 94253	+368	0.48069407	+119510	+434	0.40791022	+132241	+465
38.1	16074324	94622	369	48188917	119944	434	40923263	132708	467
38.2	16168946	94992	370	48308861	120381	437	41055971	133177	469
38.3	16263938	95364	372	48429242	120818	437	41189148	133646	469
38.4	16359302	95737	373	48550060	121257	439	41322794	134117	471
38.5	0.16455039	+ 96110	+373	0.48671317	+121696	+439	0.41456911	+134589	+472
38.6	16551149	96486	376	48793013	122138	442	41591500	135062	473
38.7	16647635	96863	377	48915151	122580	442	41726562	135537	475
38.8	16744498	97240	377	49037731	123023	443	41862099	136013	476
38.9	16841738	97620	380	49160754	123469	446	41998112	136489	476
39.0	0.16939358	+ 98000	+380	0.49284223	+123915	+446	0.42134601	+136968	+479
39.1	17037358	98382	382	49408138	124362	447	42271569	137447	479
39.2	17135740	98766	384	49532500	124811	449	42409016	137928	481
39.3	17234506	99149	383	49657311	125262	451	42546944	138409	481
39.4	17333655	99536	387	49782573	125712	450	42685353	138894	485
39.5	0.17433191	+ 99922	+386	0.49908285	+126166	+454	0.42824247	+139377	+483
39.6	17533113	100312	390	50034451	126619	453	42963624	139864	487
39.7	17633425	100701	389	50161070	127076	457	43103488	140351	487
39.8	17734126	101092	391	50288146	127532	456	43243839	140839	488
39.9	17835218	101485	393	50415678	127990	458	43384678	141329	490
40.0	0.17936703	+101879	+394	0.50543668	+128450	+460	0.43526007	+141820	+491
40.1	18038582	102275	396	50672118	128910	460	43667827	142313	493
40.2	18140857	102671	396	50801028	129373	463	43810140	142807	494
40.3	18243528	103070	399	50930401	129837	464	43952947	143301	494
40.4	18346598	103469	399	51060238	130302	465	44096248	143799	498
40.5	0.18450067	+103871	+402	0.51190540	+130769	+467	0.44240047	+144296	+497
40.6	18553938	104273	402	51321309	131236	467	44384343	144796	500
40.7	18658211	104677	404	51452545	131706	470	44529139	145296	500
40.8	18762888	105083	406	51584251	132177	471	44674435	145799	503
40.9	18867971	105490	407	51716428	132649	472	44820234	146302	503
41.0	0.18973461	+105898	+408	0.51849077	+133123	+474	0.44966536	+146807	+505
41.1	19079359	106309	411	51982200	133598	475	45113343	147313	506
41.2	19185668	106719	410	52115798	134075	477	45260656	147821	508
41.3	19292387	107133	414	52249873	134553	478	45408477	148331	510
41.4	19399520	107548	415	52384426	135033	480	45556808	148841	510
41.5	0.19507068	+107964	+416	0.52519459	+135515	+482	0.45705649	+149353	+512
41.6	19615032	108381	417	52654974	135997	482	45855002	149867	514
41.7	19723413	108801	420	52790971	136481	484	46004869	150382	515
41.8	19832214	109221	420	52927452	136967	486	46155251	150899	517
41.9	19941435	+109644	423	53064419	+137455	488	46306150	+151416	517
42.0	0.20051079		+423	0.53201874		+488	0.46457566		+521

Table of the Values of Three Elliptic Integrals, &c.—Continued.

θ	Log. K			Log. L′			Log. N		
42.0°	0.20051079	+110067	+423	0.53201874	+137943	+488	0.46457566	+151937	+521
42.1	20161146	110494	427	53339817	138435	492	46609503	152457	520
42.2	20271640	110920	426	53478252	138926	491	46761960	152981	524
42.3	20382560	111350	430	53617178	139420	494	46914941	153504	523
42.4	20493910	111779	429	53756598	139915	495	47068445	154030	526
42.5	0.20605689	+112212	+433	0.53896513	+140413	+498	0.47222475	+154558	+528
42.6	20717901	112646	434	54036926	140911	498	47377033	155086	528
42.7	20830547	113081	435	54177837	141411	500	47532119	155617	531
42.8	20943628	113518	437	54319248	141913	502	47687736	156149	532
42.9	21057146	113957	439	54461161	142416	503	47843885	156683	534
43.0	0.21171103	+114397	+440	0.54603577	+142922	+506	0.48000568	+157218	+535
43.1	21285500	114840	443	54746499	143429	507	48157786	157755	537
43.2	21400340	115283	443	54889928	143937	508	48315541	158294	539
43.3	21515623	115730	447	55033865	144447	510	48473835	158833	539
43.4	21631353	116176	446	55178312	144960	513	48632668	159376	543
43.5	0.21747529	+116626	+450	0.55323272	+145473	+513	0.48792044	+159919	+543
43.6	21864155	117077	451	55468745	145989	516	48951963	160464	545
43.7	21981232	117529	452	55614734	146505	516	49112427	161011	547
43.8	22098761	117985	456	55761239	147025	520	49273438	161560	549
43.9	22216746	118440	455	55908264	147546	521	49434998	162110	550
44.0	0.22335186	+118899	+459	0.56055810	+148068	+522	0.49597108	+162662	+552
44.1	22454085	119358	459	56203878	148592	524	49759770	163215	553
44.2	22573443	119821	463	56352470	149119	527	49922985	163771	556
44.3	22093264	120284	463	56501589	149646	527	50086756	164329	558
44.4	22813548	120750	466	56651235	150176	530	50251085	164887	558
44.5	0.22934298	+121217	+467	0.56801411	+150708	+532	0.50415972	+165448	+561
44.6	23055515	121687	470	56952119	151242	534	50581420	166011	563
44.7	23177202	122158	471	57103361	151777	535	50747431	166575	564
44.8	28299360	122631	473	57255138	152314	537	50914006	167141	566
44.9	23421991	123106	475	57407452	152853	539	51081147	167710	569
45.0	0.23545097	+123584	+478	0.57560305	+153394	+541	0.51248857	+168279	+569
45.1	23668681	124063	479	57713699	153938	544	51417136	168851	572
45.2	23792744	124543	480	57867637	154482	544	51585987	169424	573
45.3	23917287	125027	484	58022119	155029	547	51755411	170000	576
45.4	24042314	125512	485	58177148	155578	549	51925411	170577	577
45.5	0.24167826	+125999	+487	0.58332726	+156129	+551	0.52095988	+171157	+580
45.6	24293825	126488	489	58488855	156681	552	52267145	171737	580
45.7	24420313	126979	491	58645536	157237	556	52438882	172321	584
45.8	24547292	127473	494	58802773	157793	556	52611203	172906	585
45.9	24674765	+127967	494	58960566	+158353	560	52784109	+173492	586
46.0	0.24802732		+499	0.59118919		+560	0.52957601		+590

Table of the Values of Three Elliptic Integrals, &c.—Continued.

θ	Log. K			Log. L′			Log. N		
46.0	0.24802732	+128366	+499	0.59118919	+158913	+560	0.52957601	+174082	+590
46.1	24931198	128965	499	59277832	159476	563	53131683	174673	591
46.2	25060163	129466	501	59437308	160041	565	53306356	175265	592
46.3	25189629	129971	505	59597349	160609	568	53481621	175861	596
46.4	25319600	130476	505	59757958	161177	568	53657482	176457	596
46.5	0.25450076	+130984	+508	0.59919135	+161749	+572	0.53833939	+177057	+600
46.6	25581060	131495	511	60080884	162323	574	54010996	177657	600
46.7	25712555	132008	513	60243207	162899	576	54188653	178261	604
46.8	25844563	132522	514	60406106	163476	577	54366914	178866	605
46.9	25977085	133039	517	60569582	164056	580	54545780	179473	607
47.0	0.26110124	+133558	+519	0.60733638	+164639	+583	0.54725253	+180083	+610
47.1	26243682	134080	522	60898277	165223	584	54905336	180694	611
47.2	26377762	134604	524	61063500	165809	586	55086030	181308	614
47.3	26512366	135130	526	61229309	166399	590	55267338	181924	616
47.4	26647496	135658	528	61395708	166990	591	55449262	182541	617
47.5	0.26783154	+136189	+531	0.61562698	+167583	+593	0.55631803	+183162	+621
47.6	26919343	136723	534	61730281	168179	596	55814965	183785	623
47.7	27056066	137257	534	61898460	168777	598	55998750	184409	624
47.8	27193323	137796	539	62067237	169378	601	56183159	185035	626
47.9	27331119	138336	540	62236615	169981	603	56368194	185665	630
48.0	0.27469455	+138878	+542	0.62406596	+170585	+604	0.56553859	+186297	+632
48.1	27608333	139424	546	62577181	171193	608	56740156	186930	633
48.2	27747757	139972	548	62748374	171803	610	56927086	187565	635
48.3	27887729	140521	549	62920177	172415	612	57114651	188205	640
48.4	28028250	141074	553	63092592	173030	615	57302856	188844	639
48.5	0.28169324	+141629	+555	0.63265622	+173648	+618	0.57491700	+189488	+644
48.6	28310953	142187	558	63439270	174267	619	57681188	190133	645
48.7	28453140	142747	560	63613537	174889	622	57871321	190781	648
48.8	28595887	143310	563	63788426	175513	624	58062102	191431	650
48.9	28739197	143875	565	63963939	176141	628	58253533	192083	652
49.0	0.28883072	+144443	+568	0.64140080	+176771	+630	0.58445616	+192738	+655
49.1	29027515	145013	570	64316851	177403	632	58638354	193396	658
49.2	29172528	145587	574	64494254	178038	635	58831750	194056	660
49.3	29318115	146163	576	64672292	178675	637	59025806	194718	662
49.4	29464278	146741	578	64850967	179316	641	59220524	195383	665
49.5	0.29611019	+147323	+582	0.65030283	+179958	+642	0.59415907	+196050	+667
49.6	29758342	147907	584	65210241	180603	645	59611957	196720	670
49.7	29906249	148493	586	65390844	181252	649	59808677	197393	673
49.8	30054742	149084	591	65572096	181903	651	60006070	198068	675
49.9	30203826	+149675	+591	65753999	+182556	+653	60204138	+198745	+677
50.0	0.30353501			0.65936555			0.60402883		

ADDENDUM.

Since the preceding portion of this memoir was in type it has occurred to me that some of the processes might be modified with advantage.

First, the roots of the equation

$$x\,[(x-A)\,(x+C)+B^2]+B^2C\sin^2\varepsilon=0$$

can be obtained by the well-known trigonometric method. If we put

$$p=\frac{1}{3}(A-C)$$

$$q^2=p^2-\frac{1}{3}(B^2-AC)$$

$$r=\frac{1}{2}p\,(p^2-3\,q^2)+\frac{1}{2}B^2C\sin^2\varepsilon$$

$$\sin\theta=\tau=\frac{r}{q^3}$$

and if θ is taken between the limits $\pm 90^\circ$, the three quantities G, G′, and G″ are given by the equations

$$G\;=2\,q\sin\left(60^\circ-\frac{\theta}{3}\right)+p$$

$$G'\;=2\,q\sin\frac{\theta}{3}+p$$

$$G''=2\,q\sin\left(60^\circ+\frac{\theta}{3}\right)-p$$

From these equations we derive the following:

$$G\;+G''=2\sqrt{3}\,q\cos\frac{\theta}{3}$$

$$G'+G''=2\sqrt{3}\,q\cos\left(60^\circ-\frac{\theta}{3}\right)$$

$$G\;-G'\;=2\sqrt{3}\,q\cos\left(60^\circ+\frac{\theta}{3}\right)$$

If these values are substituted in the equations

$$\Gamma=\frac{F+JG'+fG'^2}{(G'+G'')\,(G-G')}\qquad\qquad \Gamma''=\frac{-F+JG''-fG''^2}{(G+G'')\,(G'+G'')}$$

we obtain

$$\Gamma = \frac{F + Jp + f(p^2 + 2q^2) + 2(J + 2fp)q\sin\frac{\theta}{3} - 2fq^2\cos\frac{2}{3}\theta}{12q^2\cos\left(60^\circ - \frac{\theta}{3}\right)\cos\left(60^\circ + \frac{\theta}{3}\right)}$$

$$\Gamma' = \frac{-[F + Jp + f(p^2 + 2q^2)] + 2(J + 2fp)q\sin\left(60^\circ + \frac{\theta}{3}\right) + 2fq^2\cos\left(120^\circ + \frac{2}{3}\theta\right)}{12q^2\cos\frac{\theta}{3}\cos\left(60^\circ - \frac{\theta}{3}\right)}$$

Or, since we have

$$\cos\theta = 4\cos\frac{\theta}{3}\cos\left(60^\circ - \frac{\theta}{3}\right)\cos\left(60^\circ + \frac{\theta}{3}\right)$$

$$\Gamma = \frac{[F + Jp + f(p^2 + q^2)]\cos\frac{\theta}{3} + (J + 2fp)q\sin\frac{2}{3}\theta}{3q^2\cos\theta} - \frac{1}{3}f$$

$$\Gamma' = \frac{-[F + Jp + f(p^2 + q^2)]\cos\left(60^\circ + \frac{\theta}{3}\right) + (J + 2fp)q\sin\left(120^\circ + \frac{2}{3}\theta\right)}{3q^2\cos\theta} - \frac{1}{3}f.$$

From these equations we derive

$$\Gamma + 2\Gamma' + f = \frac{[F + Jp + f(p^2 + q^2)]\sin\frac{\theta}{3} + (J + 2fp)q\cos\frac{2}{3}\theta}{\sqrt{3}\,q^2\cos\theta}$$

$$2\Gamma + \Gamma' + f = \frac{[F + Jp + f(p^2 + q^2)]\sin\left(60^\circ + \frac{\theta}{3}\right) + (J + 2fp)q\cos\left(60^\circ - \frac{2}{3}\theta\right)}{\sqrt{3}\,q^2\cos\theta}$$

The values of R_0, S_0, and W_0 are given by the integral

$$\frac{2}{\pi}\int_0^{\frac{\pi}{2}} \frac{[\Gamma + 2\Gamma' + f]\cos^2 T + [2\Gamma + \Gamma' + f]\sin^2 T}{(2\sqrt{3}\,q)^{\frac{3}{2}}\left[\cos\frac{\theta}{3}\cos^2 T + \cos\left(60^\circ + \frac{\theta}{3}\right)\sin^2 T\right]^{\frac{3}{2}}}\,dT$$

provided we attribute to F, J, and f the values they severally have in each case. Let us put

$$m^2 = \cos\frac{\theta}{3} \qquad n^2 = \cos\left(60^\circ + \frac{\theta}{3}\right)$$

$$a = \frac{F + Jp + f(p^2 + q^2)}{6\sqrt[4]{12}\,q^{\frac{7}{2}}} \qquad b = \frac{J + 2fp}{6\sqrt[4]{12}\,q^{\frac{5}{2}}}$$

Then the integral, just given, takes the form

$$\frac{2}{\pi}\int_0^{\frac{\pi}{2}} \frac{\left[a\sin\frac{\theta}{3} + b\cos\frac{2}{3}\theta \right]\cos^2 T + \left[a\sin\left(60^\circ + \frac{\theta}{3}\right) + b\cos\left(60^\circ - \frac{2}{3}\theta\right) \right]\sin^2 T}{\cos\theta\,[m^2\cos^2 T + n^2\sin^2 T]^{\frac{3}{2}}}\,dT$$

In the second place Gauss's processes for approximating to the values of the integrals may be employed instead of those of Legendre. The equation between definite integrals

$$\int_0^{\frac{\pi}{2}} \frac{dT}{\sqrt{(1 - c^2\sin^2 T)}} = (1 + c^0)\int_0^{\frac{\pi}{2}} \frac{dT}{\sqrt{(1 - c^{02}\sin^2 T)}}$$

may be easily transformed into

$$\int_0^{\frac{\pi}{2}} \frac{dT}{[m^2\cos^2 T + n^2\sin^2 T]^{\frac{1}{2}}} = \int_0^{\frac{\pi}{2}} \frac{dT}{[m'^2\cos^2 T + n'^2\sin^2 T]^{\frac{1}{2}}}$$

where

$$m' = \tfrac{1}{2}(m + n) \qquad n' = \sqrt{mn}$$

when we remember that

$$c^2 = \frac{m^2 - n^2}{m^2} \qquad c^0 = \frac{m - n}{m + n}$$

If this mode of transformation is continued, and we compute

$$m'' = \frac{1}{2}(m' + n') \qquad n'' = \sqrt{m'n'}$$

$$m''' = \frac{1}{2}(m'' + n'') \qquad n''' = \sqrt{m''n''}$$

$$\cdots\cdots\cdots\cdots\cdots\cdots$$

the series of quantities, m, m′, m″, etc., and n, n′, n″, etc., converge very rapidly toward a common limit μ, which Gauss has called the *arithmetico-geometrical mean* between m and n. Then,

$$\frac{2}{\pi}\int_0^{\frac{\pi}{2}} \frac{dT}{[m^2\cos^2 T + n^2\sin^2 T]^{\frac{1}{2}}} = \frac{1}{\mu}$$

The equation

$$\frac{2}{\pi}\int_0^{\frac{\pi}{2}} \frac{A + B\sin^2 T}{\sqrt{(1 - c^2\sin^2 T)}}\,dT = K\left[A + \frac{B}{2}\left(1 + \frac{c^0}{2} + \frac{c^0 c^{00}}{4} + \frac{c^0 c^{00} c^{000}}{8} + \ldots\right)\right]$$

on putting

$$A = -\frac{1}{m} \qquad B = \frac{2}{m}$$

is readily transformed into

$$\frac{2}{\pi}\int_0^{\frac{\pi}{2}} \frac{\sin^2 T - \cos^2 T}{[m^2\cos^2 T + n^2 \sin^2 T]^{\frac{1}{2}}}\,dT = \frac{1}{\mu}\left[\frac{m-n}{2(m+n)} + \frac{m-n}{2(m+n)}\;\frac{m'-n'}{2(m'+n')} + \;\ldots\ldots\right]$$

The series within the brackets may be denoted by ν. It can be transformed as follows:

$$\nu = \frac{m^2-n^2}{8\,m'^2} + \frac{m^2-n^2}{8\,m'^2}\;\frac{m'^2-n'^2}{8\,m''^2} + \frac{m^2-n^2}{8\,m'^2}\;\frac{m'^2-n'^2}{8\,m''^2}\;\frac{m''^2-n''^2}{8\,m'''^2} + \ldots.$$

$$= \frac{m^2-n^2}{8\,m'^2} + \frac{m^2-n^2}{8\,m'^2}\;\frac{(m^2-n^2)^2}{128\,m'^2\,m''^2} + \frac{m^2-n^2}{8\,m'^2}\;\frac{(m^2-n^2)^2}{128\,m'^2\,m''^2}\;\frac{(m'^2-n'^2)^2}{128\,m''^2\,m'''^2} + \;\ldots$$

As this mode of transformation may be continued indefinitely, it is plain, that if we compute the series of quantities

$$\lambda = \tfrac{1}{4}\sqrt{(m^2-n^2)} \qquad \lambda' = \frac{\lambda^2}{m'} \qquad \lambda'' = \frac{\lambda'^2}{m''} \qquad \lambda''' = \frac{\lambda''^2}{m'''}\ldots\ldots$$

we shall have

$$\nu = \frac{2\,\lambda'^2 + 4\,\lambda''^2 + 8\,\lambda'''^2 + \ldots.}{\lambda^2}$$

The equation

$$\int_0^{\frac{\pi}{2}} \frac{1 - 2\sin^2 T + c^2 \sin^4 T}{[1 - c^2\sin^2 T]^{\frac{3}{2}}}\,dT = 0$$

is readily transformed into

$$\int_0^{\frac{\pi}{2}} \frac{m^2\cos^4 T - n^2\sin^4 T}{[m^2\cos^2 T + n^2\sin^2 T]^{\frac{3}{2}}}\,dT = 0$$

Whence we conclude that

$$\frac{\pi}{2}\int_0^{\frac{\pi}{2}} \frac{\cos^2 T}{[m^2\cos^2 T + n^2\sin^2 T]^{\frac{3}{2}}}\,dT = \frac{1+\nu}{2\,m^2\,\mu}$$

$$\frac{\pi}{2}\int_0^{\frac{\pi}{2}} \frac{\sin^2 T}{[m^2\cos^2 T + n^2\sin^2 T]^{\frac{3}{2}}}\,dT = \frac{1-\nu}{2\,n^2\,\mu}$$

Substituting these values in the general integral expression for R^0, S^0, and W^0, we get

$$R^0,\ S^0,\ \text{or}\ W^0 = \frac{a}{\cos\theta}\left[\frac{1+\nu}{2\,\mu}\tan\frac{\theta}{3} + \frac{1-\nu}{2\,\mu}\tan\left(60^\circ + \frac{\theta}{3}\right)\right]$$

$$+\frac{b}{\cos\theta}\left[\frac{1+\nu}{2\,\mu}\,\frac{\cos\frac{2}{3}\theta}{\cos\frac{\theta}{3}} + \frac{1-\nu}{2\,\mu}\,\frac{\cos\left(60^\circ - \frac{2}{3}\theta\right)}{\cos\left(60^\circ + \frac{\theta}{3}\right)}\right]$$

This expression presents the inconvenience of taking the indeterminate form $\frac{0}{0}$ when the modulus c vanishes and when $\theta = -90°$. This is avoided by putting

$$\nu' = \frac{\sqrt{3}}{64}\frac{\nu}{\lambda^2}$$

where we recall that

$$\lambda^2 = \frac{1}{16}\cos\left(60° - \frac{\theta}{3}\right)$$

and transforming the expression into the shape

$$\mathrm{a}\,\frac{\sin\left(60° - \frac{\theta}{3}\right) - \nu'}{4\,\mu\cos^2\frac{\theta}{3}\cos^2\left(60° + \frac{\theta}{3}\right)} + \mathrm{b}\,\frac{\frac{1}{2} + \cos\frac{\theta}{3}\cos\left(60° + \frac{\theta}{3}\right) - \nu'\sin\theta}{4\,\mu\cos^2\frac{\theta}{3}\cos^2\left(60° + \frac{\theta}{3}\right)}$$

This may be written, if we choose, in the briefer manner

$$\mathrm{a}\,\frac{\sin\left(60° - \frac{\theta}{3}\right) - \nu'}{4\,\mathrm{m}^4\,\mathrm{n}^4\,\mu} + \mathrm{b}\,\frac{\frac{1}{2} + \mathrm{m}^2\,\mathrm{n}^2 - \nu'\sin\theta}{4\,\mathrm{m}^4\,\mathrm{n}^4\,\mu}$$

The factors of a and b in this expression are functions of τ, and their common logarithms might be tabulated with τ as the argument.

We will now put

$$\chi(\tau) = \frac{\sin\left(60° - \frac{\theta}{3}\right) - \nu'}{24\sqrt[4]{12}\,\mathrm{m}^4\,\mathrm{n}^4\,\mu} \qquad \psi(\tau) = \frac{\frac{1}{2} + \mathrm{m}^2\,\mathrm{n}^2 - \nu'\sin\theta}{24\sqrt[4]{12}\,\mathrm{m}^4\,\mathrm{n}^4\,\mu}$$

as also

$$\mathrm{V} = \frac{\mathrm{p}}{\mathrm{q}}\,\chi(\tau) + \psi(\tau)$$

Then, if

$$\mathrm{F}_1 = \frac{\mathrm{B}^2 - \mathrm{AC}}{3\,a'^2\cos^2\varphi'.\,\mathrm{q}}$$

$$\mathrm{F}_2 = -\tan\varphi'\cos\mathrm{I}.\,\frac{\mathrm{B}\sin\varepsilon}{\mathrm{q}}$$

$$\mathrm{F}_3 = -\tan\varphi'\sin\mathrm{I}.\,\frac{r}{a}\cos(v + \Pi).\,\frac{\mathrm{B}\sin\varepsilon}{\mathrm{q}}$$

$$\mathrm{J}_1 = 1 - \sin^2\mathrm{I}\sin^2(v + \Pi) - \frac{2\,\mathrm{p}}{a'^2\cos^2\varphi'}$$

$$\mathrm{J}_2 = k\alpha\frac{\tan\varphi'}{\cos\varphi'}\frac{r}{a}\sin(v + \mathrm{K}) - \frac{1}{2}\sin^2\mathrm{I}\sin 2(v + \Pi)$$

$$\mathrm{J}_3 = \sin\mathrm{I}\cos\mathrm{I}.\,\frac{r}{a}\sin(v + \Pi) - \alpha\frac{\tan\varphi'}{\cos\varphi'}\sin\mathrm{I}\sin\Pi.\,\frac{r^2}{a^2}$$

where α denotes $\frac{a}{a'}$, we shall have the following equations

$$\frac{a}{r}\,\mathrm{R}_0 = \alpha^2\, a'^2 \cos^2\varphi' .\, r\, \mathrm{q}^{-\frac{5}{2}}\, [\mathrm{F}_1\, \chi\,(\tau) + \mathrm{J}_1\, \mathrm{V}]$$

$$\frac{a}{r}\,\mathrm{S}_0 = \alpha^2\, a'^2 \cos^2\varphi' .\, r\, \mathrm{q}^{-\frac{5}{2}}\, [\mathrm{F}_2\, \chi\,(\tau) + \mathrm{J}_2\, \mathrm{V}]$$

$$\frac{a}{r}\,\mathrm{W}_0 = \alpha^2\, a'^2 \cos^2\varphi' .\, r\, \mathrm{q}^{-\frac{5}{2}}\, [\mathrm{F}_3\, \chi\,(\tau) + \mathrm{J}_3\, \mathrm{V}]$$

Why we multiply the members of these equations by $\frac{a}{r}$ will presently appear.

A third modification, which seems advantageous, is to apply the process of mechanical quadratures to the quantities $\frac{a}{r}\,\mathrm{R}_0$, $\frac{a}{r}\,\mathrm{S}_0$, and $\frac{a}{r}\,\mathrm{W}_0$, instead of applying it to the variations of the elements. If we multiply the factors of R_0, S_0, and W_0, in the expressions for the variations of the elements, by the factor $\frac{r}{a}$, they become integral functions of sin E and cos E. And thus we have

$$\left[\frac{d\,\varphi}{d\,t}\right]_{00} = \frac{m'\,n}{1+m}\,\mathrm{M}_{\mathrm{E}}\left[\cos\varphi\sin \mathrm{E}.\frac{a}{r}\mathrm{R}_0 + \left(-\frac{3}{2}e + 2\cos\mathrm{E} - \frac{e}{2}\cos 2\,\mathrm{E}\right)\frac{a}{r}\mathrm{S}_0\right]$$

$$e\left[\frac{d\,\chi}{d\,t}\right]_{00} = \frac{m'\,n}{1+m}\,\mathrm{M}_{\mathrm{E}}\left[-\cos\varphi(\cos\mathrm{E}-e)\frac{a}{r}\mathrm{R}_0 + \left((2-e^2)\sin\mathrm{E} - \frac{e}{2}\sin 2\,\mathrm{E}\right)\frac{a}{r}\mathrm{S}_0\right]$$

$$\left[\frac{d\,i}{d\,t}\right]_{00} = \frac{m'\,n}{1+m}\,\mathrm{M}_{\mathrm{E}}\left[(-\tan\varphi\cos\omega + \sec\varphi\cos\omega\cos\mathrm{E} - \sin\omega\sin\mathrm{E})\frac{a}{r}\mathrm{W}_0\right]$$

$$\sin i\left[\frac{d\,\Omega}{d\,t}\right]_{00} = \frac{m'\,n}{1+m}\,\mathrm{M}_{\mathrm{E}}\left[(-\tan\varphi\sin\omega + \sec\varphi\sin\omega\cos\mathrm{E} + \cos\omega\sin\mathrm{E})\frac{a}{r}\mathrm{W}_0\right]$$

$$\frac{m'\,n}{1+m}\,\mathrm{M}_{\mathrm{E}}\left[-2\,\frac{r}{a}\,\mathrm{R}_0\right] = \frac{m'\,n}{1+m}\,\mathrm{M}_{\mathrm{E}}\left[\left(-(2+e^2) + 4\,e\cos\mathrm{E} - e^2\cos 2\,\mathrm{E}\right)\frac{a}{r}\mathrm{R}_0\right]$$

The quantities $\frac{a}{r}\,\mathrm{R}_0$, $\frac{a}{r}\,\mathrm{S}_0$, and $\frac{a}{r}\,\mathrm{W}_0$, by the application of mechanical quadratures, must now be developed in periodic series with the argument E, so that we have

$$\frac{a}{r}\,\mathrm{R}_0 = \mathrm{A}_0^{(c)} + \mathrm{A}_1^{(c)}\cos\mathrm{E} + \mathrm{A}_1^{(s)}\sin\mathrm{E} + \mathrm{A}_2^{(c)}\cos 2\,\mathrm{E} + \ldots\ldots$$

$$\frac{a}{r}\,\mathrm{S}_0 = \mathrm{B}_0^{(c)} + \mathrm{B}_1^{(c)}\cos\mathrm{E} + \mathrm{B}_1^{(s)}\sin\mathrm{E} + \mathrm{B}_2^{(c)}\cos 2\,\mathrm{E} + \mathrm{B}_2^{(s)}\sin 2\,\mathrm{E} + \ldots$$

$$\frac{a}{r}\,\mathrm{W}_0 = \mathrm{C}_0^{(c)} + \mathrm{C}_1^{(c)}\cos\mathrm{E} + \mathrm{C}_1^{(s)}\sin\mathrm{E} + \ldots\ldots$$

where we have written only the terms whose coefficients are needed.

If the circumference, with reference to E, is divided into j parts, and the corresponding values of $\frac{a}{r}$ R_0 are $R^{(0)}$, $R^{(1)}$, $R^{(2)}$ . . . $R^{(j-1)}$, then

$$A_0^{(c)} = \frac{1}{j}\left[R^{(0)} + R^{(1)} + R^{(2)} + \dots + R^{(j-1)}\right]$$

$$\frac{1}{2}A_1^{(c)} = \frac{1}{j}\left[R^{(0)} + R^{(1)}\cos\frac{2\pi}{j} + R^{(2)}\cos\frac{4\pi}{j} + \dots + R^{(j-1)}\cos\frac{2(j-1)\pi}{j}\right]$$

$$\frac{1}{2}A_1^{(s)} = \frac{1}{j}\left[\qquad R^{(1)}\sin\frac{2\pi}{j} + R^{(2)}\sin\frac{4\pi}{j} + \dots + R^{(j-1)}\sin\frac{2(j-1)\pi}{j}\right]$$

$$\frac{1}{2}A_2^{(c)} = \frac{1}{j}\left[R^{(0)} + R^{(1)}\cos\frac{4\pi}{j} + R^{(2)}\cos\frac{8\pi}{j} + \dots + R^{(j-1)}\cos\frac{4(j-1)\pi}{j}\right]$$

$$\frac{1}{2}A_2^{(s)} = \frac{1}{j}\left[\qquad R^{(1)}\sin\frac{4\pi}{j} + R^{(2)}\sin\frac{8\pi}{j} + \dots + R^{(j-1)}\sin\frac{4(j-1)\pi}{j}\right]$$

Similar equations give the coefficients of $\frac{a}{r}$ S_0 and $\frac{a}{r}$ W_0.

In fine the following equations result

$$\left[\frac{d\varphi}{dt}\right]_{00} = \frac{m'\,n}{1+m}\left[\frac{1}{2}A_1^{(s)}\cos\varphi - \frac{3}{2}e\,B_0^{(c)} + B_1^{(c)} - \frac{e}{4}B_2^{(c)}\right]$$

$$e\left[\frac{d\chi}{dt}\right]_{00} = \frac{m'\,n}{1+m}\left[e\,A_0^{(c)}\cos\varphi - \frac{1}{2}A_1^{(c)}\cos\varphi + (1-\frac{1}{2}e^2)\,B_1^{(s)} - \frac{e}{4}B_2^{(s)}\right]$$

$$\left[\frac{di}{dt}\right]_{00} = \frac{m'\,n}{1+m}\left[\left(\frac{1}{2}C_1^{(c)} - e\,C_0^{(c)}\right)\sec\varphi\cos\omega - \frac{1}{2}C_1^{(s)}\sin\omega\right]$$

$$\sin i\left[\frac{d\Omega}{dt}\right]_{00} = \frac{m'\,n}{1+m}\left[\left(\frac{1}{2}C_1^{(c)} - e\,C_0^{(c)}\right)\sec\varphi\sin\omega + \frac{1}{2}C_1^{(s)}\cos\omega\right]$$

$$\frac{m'\,n}{1+m}M_E\left[-2\frac{r}{a}R_0\right] = \frac{m'\,n}{1+m}\left[-(2+e^2)\,A_0^{(c)} + 2\,e\,A_1^{(c)} - \frac{e^2}{2}A_2^{(c)}\right]$$

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cause. But the greater the irradiation the earlier the supposed tangency of limbs will occur at ingress. Hence, with increased irradiation the one phase will occur earlier and the other later than the moment of actual contact.

The general conclusions to which we are led are these: Between the time when, in a field of view with considerable irradiation, the body of the planet would appear as if entirely within the sun, were the limbs of both bodies continued in their circular outlines, and the time when sunlight is seen to completely encircle the planet, there occur a series of phases extending over a period of time which, under unfavorable circumstances, may approximate to an entire minute, and between which no distinct line can really be drawn in practice. With good optical power and fine definition this period of indetermination, if we may so call it, may be so reduced to twenty or even to ten seconds. An observer, taking a general view of the phenomena, and estimating a single moment of true contact from the general appearance, and not from special phenomena, may be expected on the average to note a time somewhere about the mean of the period of indetermination, and therefore a time corresponding closely to true contact. If, however, he clearly sees the formation of a thread of light before the time he assigns, his observation may be assumed to be too late and to need a correction of a few seconds. But when the phase is not described, a mean contact may be assumed to have been observed. At the same time an examination of the observations shows it unsafe to place special interpretations upon the descriptions of observers except those in which they describe the thread of light as having become very distinct.

The conclusion to be drawn from these considerations is that it is for the most part useless to attempt any investigation of phases, and that our proper course is to take an indiscriminate mean of the results of all observers irrespective of their description, except in those cases where there was something clearly different from true contact observed. It may be objected that we thus confound observations of different phases. But the same objection will lie against any observations in which observers do not agree as to the time. Whenever observers assign different times, it is certain that the phases corresponding to these times must be different.

The answer to the objection is that we must let these differences go in with the probable error of the final result. The calculation of probabilities will still apply to the final probable error of the comparison, unless we suppose different observers to have observed systematically on a different plan in different transits. If, for example, we had twenty observations of one transit and twenty of another, and if an observer of each transit was equally likely to observe some phase, *A*, or some other phase, *B*, then the chance would be that ten observers observed each phase on each occasion. The probable error arising from the possible unequal division between the phases would be determined on the same principles as the probable error of observations and would be confounded with it. Whatever errors such a course may lead to they will be inherent in the very nature of the subject, and it will be useless to attempt their elimination. Assuming that we take an indiscriminate mean without regard to the remarks of the observers, the standard phase at which we aim will not necessarily be that of true contact but that which would correspond to a mean observation by an infinite number of observers of the same kind as those who actually made the obser-

vations. The adoption of this mean is as legitimate as that of any other, provided the probable error is extended so as to include both that of observation and that of interpretation.

We now reach a difficult question in the treatment Although the course just marked out is inevitable in those cases when we do not know what distinctive phase an observer noted, should we still adopt the indiscriminate mean in the case of observers who specifically state that they observed a particular phase, for instance, that of completion of the thread of light? It might appear that since, in such cases, we know that they observed a phase different from the one aimed at, we should apply a correction to reduce it to the phase. On the other hand it may be maintained that the same phase is also noted by an observer who does not describe the observation, and that, therefore, no account should be taken of differences of descriptions.

On first commencing the work the former system was adopted as being undoubtedly founded on correct principles. Assuming that the observers who describe the completion of the thread of light differed systematically from those who did not, a correction ought undoubtedly to be applied to them, although its omission might not cause a systematic error. But it was soon found doubtful whether, in a great majority of cases, there was any real systematic difference. For example, in some of the older transits several observers describe the planet as being wholly within the sun, thus implying that contact had passed and that sunlight was seen all around the planet, but they sometimes add, to complete the description, that the limbs were in contact, which would negative the hypothesis that contact had passed.

The general impression produced by the discussion was that it was unsafe to apply any special interpretation to the language of observers, and that the different descriptions of phases were merely different ways of describing what was in reality the same thing. An exception should be made in those cases where the description is such as to render it certain that contact had passed, as, for instance, when the line of light seen around the planet was described as a band of sensible breadth.

The course finally adopted does not admit of being reduced to an absolute rule. An observation is thrown aside or corrected when it appears probable that it did not or could not correspond to the general mean. It will be seen by the discussion of the several observations, that in only a few exceptional cases is any correction applied.

One point was, however, strictly adhered to. The acceptance or rejection of the observation was not allowed to depend in any way upon the magnitude of the mean correction which would result to the tabular times. Of course, after the time was concluded upon, it might sometimes be found to need modification or rejection in consequence of discordance. The only cases of such rejection, however, are those of the doubtful times of ingress in the transits of 1740 and 1786. In both these cases it was impossible to obtain a certain result, and the observations are, therefore, rejected. Individual observations were generally rejected when they deviated from the general mean of the other observations by more than half a minute.

Another reason for adhering to the indiscriminate mean is the impossibility in practice of recognizing any observations as those of apparent contact in the sense in which that term has been defined. According to this definition a black drop is sup-

posed to be seen, and the time of apparent contact is that when, if the limbs of the sun and planet were produced through the black drop, they would touch each other. But, notwithstanding the geometric definiteness of the phase, it is, in practice, one which, under the circumstances, it is impossible to observe with any approximation to precision. As a matter of fact it may be doubted whether attempts have been made to note it, unless from some preoccupation on the part of the observer. When, therefore, the observer describes a tangency of the limbs, it cannot be inferred that he means anything different from the average contact.

As a matter of fact the only instance in which the question of this contact came in was connected with the transit of 1878, where there are several cases of geometric contact with black drop intervening.

§ 4.

Longitudes of the Stations.

When, in so great a mass of observations, made in every part of the world, absolute precision is aimed at, the determination of the longitudes of the observers would be the most troublesome and laborious part of the work. It may be doubted whether, with the most elaborate historical research, the position of the observer could in all cases be learned. Happily, the last degree of precision is not necessary, because the necessary probable error of the observations themselves being from 5^s to 10^s, their precision will not be materially diminished by a small error of longitude. Moreover, as a general rule, those observers whose longitude is unknown are otherwise entitled to the least weight. No attempt was, therefore, made at an exhaustive investigation of the observing stations. In the case of towns in France, and sometimes, also, in Germany, the table of positions in the *Comptes Rendus* was accepted without further discussion. Where there was any room for distinguishing between different points in the city, the cathedral or other central point was generally taken as most likely to be near the place of observation. The places of a few German towns were taken by measurement from the maps in the Hand-atlas of SOHR-BERGHAUS, published by CARL FLEMMING, Glogau, 1879. The British Admiralty Charts frequently furnish data for the longitude of ports, but for most maritime stations the longitudes were obtained from data in the Hydrographic Office of the Bureau of Navigation. The observations and researches of Lieutenant Commander FRANCIS M. GREEN, U. S. N., were of great value in this respect

A table of the adopted longitudes, with the authority from which each was obtained, is appended. It will aid in the application of any corrections which may be required by future investigators.

Adopted Positions of Stations.

Place.	Geog. latitude.			Longitudes. (Gr. = long. from Greenwich; Pa. = long. from Paris.)				Authorities and remarks.
	°	′	″	h.	m.	s.		
Adelaide - - - - -	−34	57	00	−9	14	21	Gr.	Am. Eph., adopted.
Altona - - - - - -	+53	32	45	−0	30	26	Pa.	Conn. des Temps.
				−0	39	47	Gr.	Adopted.
Amsterdam - - - -	+52	22	30	−0	10	12	Pa.	Conn. des Temps.
				−0	19	33	Gr.	Adopted.
Athens - - - - - -	+37	58	20.0	−1	34	55.7	Gr.	Am. Eph., adopted.
Avignon - - - - -	+43	57	13	−0	9	53	Pa.	Conn. des Temps.
				−0	19	14	Gr.	Adopted.
Bagdad - - - - - -	+33	19	50	−2	48	9	Pa.	Conn. des Temps.
				−2	57	30	Gr.	Adopted.
Batavia - - - - -	− 6	10.2		−7	7	12.5	Gr.	A. N., lxxiv, 167, adopted.
Berlin - - - - - -	+52	30	16.7	−0	53	34.91	Gr.	Am. Eph., adopted.
Bologna (St. Petrone) -	+44	29	32	−0	36	2	Pa.	Conn. des Temps.
				−0	45	23	Gr.	Adopted.
Breslau - - - - - -	+51	6	56.5	−1	8	8.71	Gr.	Am. Eph., adopted.
Brunswick - - - -	+52	16	6	−0	32	45	Pa.	Conn. des Temps.
				−0	42	6	Gr.	Adopted.
Brussels - - - - -	+50	51	10.5	−0	17	28.6	Gr.	Am. Eph., adopted.
Calcutta - - - - -	+22	33	11	−5	44	4	Pa.	Le Ver., v, 46.
				−5	53	21	Gr.	Lt. Com. F. M. Green, adopted.
Cambridge, Mass. - -	+42	22	48.3	+4	44	31	Gr.	Am. Eph., adopted.
Cambridge, Eng. - -	+52	12	51.6	−0	0	22.75	Gr.	Am. Eph., adopted.
Canton - - - - - -	+23	7.5		−7	33	8	Gr.	Admiralty Map, adopted.
Cape Town - - - -	−33	56	3.4	−1	13	55	Gr.	Am. Eph., adopted.
Celle - - - - - -	+52	37	30	−0	30	52	Pa.	From map.
				−0	40	13	Gr.	Adopted.
Cincinnati (old obs.) -	+39	6	26.5	+5	37	58.94	Gr.	Am. Eph., adopted.
Cookstown - - - -	+54	38		+0	26	52	Gr.	Map.
Copenhagen - - - -	+55	40	53	−0	40	58	Pa.	Conn. des Temps.
				−0	50	19	Gr.	Adopted.
Copenhagen (new obs.)	+55	41	13.6	−0	50	19.2	Gr.	Am. Eph., adopted.
Dantzic - - - - -	+54	21	18	−1	14	39.3	Gr.	Am. Eph., adopted.
Dorpat - - - - - -	+58	22	47.4	−1	46	53.5	Gr.	Am. Eph., adopted.
Dresden - - - - -	+51	3	39	−0	45	35	Pa.	Conn. des Temps.
				−0	54	56	Gr.	Adopted.
Durham - - - - -	+54	46	6.2	+0	6	19.8	Gr.	Am. Eph., adopted.
Edge Hill - - - - -	+53	24.4		+0	0	0.8	Gr.	
Florence - - - - -	+43	46	22	−0	35	41	Pa.	Conn. des Temps.
				−0	45	2	Gr.	Adopted.
Geneva - - - - - -	+46	11	58.8	−0	24	36.77	Gr.	Am. Eph., adopted.

Adopted Positions of Stations—Continued.

Place.	Geog. latitude.			Longitudes. (Gr. = long. from Greenwich; Pa. = long. from Paris.)				Authorities and remarks.
	°	′	″	h.	m.	s.		
Gotha (Seeberg)	+50	56	5	−0 −0	33 42	36 57	Pa. Gr.	Conn. des Temps. Adopted.
Gottingen	+51	31	47.9	−0	39	46.24	Gr.	Am. Eph., adopted.
Grantham	+52	54.9		+0	0	2.6	Gr.	
Greenwich	+51	28	38.4	0	0	0	Gr.	Am. Eph., adopted.
Haarlem	+52	22	54	−0 −0	9 18	12 33	Pa. Gr.	Conn. des Temps. Adopted.
Hague	+52	4	40	−0 −0	7 17	53 14	Pa. Gr.	Conn. des Temps. Adopted.
Hamburg	+53	33	7	−0	39	53.7	Gr.	Am. Eph., adopted.
Hartwell	+51	48	36	+0	3	24 33	Gr.	Br. Eph., adopted.
Hobart Town	−42	53	12	−9 −9	40 49	1 22	Pa. Gr.	Conn. des Temps. Adopted.
Königsberg	+54	42	50.6	−1	21	58.91	Gr.	Am. Eph., adopted.
Kremsmünster	+48	3	23.7	−0	56	32.2	Gr.	Am. Eph., adopted.
Kurnaul	+29	42.3		−4 −5	58 8	55 7	Pa. Gr	Le Ver. Lt. Com. F. M. Green, adopted.
Leiden	+52	9	20	−0	17	56.35	Gr.	Am. Eph., adopted.
Leipsic	+51	20	10	−0 −0	40 49	13 34	Pa. Gr.	Conn. des Temps. Adopted.
Lilienthal	+53	8	28	−0 −0	26 35	18 39	Pa. Gr.	Conn. des Temps. Adopted.
London (Fleet st.)	+51	30	49	+0 +0	9 0	44 23	Pa. Gr.	Conn. des Temps. Adopted.
Liverpool	+53	24	4	+0	12	17.2	Gr.	Am. Eph., adopted.
Louvain	+50	53	27	−0 −0	9 18	27 48	Pa. Gr.	Conn. des Temps. Adopted.
Malta	+35	54.8		−0	58	2	Gr.	Adopted.
Manchester	+53	29	0	−0 −0	18 27	20 41	Pa. Gr.	Conn. des Temps. Adopted.
Manheim	+49	29	13	−0 −0	24 33	31 52	Pa. Gr.	Conn. des Temps. Adopted.
Manila	+14	35	26	−8	3	49	Gr.	Lt. Com. F. M. Green, adopted.
Marseilles	+43	18	19.1	−0	21	34.64	Gr.	Am. Eph., adopted.
Marburg	+50	48	46.9	−0	35	5.0	Gr.	Am. Eph., adopted.
Milan	+45	27	35	−0 −0	27 36	24 45	Pa. Gr.	Conn. des Temps. Adopted.
Mitau	+56	39	2	−1 −1	25 34	35 56	Pa. Gr.	Conn. des Temps. Adopted.
Modena	+44	38	52.8	−0	43	42.8	Gr.	Am. Eph., adopted.
Montauban	+44	1	6	+0 −0	3 5	56 25	Pa. Gr.	Conn. des Temps. Adopted.

Adopted Positions of Stations—Continued.

Place.	Geog. latitude.			Longitudes. (Gr. = long. from Greenwich; Pa. = long. from Paris.)				Authorities and remarks.
	°	′	″	h.	m.	s.		
Montpellier	- - - - - -			−0 −0	6 15	10 31	Pa. Gr.	Conn. des Temps. Adopted.
Montevideo	−34	54	18	+3	44	49	Gr.	Lt. Com. F. M. Green, adopted.
Naples	+40	51	47	−0 −0	47 57	41 2	Pa. Gr.	Conn. des Temps. Adopted.
New Haven	+41	18	36.5	+4	51	42.19	Gr.	Am. Eph., adopted.
Nicolajeff	+46	58	20.6	−2	7	54.1	Gr.	Am. Eph., adopted.
Nienstädten	+53	33.1		−0	39	22.8	Gr.	24ˢ.2 W. of Altona (A. N., xxiii, 145).
Norristown	+40	9.7		+5	1	30.45	Gr.	From long. of Phila. and triangulations.
Nuremberg	+49	27	30	−0 −0	34 44	57.7 18.7	Pa. Gr.	 Adopted.
Padua	+45	23	45	−0 −0	38 47	11 32	Pa. Gr.	Conn. des Temps. Adopted.
Palermo	+38	6	44	−0	53	25.0	Gr.	Am. Eph., adopted.
Paramatta	−33	48	50	−9 −10	54 4	43 4	Pa. Gr.	Le Ver. (C. des T. gives 46ˢ). Adopted.
Paris	+48	50	11	0 −0	0 9	0 21	Pa. Gr.	Conn. des Temps. Adopted.
Pekin	+39	54	13	−7 −7	36 45	34 55	Pa. Gr.	Conn. des Temps. Adopted.
Pekin (*Russian Obs.*)	- - - - - -			−7 −7	36 45	20 41	Pa. Gr.	
Philadelphia	+39	57	7.5	+5	0	38.45	Gr.	Am. Eph., adopted.
Prague	+50	5	18.8	−0	57	41.4	Gr.	Am. Eph., adopted.
Princeton	+40	20	58	+4	58	37.5	Gr.	Am. Eph., adopted.
Pulkowa	+59	46	18.7	−2	1	18.67	Gr.	Am. Eph., adopted.
Quedlinburg	+51	47	32	−0 −0	35 44	29 50	Pa. Gr.	Conn. des Temps. Adopted.
Reval	+59	26	28	−1	39	6.3	Gr.	O. Struve, adopted.
Rouen (cathedral)	+49	26	29	+0 −0	4 4	58 23	Pa. Gr.	Conn. des Temps. Adopted.
Rouen (3ˢ W. of cathedral).	+49	26	29	+0 −0	5 4	1 20	Pa. Gr.	See next preceding. Adopted.
Schwerin	+53	37	38.2	−0	45	40.7	Gr.	Am. Eph., adopted.
Seftenberg	+50	5	10.1	−1	5	50.6	Gr.	Am. Eph., adopted.
Rome	+41	54	6	−0 −0	40 49	28 49	Pa. Gr.	Conn. des Temps. Adopted.
Sidney	−33	51	41	−9 −10	55 4	34 55	Pa. Gr.	Le Ver. Adopted.
St. Helena	−15	55		+0 +0	32 22	12 52	Pa. Gr.	Conn. des Temps. Adopted.
St. Petersburg	+59	56	30	−2	1	13.5	Gr.	Am. Eph., adopted.

Adopted Positions of Stations—Continued.

Place.	Geog. latitude.			Longitudes. (Gr. = long. from Greenwich; Pa. = long. from Paris.					Authorities and remarks.
	°	′	″		h.	m.	s.		
Toulouse	+43	36	33.	+	0	3	35	Pa.	Conn. des Temps.
				−	0	5	46	Gr.	Adopted.
Twickenham	+51	27	4.2	+	0	1	13.1	Gr.	Am. Eph., adopted.
Upsal	+59	51	50	−	1	1	13.3	Pa.	A. N., xi, 409.
				−	1	10	34.3	Gr.	Adopted.
Utrecht	+52	5	10.5	−	0	20	31.7	Gr.	Am. Eph., adopted.
Vienna (1′ 1″.7 S. 1s.1 E. of obs.)	+48	11	33.8	−	1	5	32.84	Gr.	Am. Eph., adopted.
Vienna (old obs.)	+48	12	35.5	−	1	5	31.74	Gr.	Am. Eph., adopted.
Viviers	+44	29	14	−	0	9	23	Pa.	Conn. des Temps.
				−	0	18	44	Gr.	Adopted.
Wanstead	+51	34	10	−	0	0	9	Gr.	
Washington College, Pa.	+40	10		+	5	21	18	Gr.	From map, adopted.
Waterloo	+53	28.4		+	0	12	4	Gr.	
University of William and Mary, Va.	+37	16		+	5	5	28	Gr.	From map, adopted.

§ 5.

External Contacts.

Experience seems to indicate that an observation of external contact, under certain conditions, can be made with nearly as much precision as one of internal contact. The observed phase will not, however, be that of tangency of limbs, but that at which the notch made by the planet passes from the stage of visibility to invisibility, or *vice versa.* This stage depends upon the state of the atmosphere, the eye of the observer, and the quality of the instrument. Yet it would seem that if we regard the effect of the differences which result from these causes as probable errors simply the total probable error will not be materially increased The fact appears to be that, in day observations upon the sun, the maximum of seeing power is nearly reached with quite a moderate-sized telescope in ordinary states of the atmosphere. No account has, therefore, been taken of differences in telescopic power, etc., except that a few observations, made with evidently insufficient means, have been rejected.

There is, however, one important point to be noted. There can be no doubt that, owing to the progressive improvements in telescopes, and in the art of observing, the phase which would be noted as external contact has continually approached nearer to that of true external tangency. It is, therefore, necessary, in the discussion, to allow for this progressive change. The mode of doing this is described in connection with the formation and solution of the equations of condition.

A very little examination shows that no reliance can be placed upon the older observations of first external contact, and very little upon the recent ones, except where the observers had first practiced upon an artificial transit. As no observations of a distinct class are of value unless they extend through a long period of time, all observations of first external contact have been rejected. Owing to the inferior weight assigned to observations of fourth contact, no attempt has been made to discuss them with the care devoted to those of internal contact. As a rule, the reductions to geometric phase have been supposed the same as in the case of internal contact. Except when the planet passes very near the limb of the sun the error of this hypothesis is insensible.

§ 6.

Explanation of the Tabular Summary of Observations.

The general construction of this summary has been so fully explained in the preceding sections that few additional explanations are necessary. Perfect symmetry of arrangement has not been aimed at, and might tend, in some respects, to mislead because of the impossibility of its according with a series of observations of so miscellaneous a character extending over two centuries.

The first two columns, giving the station and the names of observers, call for no special remark. The third column contains the description of the phase, when any is given. As a rule, the exact language of the observer is quoted, though it sometimes has to be condensed. Sometimes, also, when thus condensed, or when there can be

no doubt as to the exact meaning, his statements are expressed in English. In the majority of cases it will be seen that there is no specific description.

The fourth column contains the time as given by the observer. It is generally apparent time before 1800, and mean time since. No attempt has been made in any case to redetermine the clock correction of the observer, except in a few observations by WURZLEBAU, which, however, proved worthless.

The next column contains the reduction to the center of the earth. It is computed from the tabular data as given in the second part. The omitted terms of the second order would rarely amount to one second, and, therefore, need not be taken into account.

The transit of 1782, however, in which Mercury passed very near the sun's limb, is an exception. It was here necessary to make a rigorous reduction to the center of the earth. The details of this reduction are given in the proper place.

Next follows the concluded Greenwich mean time of geocentric contact as deduced from each observation It is obtained by correcting the observed time for equation of time, reduction to center of earth, and longitude. The three adopted corrections being all given, any error in the reduction can be readily found.

The mean time concluded from all the observations of each transit is given as the result of a separate discussion of each. It was necessary in the case of each transit to discuss the observations upon the system likely to give the nearest approximation to a general mean result.

§ 7.

Classification of Residuals with respect to Magnitude.

The classification of the errors of observed times, with respect to their magnitude, is shown in the following exhibit. The transit of 1782 is omitted. In other cases the numbers given are the differences between each individual observed time and the geocentric time concluded from the general mean of all the observations.

The algebraic signs are so applied that a positive error means that Mercury was too far from the sun's center. Hence, the differences are:

(Computed—Observed) times of contact II.

(Observed—Computed) times of contact III.

By this arrangement similar signs correspond to similar differences of phases of contact.

No allowance is made for obliquity of the path of the planet to the sun's limb.

Residuals.	Ingress.	Egress.	Total.	Residuals.	Ingress.	Egress.	Total.
s.				s.			
0	14	14	28	+1	14	16	30
−1	15	16	31	+2	15	17	32
−2	10	16	26	+3	15	15	30
−3	17	20	37	+4	7	10	17
−4	9	9	18	+5	9	6	15
−5	15	17	32	+6	9	5	14
−6	10	10	20	+7	4	9	13
−7	12	13	25	+8	8	4	12
−8	9	5	14	+9	4	7	11
−9	16	6	22	+10	6	8	14
−10	8	6	14	+11	5	4	9
−11	8	7	15	+12	4	2	6
−12	4	8	12	+13	3	3	6
−13	4	8	12	+14	5	2	7
−14	6	5	11	+15	6	6	12
−15	6	7	13	+16	1	2	3
−16	3	2	5	+17	3	2	5
−17	1	2	3	+18	2	3	5
−18	3	0	3	+19	3	3	6
−19	1	2	3	+20	1	4	5
−20	2	0	2	+21	0	4	4
−21	5	2	7	+22	2	1	3
−22	0	0	0	+23	2	1	3
−23	0	2	2	+24	0	1	1
−24	0	2	2	+25	2	1	3
−25	3	1	4	+26	2	2	4
−26	0	1	1	+27	1	0	1
−27	0	2	2	+28	3	0	3
−28	0	0	0	+29	0	0	0
−29	0	2	2	+30	3	0	3
−30	0	0	0	>30	9	14	23
>30	9	9	18				

The numbers thus presented show no tendency to divide into groups corresponding to special phases. In order to determine more definitely whether there is any such tendency we divide them into groups of five, the middle group being the sum of $-2, -1, 0, +1, +2$; the group $+5$ the sum from $+3$ to $+7$, etc. We thus have—

	Magnitude.	No. of errors.	Probable number.
Exceeding	-27 sec.	20	2
	-25 sec.	11	6
	-20 sec.	15	18
	-15 sec.	44	44
	-10 sec.	77	83
	-5 sec.	132	120
	0 sec.	147	137
	$+5$ sec.	89	120
	$+10$ sec.	52	83
	$+15$ sec.	33	44
	$+20$ sec.	23	18
	$+25$ sec.	12	6
Exceeding	$+27$ sec.	29	2

The great difference between the residuals which fall between -3^s and -7^s and those which fall between $+3^s$ and $+7^s$ is striking, but seems to arise partly from the unequal grouping; partly from the excess of positive errors of about $+20^s$; partly from the excess of instances in which observation of small weight were those of moderate negative residual. It will be remarked that in preparing the table no distinction whatever was made between different classes of results as regards quality; but every residual was enumerated. Hence, any irregularity in the distribution of weights will be shown by an irregularity in the numbers.

To compare these numbers with the probable ones, deduced from the standard law of distribution of errors, we remark that somewhat more than half the residuals are contained in the three middle groups, which again should be considered as comprising all errors between the limits $-7^s.5$ and $+7^s.5$. If we include the abnormal residuals which exceed 27 seconds, the limits between which one half the residuals are contained should be regarded as $\pm 6^s.8$. But considering only those 635 residuals which do not exceed 27 seconds, one half of them are contained between the limits $\pm 6^s.2$. Actually we have assumed $\pm 6\frac{2}{3}$ seconds as the probable error, and thus obtained the probable distribution of the residuals, as shown in the last column.

This method of deducing the probable error does not rest upon a mean of squares of errors, but is based immediately on the definition that the probable error is that quantity for which there is an equal chance that an error shall exceed it or fall short of it. The necessity for adopting such a definition arises when the law of error varies appreciably from that usually adopted, as in the present case.

It is evident, from an examination of the table, that the observations are liable to abnormal errors, since 49 of the residuals exceed $27''$, while adopting the usual law of

error the probable number should be only 4. Quite likely the abnormal error, in many of these cases, arises from such mistakes as those of a minute in the recorded time. But I think that, for the most part, they arise from the unfavorable circumstances under which observations are frequently made. It is evident that if we have a collection of observations of different degrees of probable error, in which, however, there is no way of distinguishing those of great probable error from those of small probable error, the law of the errors will not be that usually adopted, but there will be a comparative excess of large residuals. It is also evident that in such a case the arithmetical mean does not necessarily give the most probable result. For, in the case of an observation of large residual, there is evidently a preponderance of probability that it belongs to a class with large probable error, and therefore should be assigned least weight. This principal has, to a certain extent, been indirectly applied in deducing the times of geocentric contact from observation. When a result differed from the general mean by a quantity much more than half a minute it was rejected. The above table seems, however, to indicate that all residuals exceeding $25''$ should be rejected, except in cases where the other observations on the same transit were worse than the average.

That any general collection of observations of transits of Mercury must be a mixture of observations with different probable errors was made evident to the writer by his observations of the transit of May 6, 1878, which may be here described as an illustration of the subject.

The instrument used was a 4-inch telescope not moved by clock-work, and, therefore, somewhat difficult to manage. At ingress the definition of the sun's limb was fairly good, though there was a slight distortion in the figure of the planet about the time of internal contact. About the time of contact there was an interval, which I should roughly estimate as probably not less than 6^s nor more than 10^s, during which it was doubtful whether contact was or was not past. The middle of this interval was therefore taken as the time of contact, with an error which almost certainly could not exceed 5^s. About 12^s later the band of light between the limb of the sun and planet was clearly of sensible breadth. It could therefore be asserted with entire confidence that the contact took place several seconds before the last recorded moment. Yet a large number of observations gave a later time than this, when all were reduced to the center of the earth. It may, however, be remarked that the time of true contact as noted agreed closely with the general mean.

At egress, however, the circumstances were entirely different. The sun shone through a thin cirrus cloud which, although it did not seem to disturb the limb materially, yet produced such a blurring as at times almost to obliterate the view of the planet. Under these unfavorable circumstances a time of probable contact was noted. But half a minute later, during a few seconds of somewhat better definition, it appeared doubtful if contact had really taken place. Bad definition immediately followed, and the attempt to note contact had to be given up. Some seconds later the sun shone with nearly its full brilliancy; the planet then appeared to form a notch in the sun's limb the shape of the letter U, the length of which was fully double its breadth, thus showing distortion in an extreme degree. Had this state of things occurred a

minute earlier a well formed black drop would no doubt have been seen. Before the planet went off the sun its limb again became so disturbed that external contact could not be noted.

Hence it was impossible to have recorded a time of internal contact at egress which would not have been liable to an error of half a minute or more. Even during the time of best definition between third and fourth contacts an observed time would have been liable to an error of from ten to twenty seconds. And I am persuaded that at this time the sun was not more disturbed than it very often is in observed transits. The conclusion I therefore draw is that if it were possible to select from a collection those observations made under the most favorable circumstances, and by observers fully prepared for the phenomena, we might either reject, or assign but small weight to, all the other results. Unfortunately, it is, in most cases, impossible to select such observations. Still, it is not likely, in the method of treatment actually adopted, that the systematic error is considerable.

The main conclusion to be deduced from the tables is, however, that there are no distinctive features of contact which are actually noted by observers. The phases merge into each other by insensible gradations, and the mean phase is got more frequently than any phase differing from it by so much as 5″. Especially no tendency can be seen to observe anything which we may consider true contact, apparent contact, breaking of the ligament, etc., at any definite time. It may be advisable in some cases to correct an observation which the observer particularly describes as one of a special phase. But even this must be done with caution.

Tabular Summary of Observations with their Reductions.

ABBREVIATIONS.

P. T., Philosophical Transactions of the Royal Society.
Paris, Memoirs of the French Academy of Sciences.
M. A. A., Memoirs of the American Academy of Arts and Sciences (First Series).
P. A. A., Proceedings of the American Academy of Arts and Sciences.
M. A. P. S., Memoirs of the American Philosophical Society.
M. R. A. S., Memoirs of the Royal Astronomical Society.
P. R. A. S., Proceedings of the Royal Astronomical Society.
Le V., Le Verrier's Annales de l'Obs. de Paris, v.
St. P., N. C., St. Petersburg, Novi Commentarii.
B. M., Memoirs of the Academy of Berlin.
A. G. E., Zach's Allgemeine Geographische Ephemeriden.
Zach, Zach's Monatliche Correspondenz.
II. Internal contact at Ingress.
III. Internal contact at Egress.
IV. External contact at Egress.

1677, NOVEMBER 7.0.

[Equation of time: Ingress — $15^m 56^s$; Egress — $15^m 54^s$.]

Place.	Observer.	Contact and description.	Local app. time.			Red. to geocentric phase.	Corr. for phase.	Greenwich mean time of geocentric contact.			Authority and remarks.
			h.	*m.*	*s.*	*s.*	*s.*	*h.*	*m.*	*s.*	
St. Helena	Halley	I. Limbus Solis a Merc. temeratus	21	26	17	+29	....	21	33	42	Catalogus Stellarum Australium, App., p. (2).
	... do	II. Totus Merc. intra Solem	21	27	30	+29.2	....	21	34	55	
	... do	III. Limbus Merc. attigit Solis Limbum.	2	40	8	+22.0	....	2	47	28	
		IV. Solis limbus integer factus ...	2	41	54	+22	+20	2	49	34	
Avignon	Gallet	IV. Egressum e Sole.............	3	26	56	− 4	+30	2	52	14	Flamsteed, Hist. Cœl., i, 187.
In comitatu Lancastriæ.	Townley	IV. Total Egress	2	54	0	− 8	....	...	...	...	Do.

GALLET's observation may be rejected without discussion, his time being certainly more than a minute after last external contact.

TOWNLEY's time and longitude are so uncertain that his observation has not been used.

Considering the errors of the tabular times as not exceeding half a minute, it would seem that both of HALLEY's phases at ingress are a minute or more late, and both those at egress perhaps a minute too early. The most plausible explanation seems to be that, with his bad telescope and inexpertness in observing, he did not see Mercury at first contact until long after it had began to indent the solar disk, and that Mercury did not appear "totus intra Solem" until the thread of light had reached a considerable thickness. At egress the same causes would produce the reverse effects, though probably in a less degree.

These observations can, therefore, be combined with those of other observers only when this probable source of error is eliminated.

1690, NOVEMBER 10.

[Equation of time, — 15^m 40^s.]

Place.	Observer.	Contact and description.	Local app. time.	Red. to geocentric phase.	Greenwich mean time of geocentric contact.	Authority and remarks.
			h. m. s.	s.	h. m. s.	
Canton	Fontenay	Sortie certaine et entière	3 18 3	−22.5	19 28 52	Paris, vii, pt. ii, p. 870.
Nuremberg	Wurzelbau	Merc. postquam undulanti limbo Solis ad Min. temporis adhæserat exiit.	8 24 45	−61.2	....	P. T., xvii, p. 485.

According to WURZELBAU, his observation was made at 8^h 36^m by his clock, and the mean of four altitudes following it gives the correction -11^m 15^s. The result is, however, several minutes in error, and the observation is not worth discussing.

Nothing is said of the instrument with which FONTENAY'S observation was made, but his remark, "Il a paru toujours dans le Soleil comme une tache noire et fort ronde," implies a good instrument. The planet appeared half emerged at 3^h 17^m 5^s app. time.

1697, NOVEMBER 2.8.

[Equation of time, — 16^m 6^s.]

Place.	Observer.	Contact and description.	Local app. time.	Red. to geocentric phase.	Greenwich mean time of geocentric contact.	Authority and remarks.
			h. m. s.	s.	h. m. s.	
Paris	Cassini	III. Margo præcedens Mercurii pervenit ad Solis marginem præcedentem.	20 8 38	−18.1	19 42 53	Le V., p. 38.
		IV. Merc. totus emersit	20 10 24	....	19 44 39	

The second observation is quoted by FLAMSTEED, Hist. Cœl., but I have found no published record of the internal contact. The time is, therefore, adopted on the authority of LE VERRIER.

The observations were made with an 18-foot telescope, but no power is stated.

1723, NOVEMBER 9.1.

[Equation of time, — 15^m 52^s.]

Place.	Observer.	Contact and description.	Local app. time.	Red. to geocentric phase.	Greenwich mean time of geocentric contact.	Authority and remarks.
			h. m. s.	s.	h. m. s.	
Bologna	Manfredi	Merc. entirely within; limbs tangent.	3 27 45	+23	2 26 53	P. T., xxxiii, p. 228.
Paris	Maraldi	II	2 51 48	+18	2 26 53	Paris, 1723, p. 295.
	Delisle	II. Entrée totale	2 51 39	....	2 26 44	Paris, 1723, p. 309.

1723, NOVEMBER 9.1—Continued.

Place.	Observer.	Contact and description.	Local app. time.			Red. to geocentric phase.	Greenwich mean time of geocentric contact.			Authority and remarks.
			h.	m.	s.	s.	h.	m.	s.	
Paris	Cassini	Merc. était entièrement entré et son bord rasait celui du Soleil.	2	51	48	....	2	26	53	Paris, 1723, p. 261.
Wanstead	Bradley	II	2	42	38	+16	2	26	53	P. T., xxxiii, p. 228.
Greenwich	Halley	He was wholly entered, the light of the sun just beginning to appear behind his disk.	2	42	26	+16	2	26	50	Do.
London	Graham	Merc. entirely within the disk	2	42	19	+16	2	27	6	Do.

Here, four observers describe the thread of light as fully formed, or Mercury as entirely within the sun. The mean of their times is $2^h\ 26^m\ 56^s$. Three observers are silent as to the phenomenon; the mean of their times is $2^h\ 26^m\ 50^s$. Whether any correction should be applied to the first set is doubtful, for reasons given in § 6. Assuming HALLEY's to need such a correction, I shall adopt

Contact II, $2^h\ 26^m\ 52^s$.

1736, NOVEMBER 10.9.

[Equation of time: Ingress — $15^m\ 37^s$; Egress — $15^m\ 36^s$.]

Place.	Observer.	Contact and description.	Local app. time.			Red. to geocentric phase.	Greenwich mean time of geocentric contact.			Authority and remarks.
			h.	m.	s.	s.	h.	m.	s.	
Montpellier	Plantade	II. Imm. totale	21	41	27	+19	21	10	38	Mem. de Montpellier, ii.
Bologna	Manfredi	II	22	10	53	+21		10	14	
	Roversius		22	11	12	+21		10	33	
Paris	Maraldi	Entièrement entré	21	35	15	+18		10	35	
	Cassini		21	35	10	+18		10	30	
London	Graham	A few seconds between clouds; judged he was entirely within or perhaps a little more.	21	25	37	+19		10	42	P. T., xl, 102.
Montpellier	Plantade	III. Il touchait le bord	0	21	12	−75	23	48	50	
Bologna	Manfredi	III	0	51	7	−73		48	55	Ib., xl, 103.
	Algarottus		0	50	1	−73		47	49	
	Vandellius		0	50	50	−73		48	38	
Paris	Maraldi	Parait tomber	0	15	5	−77		48	51	
	Cassini	Rasait le bord	0	15	18	−77		49	4	
Montpellier	Plantade	IV	0	24	18	....	23	51	55	
Bologna	Manfredi	IV	0	53	44	−73		51	32	
	Roversius		0	54	1	−73		51	49	
	Algarottus		0	53	6	−73		50	54	
	Vandellius		0	54	6	−73		51	54	
Paris	Maraldi		0	18	11	−77		51	57	
	Cassini		0	18	18	−77		52	4	
Greenwich	Bevis		0	8	33	−79		51	38	

For second contact we may reject GRAHAM's doubtful observations, and take the mean of the remaining five. At third and fourth contact we may also reject the observations of ALGAROTTUS without question. We shall then have—

	h.	m.	s.
Contact II,	21	10	30
III,	23	48	51
IV,	23	51	50

1740, MAY 2.4.

[Equation of time, $-3^m\ 25^s$.]

Place.	Observer.	Contact and description.	Local app. time.	Red. to geocentric phase.	Greenwich mean time of geocentric contact.	Authority and remarks
			h. m. s.	s.	h. m. s.	
Cambridge, U. S...	Winthrop.......	¼ diameter entered................	4 54 59	....	9 37 28	P. T., xlii, p. 572.
		Almost within..................	5 0 40	+84.3	9 43 9	

This observation, rude and uncertain though it appears, is noteworthy, not only as being the first made on a May transit, but as belonging to that May transit in which the distance of centers was greatest. It may also be remarked that estimates of the kind here given are more accurate in comparison with contact observations than is commonly supposed. We may, therefore, see what conclusions may be drawn from them. In the first place, the tabular interval between contacts is $8^m\ 28^s$. Three-fourths of this being $6^m\ 21^s$, the reduced time of internal contact would be $9^h\ 43^m\ 49^s$. The general character of the intervals between observed contacts, as estimated by observers in those times, indicates that they often lost sight of the planet when one-fourth of it was still on the disk. The general conclusions from Professor WINTHROP's observations may, therefore, be summed up as follows:

From first estimate, time probably earlier than $9^h\ 43^m\ 49^s$.

From second estimate, time certainly later than $9^h\ 43^m\ 9^s$.

Therefore, if we assume $9^h\ 43^m\ 29^s$ as the reduced time of internal contact, the probable error of the result, mistakes aside, would seem not to exceed 20^s.

The ulterior discussion, however, shows a probable mistake in the time, since, at the last recorded moment, when WINTHROP thought internal contact had not quite arrived, the contact would seem to have been decidedly past.

1743, NOVEMBER 5.0.

[Equation of time, — 16^{m} 7^{s}.]

Place.	Observer.	Contact and description.	Local app. time.	Red. to geocentric phase.	Greenwich mean time of geocentric contact.	Authority and remarks.
			h. m. s.	s.	h. m. s.	
London, Surrey st.	Short	II. Just past internal contact. Thread of light $\frac{1}{20}$ to $\frac{1}{30}$ the diameter of Mercury.	20 30 59	− 51	20 14 27	
Paris	Lacaille	Il me parut totalement entré	20 40 38	− 50	14 20	Paris, 1743, p. 173.
	Maraldi	Je jugea qu'il était entièrement entré & que son bord rasait celui du Soleil.	20 40 46	− 50	14 28	Ib., p. 281.
	Cassini, jr.	II	20 40 34	− 50	14 16	Ib., p. 375.
Thury	Cassini, sr	Immersion totale; son bord touchait exactement celui du Soleil.	20 40 37	− 51	14 18	Ib., p. 373.
Paris	Lacaille	III. Mercure me parut toucher le bord du Soleil pour en sortir.	1 10 3	+ 17	0 44 52	
	Maraldi	Merc. me parut raser le bord du Soleil.	1 10 17	+ 17	45 6	
	Le Monnier	Merc. touche intérieurement la circonférence du disque du Soleil.	1 9 52½	+ 17	44 41	Ib., p. 360.
	Cassini, jr.	III	1 10 26	+ 16	45 14	
Thury	Cassini, sr	Son bord touchait celui du Soleil	1 10 32	+ 16	45 26	
London	Graham	III	1 0 42	+ 15	45 16	
	Bevis	III	1 0 33	+ 15	45 7	
Cambridge, U.S.	Winthrop	III	20 17 5	− 26	45 0	
Paris	Lacaille	IV	1 11 58	+ 17	0 46 47	
	Maraldi		1 12 18:	+ 17	47 7:	
	Le Monnier		1 12 2	+ 17	46 52	
	Cassini, jr.		1 12 24	+ 17	47 13	
Thury	Cassini, sr		1 12 2	+ 17	46 51	
London	Graham		1 2 16	+ 15	46 50	
	Bevis		1 2 16	+ 15	46 50	
Cambridge	Winthrop		20 18 58	− 26	46 53	

The danger of assigning special interpretations to the language of the observers may be seen in the case of MARALDI, who describes Mercury as wholly entered, while, at the same time, its limb touched that of the sun, and this a second after SHORT, at London, saw the thread of light fully formed. It would seem that in all cases the observers at ingress noted a time about that of true contact. The time required for the formation of such a thread of light as that described by SHORT would be 4^{s} to 6^{s}. If we subtract 5^{s} from his time we shall have 20^{h} 14^{m} 22^{s}. Then, the mean of all the five observations will be

$$20^{h}\ 14^{m}\ 20^{s}.8$$

In the case of third contact there is nothing to indicate any separation of phases. The general mean of all the observations is

$$0^{h}\ 45^{m}\ 5^{s}.2$$

Rejecting MARALDI's doubtful observation, we have seven observations of last contact, the mean of which is

$$0^{h}\ 46^{m}\ 53^{s}.7$$

The actual computed interval between the last two contacts is 121^s. If we suppose the observation of CASSINI the son correct, he saw the planet 20^s after the mean of the other observers, and the true internal contact could not have arrived until after 0^h 45^m 12^s. But, it seems hardly likely that the other observers should have lost sight of it so soon were his observation correct.

1753, MAY 5.9.

[Equation of time, -3^m 43^s.]

Place.	Observer.	Contact and description.	Local app. time. h.	m.	s.	Red. to geocentric phase. s.	Greenwich mean time of geocentric phase. h.	m.	s.	Authority and remarks.
Naples	Cartani	III	23	5	51	+11	22	5	17	Heinsius, St. P., N. C., vi, p. 563.
Bologna	Unknown		22	54	43	+12		5	49	Ib., mean of two observers not named.
Rouen	Bouin	Premier attouchement	22	14	18	+10		6	25	Paris, 1753, p. 424.
	Premagny	III		14	16	+10		6	23	Do.
	Le Cat	III		13	53	+10		6	0	Do.
Paris	Cassini	Le bord de Merc. me parut toucher celui du Soleil.	22	19	3	+10		6	9	Paris, 1753, p. 62.
	Le Gentil	Les bords parurent se toucher		18	47	+10		5	53	Ib., p. 272.
	Bouguer	III		18	44	+10		5	50	Ib., p. 200.
	De Merveille	Premier attouchement		18	39	+10		5	45	
	Libours			18	38	+10		5	44	
	De l'Isle	Attouchement intérieur des deux disques.		18	41	+10		5	47	
Leiden	Lulofs	III	22	28	12*	+16		6	49	Communicated by Dr. J. A. C. Oudemans, who translated from the Haarlem Memoirs.
Hague	Gabry	III	22	27	22	+16		6	41	
Haarlem	Anonymous	III	22	28	31	+16		6	31	
London	Short		22	5	35	+12		6	15	P. T., xlviii, 192; the times are mean times.
	Bird		22	5	25	+12		6	5	
Naples	Cartani	IV	23	9	5	....	22	8	32	
Bologna	Anonymous		22	57	23	....		8	29	
Rouen	Bouin		22	16	38	....		8	45	
	Premagny			16	40	....		8	47	
	Le Cat			16	26	....		8	33	
Paris	Cassini		22	21	42	....		8	48	
	Le Gentil			21	42	....		8	48	
	Delisle			21	23	....		8	29	
	De Merveille			21	35	....		8	41	
	Libours			21	46	....		8	52	
	Bouguer			21	13:	....		8	19:	
London	Short		22	8	10	....		8	50	Mean times.
	Bevis			8	6	....		8	46	
	Sisson			8	11	....		8	51	
	Bird			8	11	....		8	51	
	Canton			8	40	....		9	20	
Hague	Gabry		22	30	7	....		9	26	
Haarlem	Anonymous		22	31	25	....		9	25	

* The observer notes this phase as observed very accurately with a power of 75. There would seem to be some mistake in his time.

The discordances among the times of internal contact are such as to render it difficult to fix upon a definitive moment as that given by observation. An indiscriminate mean of the whole, rejecting only the doubtful external contact by BOUGUER, gives

	h.	m.	s.
Contact III,	22	6	4
IV,	22	8	50

But, in reality, the observations should not all have the same weight. If we take the six best known observers, CASSINI, LE GENTIL, DELISLE, SHORT, BEVIS, and BIRD, the mean results are,

	h.	m.	s.	
Contact III,	22	6	2	(5 obs.)
IV,	22	8	45	(6 obs)

In this case, the times of internal contact still have a range of 28 seconds, while the agreement in the case of external contact is fairly good.

Looking at the general agreement among the observers of external contact, it can hardly be doubted that Mercury was entirely off the sun before $9^m\ 0^s$. If this be so, there must have been an error of half a minute or more in the times of the observers at Haarlem and the Hague. One of these is entirely unnamed, the other was not an astronomer. Their results may, therefore, be rejected without question. The description of his observation given by LULOFS would indicate that it was very exact. As his time is the latest of all, and 40 seconds later than the mean of good observers, we are obliged to reject his result from the suspicion of an error in his time. The Naples observation may be placed in the same category.

We have left the five observations already cited by known astronomers, and eight others by comparatively unknown ones. For internal contact the mean of these eight results is,

Contact III, $22^h\ 5^m\ 58^s$

An indiscriminate mean of all but the Dutch observers gives $22^h\ 8^m\ 44^s.5$ as the time of external contact. The following times seem the most probable from all the observations:

	h.	m.	s.
Contact III,	22	6	0.5
IV,	22	8	45.0

1756, NOVEMBER 6.7.

[Equation of time, $-16^m\ 3^s$ for ingress; $-16^m\ 2^s$ for egress.]

Place.	Observer.	Contact and description.	Local app. time. h.	m.	s.	Red. to geocentric phase. s.	Greenwich mean time of geocentric contact. h.	m.	s.	Authority and remarks.
Pekin	Hallerstein	Ingressus totus	21	30	30	−33	13	28	6	St. P., N. C., ix, 503.
	Gaubil		21	31	54	−33	13	29	30	Paris, 1758, p. 135.
	Hallerstein	☿ cœpit egredi ex ☉	2	54	22	+8	18	52	40	
	Amiot	Attouchement inférieur	2	54	20	+8		52	38	
	Gaubil	Commencement du sortie	2	54	25	+8		52	43	
	Hallerstein	Egressus totus	2	56	6	+8		54	24	
	Amiot	Sortie totale	2	56	4	+8		54	22	
	Gaubil	Fin de la sortie	2	56	31	+8		54	49	

It is remarked by LE VERRIER that the observations of GAUBIL and AMIOT give a semi-diameter of the sun so small that he rejects them entirely. Those of HALLERSTEIN he does not refer to. The discordances will be seen from the circumstance that the tabular interval between external and internal contacts is only $1^m\ 39^s$, while that between internal contact and the least visible phase at external contact must be 10 or 20 seconds less. It is, therefore, certain that contact III was observed too early by all the observers.

The discordances at ingress are yet more strongly marked. Arranging the statements of the three observers in chronological order, they are:

h.	m.	s.	
21	29	15;	HALLERSTEIN, primum visus.
	29	49;	GAUBIL, commencé à voir Mercur.
	30	30;	HALLERSTEIN, ingressus totus.
	30	51;	GAUBIL, le centre sur le bord du Soleil.
	31	12;	AMIOT, à moitié entré.
	31	54½;	GAUBIL, Mercur. tout entré.

GAUBIL and AMIOT observed together; HALLERSTEIN in an entirely separate place. GAUBIL and HALLERSTEIN had 14-foot telescopes, AMIOT an 8½-foot telescope. That GAUBIL and AMIOT saw Mercury only half entered more than a minute after its appearance upon the disk can be attributed only to badness of their telescopes.

The best course seems to be to reject all the observations except those of last external contact, which are less affected by telescopic irradiation. Giving half weight to AMIOT's observation, the mean result will be,

Contact IV, $18^h\ 54^m\ 34^s$.

1769, NOVEMBER 9.4.

[Equation of time: Ingress, $-15^m\ 51^s$; Egress, $-15^m\ 49^s$.]

Place.	Observer.	Contact and description.	Local app. time.			Red. to geocentric phase.	Greenwich mean time of geocentric contact.			Authority and remarks.
			h.	m.	s.	s.	h.	m.	s.	
Salem, Mass	Williams	Thread of light closed in a moment.	2	56	0	......	7	24	11	M. A. A., i, 113.
Philadelphia	Williamson	II	2	37	30	+ 25	7	22	42	A. P. S., i. 78.
	Shippen			37	40	....		22	52	
	Evans			37	38	....		22	50	
	Ewing			37	30	....		22	42	
Norristown	Rittenhouse			36	35	+ 25		22	39	A. P. S., i, 159.
	Smith			36	35	......			39	
	Lukens			36	33	......			37	
Manilla	Veron	III	20	29	54	− 37	12	9	39	Le Verrier.
Batavia	Mohr	III	19	33	32	− 27	12	10	3	A. N., x. 207.
Manilla	Veron	IV	20	31	24	− 37	12	11	9	
Batavia	Mohr	IV	19	35	11	− 27	12	11	42	

Williams used a watch without a second hand, which was set by transits. Except for the possibility of systematic errors in the other observations, his result should be rejected. In view of this possibility, we may assign it the weight $\frac{1}{3}$. The results will then be,

		h.	m.	s.
Contact	II,	7	22	47
	III,	12	9	51
	IV,	12	11	26

1782, NOVEMBER 12.1.

[Equation of time, — $15^m\ 32^s$.]

Place.	Observer.	Contact and description.	Local app. time.			Red. to geocentric phase.	Greenwich mean time of geocentric contact.			Authority and remarks.
			h.	m.	s.	s.	h.	m.	s.	
Kremsmünster ...	Fixlmillner. ...	II. Après déja une ½' qu'il était dans le doute.	3	51	2	+ 197	2	42	15	
Paris	Cagnoli	II	3	0	21	+ 193		38	41	
	Le Monnier	Entrée totale		1	48	......		40	8	Paris, 1782, p. 647.
	Wallot	II		2	4	......		40	24	
	Méchain	L'entrée totale		2	8	......		40	28	
	Dagelet	II		2	32	......		40	52	
	Wallot	Mercury absolutely detached		3	46	......		42	6	
	Messier	Le deuxième bord de Mercur. parut, mais touchait encore.		4	13	......		42	33	Paris, 1782, p. 660.
	Cassini (fils)	II		4	21	......		42	41	
	Cassini	Les bords étaient séparés		4	22	......		42	42	
	Le Gentil	....		4	24	......		42	44	
	Messier..	étaient détaché		4	35	......		42	55	
	Lalande	Contact assuré		4	57	......		43	17	
Cookstown	....	Thread of light seemed completed.	2	27	43	+ 188		42	11	P. T.
Cambridge, U. S...	Winthrop	First internal contact when the thread of light was formed and Mercury recovered his roundness.	22	12	13	+ 97		42	44	M. A. A., i, 159.
	Williams	First appearance of small thread of light.	22	12	7	......		42	38	M. A. A., i, 115.
	Willard	II	22	12	37	......		43	8	
	Gauvet	II	22	12	45	......		43	16	
Chelsea	Payson	II	22	12	36	......		42	50	M. A. A., i, 127.
Ipswich	Cutter	II	22	13	37	......		42	44	
Philadelphia	Rittenhouse	II	21	40	40	+ 85		42	43	Mean time.
Paris	Cassini	III	4	17	19	−178	3	49	28	
	Wallot	....	4	17	18	......		49	27	
	Cagnoli	....	4	16	24	......		48	33	
	Méchain	....		17	46	......		49	55	
	Le Gentil	....		18	7	......		50	16	
	Duc d'Ayen	....		17	43	......		49	52	
Ipswich	Cutter	....	23	24	15	− 161		50	13	
Chelsea	Payson	....		23	31	− 161		49	29	
Cambridge	Winthrop	Mercury began to appear oblong before the second internal contact.	23	21	41	− 161	3	47	58	
		Doubtful whether the thread of light was broken.		22	44	......		49	1	

1782, NOVEMBER 12.1—Continued.

Place.	Observer.	Contact and description.	Local app. time. h.	m.	s.	Red. to geocentric phase. s.	Greenwich mean time of geocentric contact. h.	m.	s.	Authority and remarks.
Cambridge	Winthrop	Second internal contact when the thread of light was completely broken.	23	23	5	......	3	49	22	
	Williams	Second internal contact		23	8	......		49	25	
	Paine	....		22	5	......		48	22	With small telescope; power, 50.
	Willard	....	23	23	2	......		49	19	
	Gauvet	....	23	23	36	......		49	53	
New Haven	....	....	23	15	48	− 158		49	20	
Philadelphia	Rittenhouse	....	22	51	30	− 154		49	34	Mean time.
Paris	Cassini	IV	4	22	49	......	3	54	58	
	Wallot	....		22	53	......		55	82	
Rocheguyon	Several obs	....		20	47	......		52	56	
Ipswich	Cutter	....	23	30	20	......		56	18	
Chelsea	Payson	....	23	28	58	......		54	56	
Cambridge	Williams	....	23	29	19	......		55	36	
	Winthrop	....		29	10	......		55	27	
	Willard	....		29	32	......		55	49	
	Gauvet	....		29	29	......		55	46	
	Paine	....		28	6	......		54	23	
Philadelphia	Rittenhouse	....	22	57	35	......		55	39	

Among all observed transits this one is remarkable for the nearness of Mercury to the sun's limb, the least distance of centers not being 30″ less than the sun's semi-diameter. The interval between internal and external contacts was more than seven minutes, and a good opportunity was, therefore, offered for studying the phenomena of contact.

At Paris ingress occurred late in the afternoon, and egress only a quarter of an hour before sunset, so that the sun's limb was much disturbed by atmospheric vibrations. The American observations were made under much more favorable conditions, so that, if an absolute result were alone aimed at, they would be entitled to greater weight. But what we really want is not so much the time of mathematical contact as a time corresponding to the general average phase noted by other observers in other transits. Now, an examination of the descriptions of contacts shows that in this, as in the preceding transits, it seems impossible to discriminate with certainty between different phases of internal contact merely from the descriptions of the observer. Thus, MECHAIN describes "entrée totale" two minutes before any one else saw the thread of light. For ingress, one course will be to reject CAGNOLI's observation entirely, as clearly in error, and take an indiscriminate mean of all the others. This will give

$$\text{Contact II, } 2^h\ 42^m\ 16^s$$

On the other hand, we have been led to suspect that, in previous transits, observers with bad telescopes were apt to observe internal contact too late, because the

thread of light would not seem complete till after it had attained a considerable thickness. Should we reject the observations of WALLOT (int. cont.) and LE MONNIER, where it is evident the thread of light cannot have been observed, we should obtain $2^h\ 42^m\ 29^s$ as the time of contact. On the whole, however, it seems better, for the present at least, to accept the indiscriminate mean.

At the time of egress the sun had nearly set to Paris, so that the observations there were made under very unfavorable circumstances. The American observations being made nearer noon, deserve more careful consideration. In treating them, it seems advisable to take the probable skill of the observer into account. WINTHROP'S observation deserves the highest weight, because of his care in describing three well-marked phases. There seems little doubt that the time of contact from his observations should be placed between the second and third phases; perhaps midway, but more likely one-third of the way from the second to the third.

Next in order should come the observations of the practiced observers, WILLIAMS, WILLARD, and RITTENHOUSE, though, as they do not describe the phenomena, their observations have less weight than those of WINTHROP. We may, therefore, take for the three best results,

	h.	m.	s.	
WINTHROP, -	3	49	8	weight 2
WILLIAMS, - -		49	25	weight 1
WILLARD, - -		49	19	weight 1
RITTENHOUSE,		49	34	weight 1
Mean, - -	3	49	19	(1)

There remain five results of comparatively unknown American observers, including PAINE, who had but a small telescope. The indiscriminate mean of their times is

$$3^h\ 49^m\ 27^s \quad (2)$$

The indiscriminate mean of the six Paris observations is

$$3^h\ 49^m\ 35^s \quad (3)$$

If we reject CAGNOLI and D'AYEN, the mean of CASSINI, WALLOT, MECHAIN, and LE GENTIL is

$$3^h\ 49^m\ 46^s \quad (4)$$

Considering the extreme obliquity of the motion, the accordance of these four mean results is quite satisfactory. In combining them, I shall assign

To (1) the weight 4.
(2) the weight 2.
To $\frac{(3)+(4)}{2}$ the weight 1.

giving $3^h\ 49^m\ 24^s$ as the concluded time of third contact. For the fourth contact we

can do no better than take an indiscriminate mean of the eight American observations. We thus have, for the concluded times of the geocentric contacts,

	h.	m.	s.
Contact II,	2	42	17
III,	3	49	24
IV,	3	55	29

In reaching this result, however, we have used only the differential reduction to geocentric time, computed by the usual formulæ. But, in a transit occurring so near the sun's limb as this, the differential reduction will not be sufficiently accurate. A rigorous computation of the times of contacts has, therefore, been made for Paris and Cambridge by formulæ given hereafter, and these times have been compared with the geocentric times. The principal steps of the process are shown in the following table:

	Contact II. Greenwich M. T. $2^h.7$.		Contact III. Greenwich M. T. $3^h.85$.		Contact IV. Greenwich M. T. $3^h.95$.	
	Paris.	Cambridge.	Paris.	Cambridge.	Paris.	Cambridge.
	$''$	$''$	$''$	$''$	$''$	$''$
Effect of Parallax $\Delta(l-l')$	+ 2.10	− 4.87	+ 3.19	− 3.11	+ 3.26	− 2.94
Effect of Parallax $\Delta(b'-b)$	+ 8.48	+ 6.45	+ 8.31	+ 7.13	+ 8.29	+ 7.19
Effect of Parallax Δr	− 0.01	− 0.02	− 0.01	− 0.02	− 0.01	− 0.02
$l-l$	− 730.45	− 730.45	+ 151.32	+ 151.32	+ 228.02	+ 228.02
$b'-b$	− 1963.14	− 1963.14	− 2092.48	− 2092.48	− 2103.72	− 2103.72
c	2085.95	2090.29	2089.89	2090.61	2108.16	2108.58
$r+\Delta r$	2094.54	2094.53	2095.63	2095.62	2116.88	2116.87
	h.	h.	h.	h.	h.	h.
t	− 0.05072	− 0.02521	+ 0.03312	+ 0.02940	+ 0.04366	+ 0.04210
Gr. m. t. of contact	2.64928	2.67479	3.88312	3.87940	3.99366	3.99210
	h. m. s.	h. m. s.	h. m. s.	h. m. s.	h. m. s.	h. m. s.
	2 38 57.4	2 40 29.2	3 52 59.2	3 52 45.8	3 59 37.2	3 59 31.6
Geocentric times	2 41 59.5	2 41 59.5	3 50 9.5	3 50 9.5	3 57 17.3	3 57 17.3
Difference	+ 3 2.1	+ 1 30.3	− 2 49.7	− 2 36.3	− 2 19.9	− 2 14.3
Approximate reduction	3 13	1 37	− 2 58	− 2 51	. .	. .
A second approximation gives t	2 38 58.6	2 40 30.8	3 53 0.1	3 52 47.0	3 59 36.1	3 59 31.2
Geocentric times	2 41 59.5	2 41 59.5	3 50 9.5	3 50 9.5	3 57 17.3	3 57 17.3
Difference	+ 3 0.9	+ 1 23.7	− 2 50.6	− 2 37.5	− 2 18.8	− 2 13.9
Approximate reduction	+ 3 13	+ 1 37	− 2 58	− 2 51	− 2 58	− 2 51
Corr. to approximate reduction	− 12	− 8	+ 7	+ 14	+ 39	+ 37

At ingress about twice as many observations were made in Europe as in America. We may suppose the correction, -12^s, applicable to all the European observations, and -8^s to all the American ones. This will diminish the general mean by 11^s.

At egress the weights of the Paris and Cambridge observations were in the ratio

of 1 : 6. This will give + 13 seconds as the correction. We thus have, for the concluded geocentric times:

	h.	m.	s.
Contact II,	2	42	6
III,	3	49	37
IV,	3	56	6

1786, MAY 3.7.

[Equation of time — 3^m 28^s.]

Place.	Observer.	Contact and description.	Local app. time. h.	m.	s.	Red. to geocentric phase. s.	Greenwich mean-time of geocentric contact. h.	m.	s.	Authority and remarks.
Bagdad	Beauchamp	II	18	0	5	+ 43	14	59	41	St. P., N. A., ii, 281.
St. Petersburg	Rumowski	Thread of light complete	17	2	19	+ 108	14	59	25	St. P., N. A., i, 377.
	Inochodzow	II	17	3	13	....	15	0	19	St. P., N. A., ii, 268.
Mitau	Beitler		16	7	26	+ 105	15	0	48	B. M., 1786, 309–321.
Bagdad	Beauchamp	III	23	22	[illegible]2	− 35	20	21	10	
St. Petersburg	Rumowski		22	26	55	− 65		21	8	
	Tzernoi		22	27	7	....		21	20	
	Inochodzow		22	27	12	....		21	25	
Lund	(?)		21	18	48	....		21	15	St. P., N. A., ii, 274.
Upsal	Prosperin	Schr genau	21	36	40	− 80		21	21	B. J., 1789, 207.
Dresden	Köhler	Beginning of Egress	21	21	54:	− 90		22	0	P. T., lxxvi, 47.
Rome	Callendrelli		21	16	23	− 97		21	23	
Milan	Reggio		21	3	26	− 95		21	37	
	d'Cesaris		21	2	30	....		20	41	
Bologna	Matteucci		21	12	16	− 97		21	46	
Padua	Toaldo		21	13	8	− 96		20	35	
Mannheim	König		21	0	17	− 98		21	20	
Louvain	N. Pigott		20	45	41	− 102		21	43	P. T., lxxvi, 384–389.
	E. Pigott		20	45	31	....		21	33	
Paris	Messier		20	36	28	− 106		21	53	P. M., 1786, p. 123.
Toulouse	d'Arquier		20	32	39	− 111		21	34	
London	Zach (?)		20	23	23	− 110		21	58	Mean time, B. J., 1789.
London Argyle street.	Voy		20	26	51	....		21	58	St. P., N. A., ii, 274. B. J., 1789, p. 206.
Montpellier	Poitevin		20	41	45	− 108		20	58	B. J., 1789, p. 206.
Bagdad	Beauchamp	IV	23	26	48	....		25	6	
St. Petersburg	Rumowski		22	30	35	....		24	48	
	Inochodzow		22	30	15	....		24	28	
Upsal	Prosperin		21	41	20	....		26	1	
Lund			21	22	48	....		25	15	
Dresden	Köhler		21	25	23	....		25	29	
Rome	Callendrelli		21	19	18	....		24	18	
Padua	Toaldo		21	17	31	....		24	58	
Bologna	Matteucci		21	15	20	....		24	50	
Milan	Reggio		21	6	40:	....		24	51	
	Cesaris		21	6	59	....		25	10	
Manheim			21	4	14	....		25	17	
Louvain	N. Pigott		20	49	16	....		25	18	
	E. Pigott		20	49	22:	....		25	24	
Paris	Messier		20	39	58	....		25	23	
	Delambre		20	39	56	....		25	21	
London			20	29	51	....		24	58	

The fixing of a definite time of ingress from the four discordant observations is very difficult from the fact that RUMOWSKI, whose description is clear and exact, saw the ingress notably sooner than any one else. He says:

"Momentum pro contactu interno in introitu assumtum est a me illud, cum inter undulantes et tremulos limbos filum lucidum mihi sese obtulerit, id circo realis contactus aliquot minutis secundis a me observatum præcesserit necesse est."

Granting the correctness of the observation, this conclusion is sound, and geocentric contact must have occurred decidedly before $14^h\ 5^m\ 25^s$.

On the other hand, the sun was only about 8° above the horizon at St. Petersburg, while at Bagdad its altitude was considerable. Against this, however, must be placed the consideration that the longitude of Bagdad is uncertain by some seconds.

At Mitau the sun was still lower than at St. Petersburg, and the observer gives no description. The observation may, therefore, be passed over.

Some light may be thrown upon the results by the observations of external contact We have:

	h.	m.	s.
Tabular interval between contacts I and II - -		4	16
RUMOWSKI estimated bisection of disc at - - -	16	59	44
INOCHODZOW saw "contactus primus sive externus"	17	0	6

There is clearly a blunder on one side or the other; probably an error of one minute. If we assume RUMOWSKI to be correct we have nothing better to do than accept his result, and put

Contact II at $14^h\ 59^m\ 25^s$.

If, however, we assume an error of 1^m in RUMOWSKI's time, we may assign equal weight to him, BEAUCHAMP, and INOCHODZOW. We shall then have

Contact II, $15^h\ 0^m\ 8^s$.

We cannot decide *a priori* between these hypotheses.

Though the observations at egress are also unusually discordant, there is less doubt about the result. In the first place, an indiscriminate mean gives

Contact III, $20^h\ 21^m\ 26^s$.

Examining the observations critically, we may reject the observations of D'CESARIS and TOALDO on suspicion of an error of one minute, and the doubtful observation of KÖHLER. Moreover, we may suspect the two London observations to be but one, from the absolute identity of both third and fourth contacts. We may also assign superior weight to the observations of RUMOWSKI, INOCHODZOW, PROSPERIN, and MESSIER. The mean of their times is $20^h\ 21^m\ 27^s$. The mean of the remaining unsuspected twelve observations is $20^h\ 21^m\ 28^s$. We therefore have

	h.	m.	s.
Contact III,	20	21	27
IV,	20	25	3

1789, NOVEMBER 5.2.

[Equation of time = $-16^{m}\ 11^{s}$.]

Place.	Observer.	Contact and description.	Local app. time. h.	m.	s.	Red. to geocentric phase. s.	Greenwich mean time of geocentric phase. h.	m.	s.	Authority and remarks.
Vienna	Triesnecker	II	2	15	6	−22	0	53	1	B. J., 1794, 136.
Prague	Gerstner	Sure to 2 seconds	1	51	16	−24	0	53	11	B. J., 179[illegible], 110, mean time.
Marseilles	De Thulis	Gewiss	1	31	7	−24	0	53	4	B. J., 179[illegible], 124.
Viviers	Flaugergues	....	1	28	40	−25		53	20	
Paris	Messier	Mercur. touchait encore	1	18	47	−28		52	47	
	do	Je commença à voir un filet de lumière.	1	18	56	......		52	56	
	Méchain	Absonderung der Ränder	1	19	0	......		53	0	
	Cassini	....	1	19	5	......		53	5	
	Delambre	....	1	19	2	......		53	2	
Montauban	De la Chapelle	....	1	15	14	−26		53	12	
Cambridge	Willard	....	20	25	52	−46		53	22	
Philadelphia	Rittenhouse	....	19	53	20	−46		53	12	Mean time.
Washington College.	Smith	....	20	5	0	−46		52	13	
Montevideo	Galliana	III	2	15	11	+21	5	44	1[illegible]	B. J., 1794, 136.
Cambridge	Willard	....	1	15	44	+12		44	13	
Philadelphia	Rittenhouse	....	0	43	24	+10		44	12	
Washington College.	Smith	....	0	55	10	+7	......	......	......	
William and Mary.	Madison	....	0	53	42	+2		44	11	
	Andrews	....	0	53	48	......		44	17	
Cambridge	Willard	IV	1	17	36	......	5	46	[illegible]5	
Philadelphia	Rittenhouse	....	0	45	4	......		45	52	
Washington College.	Smith	....	0	56	35	......		44	41	
William and Mary.	Andrews	....	0	55	19	......		45	48	

The "Washington college" observations are assumed to have been made at an institution of that name in Chestertown, Md.; but this is purely conjectural. No locality is mentioned. Their systematic discordance indicates an error in the time, and they are not used.

The ingress affords a case in which the astronomer must feel in doubt what conclusion ought to be drawn from the combined observations. Supposing the observations of MESSIER and MECHAIN correct, the true contact must have occurred *before* $0^{h}\ 53^{m}\ 0^{s}$. But every one of the other observers assigns a time later than this. The general mean results are:

	h.	m.	s.
True contact, from observations of MESSIER and MECHAIN -	0	52	56
Indiscriminate mean of all the observers - - - - - - -	0	53	8

If we employed no transits except those in which we could deduce a time of true contact from the descriptions of the observers, we should, no doubt, use only the first

result. But, being obliged, in many cases, to use indiscriminate means, the last result is not to be neglected. On the whole, it would seem that the mean of the two is about the phase we want.

The third contacts offer no difficulty. In the fourth we give double weight to WILLARD and RITTENHOUSE. The concluded results are, therefore,

	h.	m.	s.
Contact II,	0	53	2
III,	5	44	12
IV,	5	46	8

1799, MAY 7.1.

Equation of time: Ingress, — $3^m\ 43^s$; Egress, — $3^m\ 44^s$.

Place.	Observer.	Contact and description.	Local app. time. h.	m.	s.	Red. to geocentric phase. s.	Greenwich mean time of geocentric contact. h.	m.	s.	Authority and remarks.
St. Petersburg	Rumowski	II	23	14	26	— 4	21	9	25	A. G. E., iv, 172.
	Henry			14	31	....			30	Ibid., iv, 465.
Krakau	Anonymous		22	33	54	— 21		9	60	
Breslau	Jungnitz I	Inner contact	22	21	46	— 24		9	30	
		Lichtfaden		22	47	....		10	31	
	Hoffman			22	51	....		10	35	
	Ender			22	28	....		10	12	Seine erste ast. Beobachtung.
Vienna	Triesnecker	Tropfen	22	15	43	— 27		9	44	A. G. E., iv, 66. Mean time.
		Lichtfaden		15	45	....		9	46	Mean time.
	Bürg	do		15	47	....		9	48	
	Vega	do		15	52	....		9	53	
Prague	David		22	7	49	— 28		9	40	Mean time.
	Schonau			7	55	....		9	46	A. G. E., iv, 172.
	Strnad			8	10	....		9	61	
Dresden	Köhler	Kein Lichtfaden sondern ein kleiner Tropfen.	22	5	7	— 28		9	43	B. A. J., 1802, 214. Mean time.
		Tropfen verschwunden		5	16	....		9	52	
	v. Gesler		22	5	20	....		9	56	
Kremsmünster	Derfflinger		22	4	12	— 28		7	12	Mean time.
	Oettel			4	8	....		7	8	A. G. E., iv, 68.
Berlin	Bode		22	3	46	— 27		9	44	B. A. J., 1803, 114. Mean time.
Leipzig	Rudiger		22	1	41	— 28		11	39	Ibid. Mean time.
Padua	Chiminello		22	1	5	— 35		9	16	A. G. E., iv, 464.
Hamburg	Reinke		21	53	42	— 29		9	36	A. G. E., iv, 65.
	Eimbcke			53	25	....			19	
Gotha	Anonymous		21	53	17	— 32		9	54	A. G. E., iv, 217.
Paris	Méchain	Erste innere Berührung		23	14	— 42		9	28	A. G. E., iv, 171.
		Lichtfaden sehr kenntlich		23	36	....		9	50	
	Bouvard			23	14	....		9	28	
	Lalande			23	43	....		9	57	
	de Lambre			23	53	....		10	7	
Amsterdam (Felix Meritis.)	v. Beeck	Limbs clear; the planet entered this moment.	21	33	11	— 35		9	20	
Utrecht	Utenhoner	Planet very faint; observations a little late.	21	34	43	— 35		9	53	
Marseilles	Vidal		21	32	10	— 44		9	58	M. C., viii, 116.

1799, MAY 7.1—Continued.

Place.	Observer.	Contact and description.	Local app. time. h.	m.	s.	Red. to geocentric phase. s.	Greenwich mean time of geocentric phase. h.	m.	s.	Authority and remarks.
St. Petersburg	Rumowski	III	6	33	34	+111	4	30	27	
	Henry			33	26			30	19	
	Ein See-Officier			33	40			30	33	
Breslau	Jungnitz I	Schwarzeben Tropfen	5	40	35	+110		30	32	
		Berührung		40	41			30	38	
	Hoffman			39	56			29	53	HOFFMAN, professor of theology, used a power of only 24.
	Jungnitz II			40	48			30	45	
	Ender			39	46			29	43	JUNGNITZ II seems to have been unable to make a good observation; and ENDER was inexperienced.
Dantzic	Koch		5	47	2	+110		30	30	
Dresden	Köhler	Tropfen enstelit	5	23	37	+109		30	30	
		Tropfen verschwunden		23	48			30	41	
	v. Gesler			23	36			30	29	Mean time.
Berlin	Bode		5	22	17	+109		30	31	
Hamburg	Reinke		5	12	20	+107		30	29	
	Eimbcke			12	16				25	
Paris	Mechain	Ein schwarzer Punckt	4	41	52	+104		30	31	
		Sceinbare Vereinigung d. Ränder		42	2			30	41	
	Burckhardt			41	51			30	29	
	Messier			42	10			30	49	
	De Lambre			41	48			30	27	
	Bouvard			41	42			30	21	
Greenwich	Maskelyne		4	28	43	+103		30	26	Mean time.
	Wilson			28	33			30	16	
	Nisbet			28	53			30	36	
	T. F			28	47			30	30	
London	Troughton		4	28	14	+103		30	22	A. G. E., iv, 172.
Manheim	Barry		5	2	28	+107		30	25	Mean time. A. G. E., iv, 172.
Upsal	Anonymous		5	39	14	+108		30	32	M. C., viii, 116.
Marseilles	Vidal		4	50	25	+105		30	42	A. G. E., iv, 218.
St. Petersburg	Rumowski	IV	6	35	53	+111	4	32	46	
	Henry			36	17			33	10	
	See-Officier			36	6			32	59	
Breslau	Jungnitz I		5	43	36	+110		33	33	
	Jungnitz II			43	29			33	26	
	Hoffman		5	42	6			32	3	
	Ender			42	46			32	43	
Dantzic	Kock		5	49	38	+111		33	6	
Dresden	Köhler		5	26	34			33	27	
	v. Gesler			26	31			33	24	
Berlin	Bode		5	25	30			33	44	
Hamburg	Reinke		5	14	16			32	25	Cloudy; doubtful.
	Eimbcke			14	10			32	19	
Paris	Mechain		4	45	3	+104		33	42	
	De Lambre			44	49			33	28	
Greenwich	Wilson		4	31	20			33	3	
	T. F			31	8			32	51	

The discordances at ingress are striking; but do not prevent us approximating to a definite result. In the first place, we have four observers who distinguish between

inner contact and thread of light. From these we may reject TRIESNECKER, from the extraordinary interval of 61 seconds between the phenomena. Taking the mean between the times of the two phenomena in the other cases, the results for contact II are

	h.	m.	s.
TRIESNECKER	21	9	45
KÖHLER - -		9	48
MECHAIN - -		9	39
Mean - -	21	9	43

We note also that the mean of the three intervals between the two phenomena is 6^s.

Next, we may take the well-known observers who do not describe the phenomena, BÜRG and VEGA, who noted the thread of light, and v. BEECK, who seems to have made a satisfactory observation. From the times of the three last named we may subtract 5^s to reduce them to probable true contact. We then have the following results:

	h.	m.	s.
RUMOWSKI -	21	9	25
BÜRG - -		9	43
VEGA - -		9	48
DAVID - -		9	40
BODE - -		9	44
BOUVARD -		9	28
LALANDE -		9	57
DELAMBRE -		9	67
v. BEECK -		9	15
Mean - -	21	9	41

Should we reject the three Paris observations and that at Amsterdam, on account of their discordance, the mean result would be

$$21^h\ 9^m\ 40^s.$$

From the remaining observations we may reject those of Kremsmünster and Leipzig without question, as well as that at Utrecht, where clouds rendered the observation late. The indiscriminate mean of the remaining ones is

$$21^h\ 9^m\ 48^s.$$

But there is little doubt that the Breslau observations should be rejected from any mean. We should then have

Mean of 10 observations, $21^h\ 9^m\ 41^s$.

The different classes of observations seem to group themselves so clearly around the mean $21^h\ 9^m\ 42^s$ that we may adopt this as the time of contact.

Treating the observations of egress in the same general way, we note that three observers observed separately the formation of the black drop and the internal contact. Moreover, the means of their times agree almost perfectly, and give

Contact III, $4^h\ 30^m\ 35^s.5$

Other experienced observers, who do not describe any phenomena, give the results:

	h.	m.	s.
RUMOWSKI -	4	30	27
BODE - - -		30	31
BURKHARDT -		30	29
MESSIER - -		30	49
DELAMBRE -		30	27
BOUVARD - -		30	21
MASKELYNE -		30	26
WILSON - -		30	16
NISBET - -		30	36
T. F. - - -		30	30
TROUGHTON -		30	22
Mean - -	4	30	29

The indiscriminate mean of the remaining twelve results is

$4^h\ 30^m\ 24^s.$

But there is little doubt that we should exclude the three observations at Breslau. The mean result will then be

$4^h\ 30^m\ 29^s.$

The mean result to be adopted may be fixed at $4^h\ 30^m\ 32^s$.

For fourth contact we reject the observations of HOFFMAN, JUNGNITZ II, and ENDER, as well as the doubtful ones at Hamburg. We thus have the following geocentric times for the three contacts:

	h.	m.	s.
Contact II,	21	9	42
III,	4	30	32
IV,	4	33	16

1802, NOVEMBER 9.0.

[Equation of time = — $16^m\ 0^s$.]

Place.	Observer.	Contact and description.	Local app. time. h.	m.	s.	Red. to geocentric phase. s.	Greenwich mean time of geocentric contact. h.	m.	s.	Authority and remarks.
Prague	David	III	0	54	57	−13	23	41	3	Paris, 1806, p. 55.
Naples	Cassela		0	54	8	−10		40	57	
Copenhagen	Bugge		0	47	44	−15		41	10	
Leipzig	Rudiger		0	46	51	−14		41	3	
Gotha	Zach		0	40	30	−14		41	25	Zach. Monat. Corr., VI, 567.
	Sein Bruder			40	33	....		41	28	
	Catenelli			40	19	....		41	14	
Brunswick	Gauss		0	39	16	−15		40	57	
Celle			0	21	41m	−15		41	13	
Lilienthal	Schröter		0	17	3m	−16		41	8	
	Harding		0	16	58	....		41	3	
Quedlinburg	Fritsche		0	26	8m	−14		41	17	
Marseilles	Thulis		0	18	5	−15		40	22	
Viviers	Flaugergues		0	15	50	−16		40	50	Ib., VII, p. 81.
Paris	Lalande		0	6	29	−18		40	50	
	Lalande, Nev			6	44	....		41	5	
	Bouvard			6	54	....		41	15	
	Messier			6	49	....		41	10	
	Mechain			6	45	....		41	6	
	Burckhardt			6	45	....		41	6	
Greenwich	T. F		23	57	21	−20		41	1	
	Best		23	57	21	....		41	1	
Naples	Cassela	IV	0	55	50	....	23	42	39	
Copenhagen	Bugge		0	49	9	....		42	35	
Leipzig	Rudiger		0	48	9	....		42	21	
Brunswick	Gauss		0	40	48	....		42	29	
Lilienthal	Schröter		0	18	33	....		42	38	
	Harding		0	18	36	....		42	41	
Quedlinburg	Fritsche		0	27	41	....		42	55	
Marseilles	Thulis		0	19	58	....		42	15	
Viviers	Flaugergues		0	17	13	....		42	13	
Paris	Lalande		0	7	56	....		42	17	
	Messier			8	20	....		42	41	
	Lalande, Nev			8	19	....		42	40	
	Bouvard			8	19	....		42	40	
	Mechain			8	30	....		42	51	
	Burckhardt			8	20	....		42	41	
Greenwich	T. F		23	59	1	....		42	41	
	Best		23	58	57	....		42	37	

Giving double weight to each of the Paris and Greenwich observers, and to SCHRÖTER and HARDING, the mean result for

	h.	m.	s.
Internal contact is	23	41	5
External contact is	23	42	34

1822, NOVEMBER 4.5.

Place.	Observer.	Contact and description.	Local mean time. h.	m.	s.	Red. to geocentric phase. s.	Greenwich mean time of geocentric contact. h.	m.	s.	Authority and remarks.
Calcutta	Hodgson	II	18	56	16	−43	13	2	8	M. R. A. S., iii, 110; uncertain to 4 or 5 seconds.
Paramatta	Rümker	Complete immersion	23	7	20	+26		3	42	P. T., 1829, app. p. 30; A. N., ii, 210.
Sydney	Brisbane	....	23	8	6	+27		3	42	
Calcutta	Hodgson	III	21	38	34	− 6	15	45	3	Hodgson used a power of 45 at ingress, and 60 at egress.
	Herbert	....		38	42	....			11	
Kurnaul	Euwer	....	20	53	46	−11		45	28	
Paramatta	Rümker	....	1	49	8	+14		45	18	
Sydney	Brisbane	....	1	50	2	+14		45	25	
Calcutta	Hodgson	IV	21	40	56	....	15	47	25	
	Herbert	....	21	40	55	....		47	24	
Kurnaul	Euwer	....	20	56	16	....		47	58	
Paramatta	Rümker	....	1	52	7	....		48	17	
Sydney	Brisbane	....	1	53	0	....		48	23	

The discordance between HODGSON's observation of ingress and the observations of RÜMKER and BRISBANE is embarrassing. If, as appears to be the case, BRISBANE never published his observations himself, it is likely he assigned them little weight. As a combination of HODGSON's observation with the other two is out of the question we must for second contact adopt

$$13^{h}\ 3^{m}\ 42^{s}$$

with a suspicion that it may be too late.

For third contact there seems no better course than to combine all, giving greater weight to RÜMKER and BRISBANE. The result,

$$\text{Contact III, } 15^{h}\ 45^{m}\ 18^{s},$$

seems most probable.

The fourth contacts observed at Calcutta again are troublesome, because it hardly seems probable that the planet should have disappeared from sight more than a minute before external contact. We shall therefore reject their results. The mean of the remaining three observations is

$$\text{Contact IV, } 15^{h}\ 48^{m}\ 13^{s}.$$

All these results must be regarded as more doubtful than usual.

1832, MAY 5.0.

Place.	Observer.	Contact and description.	Local mean time. h.	m.	s.	Red. to geocentric phase. s.	Greenwich mean time of geocentric contact. h.	m.	s.	Authority and remarks.
Königsberg	Bessel	II	22	24	39	+ 51	21	3	31	Ast. Nach., x, 186–7.
Cape	Henderson		22	19	13	−100		3	38	
Breslau	Boguslawski		22	11	3	+ 43		3	37	
Prague	David		22	0	25	+ 40		3	24	
Padua	Santini		21	49	59:	+ 30		3	0:	
Modena	Bianchi		21	46	50	+ 28		3	35	
Altona	Schumacher		21	42	41:	+ 58		3	53:	
	Peterson			42	7			3	19	
	Nyegaard			42	29			3	41	
Milan	Four obs.	Mean of all	21	39	55	+ 28		3	37	Eph. Milano, 1833, p. 105.
Marburg	Gerling		21	38	14	+ 39		3	48	
Manheim	Nicolai		21	36	47	+ 36		3	33	
Marseilles	Gambart	App. tangency	21	24	26	+ 23		3	14	
		Filet de lumière		24	44			3	32	
Utrecht	Moll		21	23	25	+ 40		3	33	
	Foekens		21	23	25			3	33	
	Van Beck		21	23	25			3	33	
Leiden	Uglenbrock		21	20	42	+ 40		3	26	
	Kaiser		21	20	50			3	34	
Konigsberg	Bessel	III	5	7	38	+ 64	3	46	43	
	Argelander		5	7	39			46	44	
	Busch		5	7	39			46	44	
Dantzig	Bille		4	59	56	+ 65		46	22	
	Weinholdt		5	0	0			46	26	
Cape	Henderson		4	58	26:	+104		46	15:	☉ only 1° high.
Breslau	Boguslawski		4	53	39	+ 70		46	40	
Prague	David		4	43	14	+ 71		46	43	
	Hallaschka		4	43	14			46	42	
Berlin	Mädler		4	39	3	+ 66		46	34	Nicht sehr zuverlässig.
Padua	Santini		4	33	2	+ 77		46	50	
Modena	Bianchi		4	28	49	+ 78		46	24	
Gottingen	Gauss		4	25	32	+ 66		46	52	
Altona	Schumacher		4	25	28	+ 63		46	45	Through clouds.
	Peterson		4	25	41			46	58	
Milan	Four obs	Mean of all	4	22	4	+ 76		46	34	
Marburg	Gerling	App. tangency of limbs after formation of black drop.	4	20	28	+ 67		46	30	
Mannheim	Nicolai		4	19	37	+ 69		46	57	
Marseilles	Gambart	Disparition du filet	4	6	51	+ 76		46	33	
		Tangency	4	7	19			47	1	
Amsterdam	Vouté		4	5	14			46	44	A. N., x, 209.
Utrecht	Foekens		4	6	7	+ 63		46	38	
	Van Beck		4	5	57			46	28	
	Van Rees		4	6	8			46	39	
Brussels	Quetelet		4	2	52	+ 64		46	28	
Königsberg	Bessel	IV	5	11	1	+ 65	3	49	67	
	Argelander		5	10	48			49	54	
	Busch		5	10	34			49	40	

1832, MAY 5.0—Continued.

Place.	Observer.	Contact and description.	Local mean time.	Red. to geocentric phase.	Greenwich mean time of geocentric contact.	Authority and remarks.
			h. m. s.	s.	h. m. s.	
Breslau	Boguslawski		4 56 58	+ 70	49 59	
Dantzig	Bille		5 3 30	+ 64	49 55	
	Weinhault		5 3 30	......	49 55	
Prague	David		4 46 27	+ 71	49 57	
	Hallaschka		4 46 25	......	49 55	
Berlin	Mädler		4 42 18	+ 66	49 49	
Padua	Santini		4 35 57	+ 77	49 45	
Modena	Bianchi		4 32 14	+ 78	49 49	
Gottingen	Gauss		4 28 22	+ 66	49 42	
Altona	Schumacher		4 28 41	+ 63	49 58	
	Peterson		4 28 44	......	49 61	
	Nyegaard		4 28 49	......	49 66	
	Selander		4 28 47	......	49 64	
Milan	Mean of 5 obs.		4 25 24	+ 76	49 54	
Marburg	Gerling		4 23 42	+ 67	49 44	
Utrecht	Foeken		4 9 15	+ 63	49 46	
	Van Rees		4 9 18	......	49 49	
	Van Beck		4 9 10	......	49 41	
Marseilles	Gambart		4 10 17	+ 76	49 58	
Brussels	Quetelet		4 6 1	+ 64	49 36	

The observations afford little ground for discussion. In ingress we may omit the doubtful observations of SCHUMACHER and SANTINI, and in egress that of HENDERSON, and take an indiscriminate mean of all the other quoted results. We thus have

		h.	m.	s.
Contact II,	May 4,	21	3	32
III,	May 5,	3	46	40
IV,	May 5,	3	49	52

1845, MAY 8.

Place.	Observer.	Contact and description.	Local app. time.	Red. to geocentric phase.	Greenwich mean time of geocentric contact.	Authority and remarks.
			h. m. s.	s.	h. m. s.	
Pulkowa	Struve	II. No distortion	6 24 19	+48	4 23 48	Communicated by O. Struve.
	Döllen		6 24 26	......	55	
Reval	Sivens		5 52 8	+50	23 52	
	Gallenskap		5 52 15	......	59	
Dorpat	Mädler		6 9 31	+52	23 30	
Seftenberg	Hackel		5 28 9	+69	23 27	
Nienstedten	Schumacher		5 2 20	+61	23 58	A. N., xxiii, 146.
	Peterson		5 2 21	......	23 59	
	R. Schumacher		5 1 57	......	23 35	

1845, MAY 8—Continued.

Place.	Observer.	Contact and description.	Local mean time. h.	m.	s.	Red. to geocentric phase. s.	Greenwich mean time of geocentric contact. h.	m.	s.	Authority and remarks.
Hamburg	Rümker	He thinks too early	5	2	26	+ 61	4	23	33	
	Götze		5	2	43	....		23	50	
	Funk		5	2	46	....		23	53	
	Olde		5	2	50	....		23	57	
Geneva	Plantamour		4	47	22	+ 74		23	59	
Marseilles	Valz		4	44	9	+ 79		23	53	
Brussels	Quetelet		4	40	29	+ 64		24	4	
	Houzeau		4	40	29	....		24	4	
	Bouvy		4	40	26	....		24	1	
	Liagre		4	40	22	....		23	57	
Greenwich	H	Through clouds	4	21	59	+ 61		23	0	
	....	Ingress certain	4	22	21	....		23	22	
Portland, Me	C. H. Davis		23	43	54	− 14	4	24	41	The American observations are almost entirely from the unpublished records of the U. S. Coast Survey.
Princeton	Alexander	Internal contact	23	25	18	− 21		23	35	
		Penumbra or black drop broke	23	25	26	....		23	43	
New York	Loomis	A faint line of light began to show itself between the limbs of the planet and sun.	23	27	58	− 21		23	35	
Cobb Hill, near Baltimore.	J. H. Perry?		23	18	22	− 24	....			
West Point	Bartlett		23	28	33	− 20		24	3	
Charleston, S. C	Gibbes		23	4	51	− 30		24	5	
Cincinnati, Ohio	Mitchell		22	46	18	− 38		23	39	
Portland, Me	C. H. Davis	III. The disc of the planet appeared to elongate or make a bead upon the edge of the sun. Very uncertain.	6	5	36	+123	10	48	40	
Nantucket	Mitchell	Planet sharply defined	6	6	31	+122		48	57	
Cambridge	W. C. Bond		6	2	32	....		49	5	
	G. P. Bond		6	2	31	....		49	4	
Middletown	Aug. W. Smith		5	56	33	+121		49	11	
Princeton	Alexander	Penumbra reappeared	5	48	33	+119		49	10	
	....	Internal contact	5	48	40	....		49	17	
New York	Loomis	A mean of three phases	5	50	56	+119		48	53	
Providence	Caswell		6	1	44	+121		49	21	
Great Meadow Station, Mass.	Bache	Disc of planet appeared to unite into that of the sun.	6	2	20	+121		49	13	
		Real contact?	6	2	48	....		49	41	
Cobb Hill	Perry?		5	41	8	+117	....			
Baltimore	Gould		5	40	46	+117	....			
St. Mary's College, Annapolis, Md.	Veraut		5	40	55	+117	....			
West Point	Bartlett		5	51	28	+120		49	18	
Charleston	Gibbes		5	27	40	+108		49	12	
Cincinnati	Mitchell		5	9	17	+112		49	8	
		FOURTH CONTACTS.								
Princeton	Alexander	Probably last contact; a drop still adhering.	5	51	48	....	10	52	25	
	....	Merc. certainly disappeared	5	52	8	....		52	45	
New York	Loomis	Planet ceased to make a sensible impression.	5	54	29	....		52	26	

1845, MAY 8—Continued.

Place.	Observer.	Contact and description.	Local mean time.			Red. to geocentric phase.	Greenwich mean time of geocentric contact.			Authority and remarks.
			h.	m.	s.	s.	h.	m.	s.	
Providence	Caswell		6	5	14	....	10	52	51	
Portland	Davis		6	8	38	......		51	42	
West Point	Bartlett		5	54	55	....		52	45	
Middletown	A. W. Smith		5	59	58	.....		52	36	
Nantucket	Mitchell		6	9	56	....		52	22	
Charleston	Gibbes		5	30	55	....		52	27	

Studying the results for ingress we may act on the following conclusions:

1. No doubt of a mistake of 10^m in copying the REVAL observations.
2. As RÜMKER considered his observation too early we may reject it.
3. H.'s observation through clouds should be rejected; his certain one retained.
4. DAVIS's observation is probably 1^m in error, but as he gives no description, cannot be safely corrected. We therefore reject it.
5. ALEXANDER's two observations should be separately retained, the sun being high and his description clear.

The mean of the 25 observations of first internal contact is $4^h\ 23^m\ 50^s$.

At egress DAVIS describes his observation as very uncertain, and the time that of first elongation of the planet. The average correction for this phase is $+10^s$; we may apply this and assign ½ weight to his results. We may also assign double weight to MITCHELL, of Nantucket, and the BONDS.

From BACHE's diagram I judge the most probable time of contact is found by applying $+5^s$ to his first observation. The mean, with these modifications, becomes $10^h\ 49^m\ 7^s$. We thus have,

	h.	m.	s.
Contact II,	4	23	50
III,	10	49	7

For fourth contacts we may include ALEXANDER's two observations separately and reject the Portland observation as certainly in error. We then have

Contact IV, $10^h\ 52^m\ 35^s$.

1848, NOVEMBER 8.

Place.	Observer.	Contact and description.	Local mean time. h.	m.	s.	Red. to geocentric phase. s.	Greenwich mean time of geocentric contact. h.	m.	s.	Authority and remarks.
Hamburg	Rümker	Er hält das Moment für genau	23	46	40	− 10	23	6	36	A. N., xxviii, 106—108.
	Weyer	II		46	46			6	42	
	Jürgensen			46	58			6	54	
	Breymann			46	52			6	48	
Altona	Schumacher		23	46	45	− 10		6	48	From 34s to 52s contact doubtful. Then permanent separation.
	Peterson			46	9			6	12	During meridian transit.
	Sonntag			46	33			6	36	
	Olde			46	41			6	44	
Geneva	Plantamour		23	31	33	− 12		6	45	Ib., p. 121.
	Bruderer			31	25	− 11		6	37	
Leiden	Oudemanns		23	24	59	− 12		6	50	A. N., xxix, 154.
Brussels	Quetelet		23	24	29	− 13		6	48	
	Bouvy			24	27			6	46	
	Houzeau	Notablement trop tard		24	59			6	78	Obs. on screen.
Greenwich	Airy		23	7	14	− 14		6	60	Greenwich obs., 1848.
	B	Breaking of drop		7	15			6	61	
	H			6	59			6	45	
	E	Ingress complete		7	4			6	50	
	R			7	13			6	59	
	D	No distortion		7	7			6	53	
	H. B			6	19			6	5	Rejected.
Regent's Park	Hind		23	6	26	− 15		6	48	A. N., xxviii, 110.
Liverpool	Hartnup		23	6	54	− 16		6	38	Greenwich m. t.
Durham	Thomson		23	6	56	− 15		6	41	Do.
Cambridge	Challis		25	6	48	− 14		6	34	M. N., R. A. S., ix, 3.
	Breen			6	47			6	33	Greenwich mean time.
Hartwell	Dell	Well observed	23	3	57	− 15		6	66	M. N., R. A. S., ix, 22.
Princeton	Alexander	III. Dark fringe	23	29	52	− 22	4	28	8	A. N., xxviii, 151.
Princeton	Alexander	IV	23	31	36		4	29	52	
	Loomis	IV	23	31	32		4	29	48	

Rejecting the doubtful observations of PETERSON and HOUZEAU, and the discordant one of H. B. at Greenwich, we may take the mean of all the others. We thus have—

			h.	m.	s.	
Contact	II,	Nov. 7,	23	6	47	
	III,	Nov. 8,	4	28	8	(only one observer).
	IV,		4	29	40	

1861, NOVEMBER 11.8.

Place.	Observer.	Contact and description.	Local mean time. h.	m.	s.	Red. to geocentric phase. s.	Greenwich mean time of geocentric contact. h.	m.	s.	Authority and remarks.
Sydney	Scott	II	3	24	34	+ 30	17	20	13	
Hobart Town	Abbot	Doubtful	3	9	36	+ 22		20	37	
Adelaide	Todd	Very exact	2	34	12	+ 23		20	14	
Nicolajew	Knorre		19	29	15	− 19		21	1	A. N., lvi, 336.
Batavia	Oudemanns	On screen	0	27	24	+ 14		20	18	A. N., lvii, 158.
Batavia	Oudemanns	III. On screen	4	25	21	+ 21	21	18	22	Do.
Nicolajew	Knorre, sr		23	27	1	− 48		18	19	A. N., lvi, 336.
	Knorre, jr		23	27	3			18	21	
Pulkowa	Wagner		23	20	23	− 49		18	15	A. N., lvi, 303. Thick clouds. Planet faint.
	Kortazzi		23	20	28			18	20	
Athens	Schmidt		22	54	6	− 47		18	24	A. N., lvi, 315.
Vienna	Werdmuller		22	23	54	− 52		17	30	A. N., lvi, 255.
Malta	Lassell	Good	22	18	6	− 48		19	16	
Berlin	Encke		22	12	51	− 54		18	22	A. N., lvii, 44.
	Förster			12	58			18	29	Do.
	Tietjen			12	48			18	19	Do.
	Romberg			12	56			18	27	Do.
Copenhagen	d'Arrest	Black band 20s before actual contact.	22	9	52	− 54		18	39	
	Schjellerup			9	51			18	38	
Rome	Secchi		22	9	9	− 51		18	23	A. N., lvi, 329.
	Rosa			9	16			18	30	Do.
Leipzig	Bruhns		22	8	36	− 54		18	8	A. N., lvi, 345.
	Engelmann			8	41			18	13	
	v. Zahn			8	42			18	14	
	Auerbach			8	34			18	6	
Padua	Michez		22	6	34	− 52		18	13	N., lvii, 5.
Durham	Chevalier		21	19	13	− 56		18	17	Greenwich mean time.
	Marth			19	14			18	18	Do.
Waterloo	Joynson			19	14	− 55		18	19	Do.
Edge Hill	Jee			20	00	− 55				Greenwich, 2½″ aperture.
Grantham	Jeans			19	18	− 55		18	23	Greenwich mean time.
Manchester	Baxendell			19	9	− 55		18	14	Do.
Liverpool	Hartnup	Line of light formed and broken several times.		19	14	− 56		18	18	Do.
Batavia	Oudemanns	IV	4	27	24	+ 21				
Nicolajew	Knorre, sr		23	29	7	− 48	21	20	25	
	Knorre, jr			29	7			20	25	
Leipzig	Bruhns		22	11	2	− 54		20	34	
	Engelmann			10	49			20	21	
	Auerbach			11	3			20	35	
Pulkowa	Struve		23	22	46	− 49		20	38	
	Kortazzi			22	35			20	27	
Vienna	Werdmuller		22	26	17	− 52		19	53	
Malta	Lassell		22	20	21	− 48		21	31	

1861, NOVEMBER 11.8—Continued.

Place.	Observer.	Contact and description.	Local mean time. h.	m.	s.	Red. to geocentric phase. s.	Greenwich mean time of geocentric contact. h.	m.	s.	Authority and remarks.
Berlin	Encke	IV.	22	15	0	−54	21	20	31	
	Förster			14	58	....		20	29	
	Tietjen			15	3	....		20	34	
	Romberg			14	56	....		20	27	
Copenhagen	d'Arrest		22	41	37	−54		20	24	
	Schjellerup			11	36	....		20	23	
	Thiele			11	44	....		20	31	
Rome	Secchi		22	11	17	−51		20	31	
	Rosa		22	11	12	....		20	26	
Padua	Micher		22	8	42	−52		20	21	
	Segnovi			8	44	....		20	23	
Durham	Chevalier		21	21	13	−56		20	17	Greenwich mean time.
	Marth			21	19	−55		20	23	
Waterloo	Joynson			21	9	....		20	14	
Edge Hill	Jee			21	20	....		20	25	
Grantham	Jeans			21	24	....		20	29	
Manchester	Barendall			21	23	....		20	28	
Liverpool	Hartnup			21	26	....		20	30	

The observations of second contact are so scanty and uneven as to require some care in treatment. KNORRE's observation is indicated as very uncertain, which might well be, as the sun had risen only 23 minutes before, and its altitude was less than 4°. Owing to this, and its discordance, the observation has been rejected. OUDEMANNS observed on a screen, and lost the moment of actual contact by a cloud. But he was able to obtain what he deemed a satisfactory result by subtracting $13^s.8$ from the time when he concluded the thickness of the thread of light to be 1″. The best combination appears to be to assign weights as follows: SCOTT 1, ABBOT $\frac{1}{3}$, TODD 2, OUDEMANNS 1. The mean result will then be $17^h\ 20^m\ 16^s$.

Among the third contacts the Edge Hill observation is clearly too late, while it may be assumed that the observation of LASSELL is affected by an error of one minute. We might suspect the same error in WERDMÜLLER were it not that his observation of external contact cannot be thus reconciled with the others. Correcting LASSELL, and rejecting JEE and WERDMÜLLER, the agreement is excellent and the general mean result is $21^h\ 18^m\ 20^s$.

The concluded results now become—

		h.	m.	s.
Contact	II, Nov. 11,	17	20	16
	III,	21	18	20
	IV,	21	20	27

TRANSIT OF NOVEMBER 4, 1868.

In this transit the attention of observers was attracted to the optical phenomena of contact more fully than in any preceding one. Although it had been long wel understood that the outlines of Venus and the sun did not always preserve their geometric form at the time of contact, few observers were conscious of the necessity of noting and describing any distortion that might be perceived, and of stating the exact appearance of the planet at the moment cited as the time of internal contact. In a paper published in the American Journal of Science and Arts for July, 1870,* I have given a collection of the principal observations of egress made at this transit, arranging them in the order of reduced geocentric time. A similar but more complete table is also given by ANDRÉ in his paper on the observation of contacts in transits of Venus and Mercury.†

For our present purpose, the best course seems to be, so far as egress is concerned, to follow the same plan, tabulating the observations in the order of reduced geocentric time, and indicating the observer's description of the phenomena in each case. In general, no distortion was observed at stations where the altitude of the sun was considerable and the atmosphere steady. In such cases there would be no especial phenomena for the observer to describe, and the time noted would naturally be that of true contact.

* American Journal of Science and Arts, second series, vol. l, page 80.
† Annales de l'Observatoire de Paris, vol. x, p. B. 2.

1868, NOVEMBER 4.8.

Place.	Observer.	Contact and description.	Local mean time.	Red. to geocentric phase.	Greenwich mean time of geocentric contact.	Authority and remarks.
			h. m. s.	s.	h. m. s.	
Adelaide	Todd	II. Good	2 41 30	−16	17 26 53	M. N., R. A. S., xxix, 89.
Cape	Mann	Do	18 41 35	−34	17 27 06	M. N., R. A. S., xxix, 197.
Pekin	Lepissier	II	1 14 31:	+46	17 29 36:	C. R. Image undulating excessively.
Göttingen	Koldewey	III	21 39 31.0	....	20 59 49	A. N., vol. lxxiii, 95.
Helsingfors	Fabritius	No description	22 39 26.5	+17.2	54	A. N., vol. lxxiii. 191.
Bonn	Oppenheim		21 28 16.5	+2.1	55	A. N., lxxii, 355.
Vienna	Oppolzer	Thread of light broke	22 5 14.5	+6.0	56	Images very bad. A. N., vol. lxxii, 347.
Paris	Rayet	Regular contact	21 9 19.4	−2.0	57	C. R., vol. lxvii, p. 948.
Marseilles	Le Verrier	Sudden black drop	21 21 35.7	−4.0	58	Ibid, p. 922.
Dunkerque	Torquem		21 9 28.5	−0.4	58	
Leiden	P. J. Kaiser	No description	21 17 55	+1.6	58	Images very bad. A. N., vol. lxxiii, 214.
Greenwich	Lynn	First contact of filament	21 0 0.7	−1.4	59	
San Fernando	La Flor		20 35 31.3	−22.0	59	Le Verrier, Annales x, p. B 2.
	Lopez		20 35 31.8	....	59	
	Ruiz		20 35 34.3	....	21 0 1	
	Garrido		20 35 33.8	−22.0	1	

1868, NOVEMBER—Continued.

Place.	Observer.	Contact and description.	Local mean time.	Red. to geocentric phase.	Greenwich mean time of geocentric contact.	Authority and remarks.
			h. m. s.	s.	h. m. s	
Christiania	Geelmuyden		21 42 44.1	+ 9.8	21 0 0	A. N., lxxii, 345.
Göttingen	Copeland		21 39 44.4	... .	2	A. N., lxxiii, 95.
Königsberg	Tischler		22 21 48.6	+12.4	2	A. N., lxxiv, 104.
Pulkowa	Rosen	Regular contact	23 1 2	+19.9	3	Comptes Rendus, 1868, ii, p. 1284
Greenwich	Stone	Very fine dark filament	21 0 55	− 1.4	4	
Paris	André	Regular contact	21 9 27.3	− 2.0	4	
Atalaïa	Liais	Limbs perfectly in contact; no distortion.	18 8 46	−64.3	4	A. N., lxxiii, 209.
Bonn	Wolff	No description	21 28 26	+ 2.3	5	A. N., lxxii, 355.
Rome	Lais	Thread broke.	21 49 59	+ 0.3	5	A. N., lxii, 367.
Pulkowa	Kortazzi	Regular contact	23 1 4	+19.9	5	Comptes Rendus, 1868, ii p 1284.
	Wagner	Do.	23 1 4	+19.9	5	
	Nyrén	Do.	23 1 4	+19.9	5	
	Fuss	Do.	23 1 4	+19.9	5	
Greenwich	Dunkin	Planet suddenly pear-shaped	21 0 6.2	− 1.4	5	
Königsberg	Lorek		22 21 51.9	+12.4	5	A. N., lxxiv, 104.
Paris	Villarceau		21 9 28.9	− 2.0	6	
Altona	C. F. W. Peters	No description	21 39 47.3	+ 6.0	7	Bad images.
Edinburgh	Smyth	Very bad definition	21 0 7	+ 0.4	7	
Christiania	Mohr		21 42 50.8	......	7	A. N., lxii, 345.
Helsingfors	Krüger		22 39 40.7	......	8	A. N., lxxiii, 191.
Hamburg	Kampf		21 39 57	+ 5.6	9	A. N., lxxiv, 43.
Pulkowa	Lebedeff		23 1 8	+19.9	9	
	Döllen		23 1 8	+19.9	9	
	Miroschnit-Schenko		23 1 8	+19.9	9	
	Leskinen		23 1 8	+19.9	9	
Rome	Mancini	Thread broke	21 50 3.9	+ 0.3	9	A. N., lxii, 367.
Bonn	Argelander	No description	21 28 30	+ 2.3	9	A. N., lxxii, 355.
Christiania	Fearnley		21 42 52.8	+ 9.8	9	A. N., lxxii, 345.
	Throndsen		21 42 54.8	......	11	Do.
	Pihl		21 42 56	......	12	Do.
Göttingen	Börgen		21 39 52.9	......	11	A. N., lxxiii, 95.
Greenwich	H. J. Carpenter	Light ceased between limbs	21 0 11.1	− 1.4	10	
Pulkowa	Kasarinoff		23 1 10	+19.9	11	
Greenwich	Criswick	Sudden rupture of ring	21 0 12.8	− 1.4	11	
Paris	Wolf		21 9 33.8	− 2.0	11	
Vienna	Oppolzer	Discs tangent	22 5 31.5	+ 6	12	A. N., lxxii, 347.
Leiden	F. Kaiser	Cont. of elongated image in double image micrometer.	21 18 6.8	+ 2.0	12	A. N., lxxiii, 213.
Rome	Secchi	Thread broke	21 50 5.9	+ 0.3	12	A. N., lxxii, 367.
Durham	Plummer		21 0 12.0	+ 0.2	12	Greenwich mean time.
Marseilles	Stephan		21 21 51.3	− 4.3	12	
Greenwich	J. Carpenter	Thread broke suddenly.	21 0 14.1	− 1.4	13	
Altona	C. A. F. Peters		21 39 53	+ 6.0	13	A. N., lxii, 327.
Vienna	Weiss	No particular phenomenon	22 5 38.5	+ 6.5	13	A. N., lxxiii, 173.
Pulkowa	Struve		23 1 13	+19.9	14	

1868, NOVEMBER—Continued.

Place.	Observer.	Contact and description.	Local mean time. h.	m.	s.	Red. to geocentric phase. s.	Greenwich mean time of geocentric contact. h.	m.	s.	Authority and remarks.
Madrid	Merino	Breaking of thread	20	45	40.4	− 12			14	Uncertainty of three seconds, A. N., lxxii, 356.
Lund	Dunér	Thread broke	21	52	51.6	8.8			15	He adds that at 21 52 43 Mercury had not reached the sun's limb.
Walworth	Buckingham	Sudden black ligament.	21	0	18.8	− 1.4			17	A. N., lxii, 377.
Paris	De la Grye	Very accurate	21	9	39.7	− 1.8			17	
Pulkowa	Sokolov	....	23	1	18	+19.9			19	
Kahlenberg (Vienna.)	Pohl	Mercur vollkommen rund	22	5	45.4	+ 6			20	A. N., lxxiii, 77.
Leiden	Kam	No distortion of planet	21	18	15	+ 2.0			21	A. N., lxxiii, 214.
Madrid	Ventosa	Contact formed suddenly	20	45	47.4	......			21	Observations de confiance. A. N., lxxii, 356.
Göttingen	Klinkerfues	....	21	40	5.6	+ 4.3			24	A. N., lxxiii, 95.
Cuckfield	Knott	....	21	0	25.9	− 1			25	Greenwich mean time.
Leiden	Kaiser	Round images in contact	21	18	19.8	+ 2.0			25	A. N., lxxiii, 213.
Pulkowa	Lindemann	....	23	1	24	+19.9			25	
Uckfield	Prince	Apparent contact (?)	21	0	30	− 1.5	21	0	28	Greenwich mean time.
Bologna	Palagi	....	21	45	54.0	+ 1.0			30	A. N., lxiii, 75.
Greenwich	Lynn	Contact of limbs established. Very doubtful.	22	0	32.6	− 1.4			31	
Maidenhead	Lassell	....	21	0	37	− 1.0			36	Greenwich mean time.
Wimbledon	Penrose	Thread of light interrupted	21	0	53.7	− 1.4			52	Greenwich mean time.

The Peking observation is given in so confused a manner that no use has been made of it. The remaining two observations of contact II seem entitled to equal weight.

For contact III there are several sources of doubt connected with the San Fernando observations; they are, therefore, regarded as forming but a single observation. The Wimbledon observation may be rejected as certainly affected with some abnormal error. The mean of the remaining results is taken without discrimination, since no discussion would materially affect the general result to be derived from the whole.

No separate reduction of fourth contacts has been attempted, but the mean result has been taken from the paper by Wolf and André already cited. The concluded results are:

	h.	m.	s.
Contact II,	17	27	0
III,	21	0	9.8
IV,	21	2	33.

TRANSIT OF 1878, MAY 6.

EUROPEAN OBSERVATIONS OF INGRESS.

Place.	Observer.	Contact and description.	Local mean time. h.	m.	s.	Red. to geocentric phase. s.	Greenwich mean time of geocentric contact. h.	m.	s.	Authority and remarks.
Kiel	Weyer	II	3	54	43	+ 94	3	15	41	A. N., xcii, 191.
	C. A. F. Peters			54	42				40	
	C. F. W. Peters	Mercur rund		54	40				38	
		Deutlicher Lichtfaden		54	55				53	
	R. Schumacher	II		54	39				37	
	J. Lamp			54	50				48	
Göttingen	Klinkerfues		4	7	36	+ 94			35	A. N., xcii, 199.
	Boeddicker			7	49				48	Berlin mean time.
	Heidorn			7	49				48	
Anvers	Boë		3	14	15	+ 91			46	A. N., xcii, 223.
	Van Litborn			14	16				45	Greenwich mean time.
Vienna (Josephstadt.)	Oppolzer		4	19	36	+ 98			49	A. N., xcii, 223.
	Kühnert			19	33				46	
	Anton			19	40				53	
Christiania	Geelmuyden	Geschätze oder erwartete Berührung.	3	57	2	+ 94			42	A. N., xcii, 237.
	Mohn	Scheinbare Berührung eher etwas spät angesetzt.		57	5				45	
	Geelmuyden	Erste Spur vom Lichtfaden.		57	12				52	
	Fearnley	Bildung der Lichtbrücke		57	14				54	
	Mohn	Sichere Trennung		57	15				55	
Lund	Dunér	Apparent contact	4	6	41	+ 96			32	A. N., xcii, 283.
	Linstedt	do		6	39				30	
	Dunér	Aeusserst zarter Lichtfaden		6	52				43	
	Linstedt	do		6	52				43	
Breslau	Galle	Geometrische Berührung	4	22	18	+ 98			47	A. N., xcii, 287.
		Völlige Trennung		22	41				70	
	Neugebauer	do		22	40				69	
Prague	Wenzel	II	4	11	31	+ 97			27	A. N., xcii, 287.
	Seydler			11	50				46	
Krakau	Karlinski		4	33	49	+100			39	A. N., xcii, 299.
Königsberg	Luther		4	36	11	+ 99			51	A. N., xcii, 301.
	Franz			36	1				41	
	Benecke			36	14				54	
Berlin	Förster	Planet round and in contact	4	7	38	+ 96			39	A. N., xcii, 319.
		Breiter Lichtfaden		7	56				57	
	Tietjen			7	45				46	
	Bruns	Volkommen kreisförmig		7	52				53	
	Becker	II		7	45				46	
		Deutlich Lichtstreifen		8	3				63	
	Knorre	Apparent contact		7	37				38	
		Erscheinen eines Lichtfadens		7	43				44	
	Oppenheim	II		7	51				52	
Vienna (Observatory.)	Holetschek		4	19	46	+ 98			52	A. N., xcii, 365.
	Zelbr			20	8				74	
	Lukas			19	51				(57)	Aperture, 3″; power, 15.
	Weiss	Planet pear-form		19	24				30	Images very bad.
		Ligament formed		19	29				35	
		Ligament broke		19	53				59	
Gotha	Krueger	Dunkelgraues Band zwischen Mercur und Sonnenrand.	3	56	54	+ 94			47	A. N., xciii, 63.
	Donner	II		56	59				52	

TRANSIT OF 1878, MAY 6—Continued.

Place.	Observer.	Contact and description.	Local mean time. h.	m.	s.	Red. to geocentric phase. s.	Greenwich mean time of geocentric contact. h.	m.	s.	Authority and remarks.
Strassburg	Winnecke	Innere (geometrische) Berührung; keiner der Beobachter hat jedoch irgend welche auffallende Erscheinungen notirt.	3	45	15	+ 92	3	15	44	A. N., xcii, 337.
	Küstner			45	17	....			46	
	Hartwig			45	17	....			46	
	Elkin			45	33	....			62	
Wimbledon	Penrose	Planet apparently circular but clinging to limb.	3	14	32	+ 88			60	M. N., xxxviii, 404.
		Two thin threads of light		14	36	....			64	Greenwich mean time.
Orwell Park	Plummer	Light permanently established in rear of planet.	3	14	20	+ 88			48	M. N., xxxviii, 413.
Dunecht	Ranyard	The disks in geometrical contact	6	2	16	+ 86			60	M. N., xxxviii, 414.
	H. J. Carpenter	Mercury fully on. Thread of light		2	14	....			58	Sidereal time.
	Lohse	Contact certainly past; probably late by 15s to 20s.		2	17	....			61	
	Copeland	Apparent geometric contact		2	7	....			51	
		Distant rupture of ligament		2	19	....			63	
O-Gyalla	Horváth	....	3	14	31?	+ 99			70:	M. N., xxxviii, 427.
	Cvet	....		13	58?	....			37:	Greenwich mean time.
	Kaiser	....		13	44	....			23	
	Schrader	....		13	45	....			24	
	Konkoly	....		13	45	....			24	
Glasgow	Bowden		3	14	16	+ 85			41	M. N., xxxix, 167.
Toulouse	Perrotin	....	3	19	52	+ 80:			21	
Palermo	Tacchini	....	4	7	47	+ 86:			47	
Pulkowa	O. Struve	....	5	15	32	+ 102			55	St. P. Mélanges, v, 551.
	Dubjago	Tropfenphänomen		15	32	....			55	
	Lewitski	do		15	35	....			58	
	Lindemann	....		15	31	....			54	
	Nyrén	....		15	35	....			58	
	H. Struve	....		15	30	....			53	
	Döllen	....		15	33	....			56	
	Gladyschew	....		15	24	....			47	
	Zinger	....		15	33	....			56	
	Romheny	....		15	35	....			58	
	Wagner	....		15	21	....			44	
	Hechet	....		15	35	....			58	
	Naraview	....		15	33	....			56	
	Baranow	....		15	18	....			41	

Among these observations, the following may be set down as too late to correspond to any observation of contact as usually made, namely: MOHN, "Sichere Trennung"; GALLE and NEUGEBAUER, "Völlige Trennung"; FÖRSTER, "Breiter Lichtfaden"; BECKER, LOHSE, "contact certainly past."

Nearly the same thing may be said of the "Deutlicher Lichtstreifen" of BECKER and PETERS, but as we have been under the necessity, in other transits, of retaining observations of the threads of light, we may include these in the same class, and in each case, take the mean of the two times given.

In all the remaining cases we may take the indiscriminate mean of all the separate times noted, with the following exceptions.

The observation of LUKAS, at Vienna, is rejected, owing to the totally insufficient

optical power employed. The mean of WEISS's three times is taken as a single observation. Half weight is assigned to the doubtful O-Gyalla observations of HORVÁTH and CVET. We thus find:

Mean result of 73 European observations, $3^h\ 15^m\ 47^s.3$, G. M. T.

Let us now compare this result with that of the American observations. In Appendix II to the Washington Observations for 1876 is an exhaustive discussion by Professor EASTMAN and Mr. H. M. PAUL of 109 observations of this transit made in the United States. The classification of the times of the different phenomena, as described by the observers, is especially complete and instructive. I therefore transcribe the mean results, reduced to Greenwich time.

CONTACT II.

	Times. h. m. s.	No. of obs.
1. Geometric contact with black drop	3 15 26.7	4
2. Phase I	31.7	8
3. Geometric contact without black drop	42.6	3
4. Geometric contact with no statement of black drop	42.9	9
5. First glimmer of light behind planet	49.4	12
6. Phase II	50.2	16
7. No description of phase	51.8	27
8. Breaking of ligament or black drop	52.8	7
9. Closing of line of light	57.0	11
10. Phase III	61.2	14

In this table phases I, II, and III refer to the appearance of the planet at three different times, as shown on a large diagram which had been circulated among the observers.

Phase I, in this diagram, represented the planet as it would appear 12^s before first interior contact, the cusps being separated by nearly one-half the diameter of the planet.

Phase II represented the appearance of the planet at the exact moment of true interior contact.

Phase III represented the appearance 12^s after contact, a broad line of light being formed between the planet and the limb of the sun.

The size of the diagram was such that when placed at the distance of half a mile, the angular magnitude of the planet on the diagram would be the same as that of the real planet in the heavens.

Mr. PAUL's discussion, however, showed that the observations of these outside phases, I and III, were much less accurate than in the case of ordinary contacts, so that they had to be rejected entirely. But he also rejects phase II, which is that of true internal contact, for reasons which he does not fully state. The only phases which he retains in his discussion are those of $\frac{1}{2}(5+9)$, 8 and 7. From these he deduces

Contact III, $3^h\ 15^m\ 52^s.7 \mp 0^s.46$.

Whatever we may say of the correspondence of this result with the time of true contact, it is cannot be considered as the time to be compared with observations of previous transits by other observers. Since we have been obliged to include all observations,

those in which phases were not described as well as those in which phases were described, our proper course is to take the same sort of a mean which would have been taken had the observations been treated in the same manner and not classified as they are. Let us take up the phases in order.

The four observations of geometric contact with black drop, occurring as they did 5^s before phase I, when the cusps were separated by more than the radius of the planet, should be rejected entirely if anything like true contact is sought for. But it may be that they correspond to observations in other transits, and therefore should be retained.

Phase I and III, for reasons already given, may be thrown aside, although their mean corresponds closely to the general mean of the observations. All the other observations seem to correspond closely to the usual observed phases of contacts: the general mean may, therefore, be taken.

It is, however, to be remarked that Mr. PAUL's means, as given above, are taken by weighting the observations according to atmospheric and other conditions. Although this process has not been employed to any considerable extent in the discussion of the preceding transits, we may adopt Mr. PAUL's weighted results as being better, or at least, no worse than indiscriminate means, even for purposes of comparison. We therefore first give to each of the results, from 3 to 9, inclusive, a weight proportional to the number of observations on which it depends, and thus obtain the first result which follows. Next, we show the result when the four observations of geometric contact with black drop:

	h.	m.	s.	
Contact II,	3	15	50.7,	85 obs. (rejecting class 1)
		15	49.6,	89 obs. (retaining class 1)
		15	47.3,	73 obs. (European)

What mean between these results we should choose is largely a matter of judgment. That the means of two so large bodies of observers should differ by 3 seconds shows it hopeless to expect a probable error of less than 2 seconds in the best possible results from observation. In view of the fact that the American observations were generally made with the sun at a higher altitude than in Europe, I have taken $3^h\ 15^m\ 49^s.2$ as the most probable mean.

CONTACT III.

For egress we have the following results from Mr. PAUL's tabular summary:

	Times. h.	m.	s.	No. of obs.
1. Phase III - - - - - - - - - - - - - - - - - -	10	43	15.7	5
2. Phase II - - - - - - - - - - - - - - - - -			36.4	5
3. Geometric contact with nothing about black drop			36.6	8
4. Formation of ligament or drop - - - - - - -			39.2	10
5. Breaking of line of light - - - - - - - - - -			39.6	7
6. Geometric contact without black drop - - - -			39.8	2
7. No description of phase - - - - - - - - - - -			42.2	27
8. Last glimmer of light - - - - - - - - - - -			42.8	8
9. Phase I - - - - - - - - - - - - - - - - -			52.3	5
10. Geometric contact with black drop - - - - -			57.7	7

The curious reversal of the supposed normal order of phases, geometric contacts being noted earlier than last glimmer of light, is noteworthy. The early occurrence of "phase II" (true contact) is also remarkable. Whatever conclusions we may draw from these anomalies, the only course open to us is to take a mean of those phases which we may suppose to correspond to phases observed in preceding transits. We therefore reject phases I and III. It is doubtful whether geometric contact with black drop should be retained or rejected.

	h.	m.	s.	Obs.
Retaining (10) we have - - - -	10	43	42.0	74
Rejecting (10) we have - - - -		43	40.4	67

We may accept the mean of the two as the result to be accepted.

For external contact an indiscriminate mean is taken in the usual way, rejecting a few observations with very insufficient optical power. The results then are:

	h.	m.	s.
Contact II, 1878, May 6,	3	15	49.2
III,	10	43	41.2
IV,	10	46	23

TRANSIT OF 1881, NOVEMBER 7.6.

Place.	Observer.	Contact and description.	Local mean time. h.	m.	s.	Red. to geocentric phase. s.	Greenwich mean time of geocentric contact. h.	m.	s.	Authority and remarks.
Melbourne........	Ellery..........	II. Internal contact; good........	19	58	44	− 10	10	18	39	Communicated by Mr. ELLERY;
		Black drop broken................		59	8	....		19	3	also, M. N., xlii, 101.
	White..........	Thread of light..................		58	47	....		18	42	
	Moerlin........	Complete separation		59	43	....		19	38	
Windsor..........	Tebbutt........	Contact nearly made..............	20	22	10	− 11		18	37	Communicated by Mr. TEBBUTT;
		Band of light distinct...........		22	16	....		18	43	also, l. c.
Sydney..........	Russell........	Unsatisfactory...................		23	36	− 11		18	34	Contact made and broken several times during 10ˢ.
	Lenehan.......	No description...................		23	43	....		10	41	
	Wright.........	......do.........................		23	10	....		18	8	
	Morrice.......	......do.........................		23	46	....			44	
	Hargrave......	....		23	41	....		18	39	Definition good.
	Bladen.........	Contact expected.................		23	25	....		18	23	The Sydney observations were
		Band of light distinct...........		23	43	....		18	41	all communicated by Mr. RUSSELL.
	Conder.........	Limbs tangential.................		23	36	....		18	34	
	Brooks.........	First indication of white line.....		23	86	....		18	34	
Honolulu.........	Rockwell.......	II...............................	22	18	26	− 6		18	10	
M. Hamilton......	Holden.........	II...............................	2	12	26	− 5		18	53	Power 50. Communicated by
	Burnham.......	II...............................	2	12	6	− 5		18	33	Power 300. Professor HOLDEN.
Melbourne........	Ellery..........	III. Thin flickering line connecting planet and limb.	1	15	6	+ 16	15	35	27	
		Internal contact.................	1	15	33	....		35	54	
	White..........	....		15	30	....			51	
	Moerlin........	....		15	34	....			55	
	Turner.........	Filament assumed a solid form....	1	15	23	....			44	
		Internal contact past............		16	4	....			85	
Windsor..........	Tebbutt........	Thread of light very fine.........	1	38	54	− 20			52	
		Contact complete.................			58	....			56	

TRANSIT OF 1881, NOVEMBER 7—Continued.

Place.	Observer.	Contact and description.	Local mean time.			Red. to geocentric phase.	Greenwich mean time of geocentric contact.			Authority and remarks.
			h.	*m.*	*s.*	*s.*	*h.*	*m.*	*s.*	
Sydney	Russell	Contact good	1	40	27	+ 20			56	Clean contact and rupture of band of light.
	Lenehan	Definition not good		40	29				58	
	Wright			40	23				52	
	Hargrave			40	10				39	Moderately good.
	Bladen	Contact surely complete		40	44				73	Boiling badly.
	Brooks	No black drop		40	27				56	
Honolulu	Rockwell	III	15	35	36	+ 36	15	36	12	Comm. by Mr. ROCKWELL.
Melbourne	Ellery	IV	1	17	13			37	34	
	White		1	17	10			37	32	
	Moerlin		1	17	6			37	28	
	Turner		1	17	7			37	28	
Windsor	Tebbutt		1	40	56			37	35	
Sydney	Russell		1	42	9			37	38	
	Lenehan			42	4			37	33	
	Wright			42	3			37	32	
	Morrice			42	16			37	45	
	Hargrave			42	0			37	29	
	Bladen			42	24			37	53	
	Brooks			42	9			37	38	
Honolulu	Rockwell			36	23			37	4	

Guided by the light thrown on the case by previous observations, we shall endeavor to deduce a time of contact from the statements of each observer, and assign weights according to the apparent certainty of the results.

Whether Mr. ELLERY'S "good" internal contact is to be regarded as a true mean contact it is difficult to say. The breaking of the black drop may be regarded as certainly later than mean contact. On the whole, the most probable result seems to be that obtained by adding to the time of the first observation one-third of the interval between it and the second. This will give $10^h\ 18^m\ 47^s$, Greenwich time.

From Mr. WHITE'S observation we may subtract 5^s, giving 37^s.

MOERLIN'S "complete separation" is clearly too late.

TEBBUTT'S description is excellent; his mean time is 40^s.

During the 10^s, from 24^s to 34^s, RUSSELL saw the thread of light formed and broken several times. This is what should take place in the situation corresponding to true contact. We may, therefore, take 29^s as his result instead of 34^s.

The four observers following RUSSELL give no indications for judging their results. WRIGHT is so obviously too early that it is doubtful whether his result should not be rejected. A mean course will be to assign him half weight. The mean result of the four will then be 37^s.

BLADEN'S two descriptions clearly bound the possible times of contact. The mean 32^s seems good. The next two results may also be accepted unchanged.

With ROCKWELL'S Honolulu observation we have the same trouble as with WRIGHT'S at Sydney. In doubt whether to reject it, we can assign it only small weight.

We now have the following single or combined results for probable mean contact II:

	h.	m.	s.	Wt.
ELLERY	10	18	47	2
WHITE			37	2
TEBBUTT			40	3
RUSSELL			29	1
Four other observers			37	3
COUDER			34	2
BROOKS			34	2
ROCKWELL			10	½
HOLDEN			53	2
BURNHAM			33	2
Mean	10	18	38	

Of the third contacts, only the following call for special remark.

Mr. ELLERY's "thin flickering line" was probably the result of atmospheric softening of the thin line of light just before contact, and so not to be regarded as a true contact of any kind.

TURNER's description corresponds accurately to a true contact.

TEBBUTT's description close limits the time of contact.

BLADEN's result seems valueless. The images were too bad to permit of an obser vation of contact.

With ROCKWELL's we have the same trouble as at ingress, only he is now late. We should, perhaps, treat his observation in the same way as at ingress.

In the combination we give double weight to TEBBUTT and RUSSELL. The mean result is then $15^h\ 35^m\ 54^s$.

The results for geocentric contact then are—

	h.	m.	s.
Contact II, 1881, Nov. 7,	10	18	38
III,	15	35	54
IV,	15	37	33

PART II.

COMPUTATION OF TABULAR ELEMENTS.

LEVERRIER's tables of Mercury and the sun have been adopted as the medium for obtaining the results of the preceding observations, in so far as the elements to be corrected depend upon the relative heliocentric positions of Mercury and the earth. The method of deriving the times of contact and other results from the tabular positions is not the usual one, since the computation of the geocentric place of the planet is entirely dispensed with. Not only would the computation of this place involve considerable additional labor with great liability to errors, but the symbolic expressions of the corrections to the theory would also have been more troublesome in computation. A method has, therefore, been adopted which is founded on BESSEL's theory of eclipses. By this method the time of contact is defined as that when the observer is on the conical surface touching the planet and the sun. Of the two circumscribing cones, that whose vertex is between the interior planet and the earth will correspond to internal contact, and that whose vertex is between the planet and the sun to external contact.

§ I

Determination of times of contact during transits of an inferior planet by the heliocentric method, using the conceptions of BESSEL's *method of eclipses.*

By this method the time of contact is fixed as that when the observer is upon the surface of one of the two shadow cones touching the sun and the planet. The axis of the cone is the line joining the centers of the sun and planet. The fundamental plane of reference, or the plane of XY, passes through the center of the earth perpendicular to the axis of the cone.

The axis of X in the present case is formed by the intersection of the ecliptic with the fundamental plane, its positive direction being toward the west or the opposite of that in which planets are moving. The axis of Y is, as usual, toward the north.

Let us put:

r, b, l, the heliocentric radius vector, latitude, and longitude of the planet Mercury or Venus.

r', b', l', the same quantities for the earth.

c, the angular distance of the planet and earth as seen from the center of the sun.

ω, the position angle of the point in which the shadow-axis intersects the plane of XY, counted from the axis of X towards that of Y.

R′, the linear radius of the sun.
R, the linear radius of the planet.
f, the semi-angle of the shadow cone.
ρ_0, the radius of the cone on the plane of reference.

EFFECT OF ABERRATION.

The preceding quantities require some modification in their employment owing to the motion of the planets during the interval required by light to reach them from the sun. When we compute the place of the sun from the tables for a time T, the tables are so constructed as to give its apparent direction at this moment. If τ_2 be the time required for light to reach the earth from the sun, the position we shall get from the tables for the time T will be the true one at the time $T - \tau_2$. By subtracting 180° from the longitude and changing the algebraic sign of the latitude we shall have the true direction of the earth from the sun at the moment $T - \tau_2$.

The condition that the planet, as seen from the earth, shall appear projected upon a given point of the sun's disk at a moment T′, is that a ray of light, emanating from the given point at a certain moment, shall pass through the planet and through the earth, reaching the latter at the moment T'. So, putting τ_1 for the time required for the ray to reach the planet, and referring positions to the sun, the true heliocentric position of the planet at the moment $T' - \tau_2 + \tau_1$ and that of the earth at the moment T′, must be in the same straight line from the given point on the sun's disk.

Therefore, in order that the positions of the earth and planet may be comparable for a moment T′, we must use the true heliocentric position of the earth at this moment. This is obtained either by computing the apparent position from the tables for the moment $T' + \tau_2$, or taking the apparent position as computed for the time T′, and increasing the longitude by the aberration in longitude.

The corresponding position of the planet must be the true one for the moment $T' - \tau_2 + \tau_1$. If, therefore, it is computed from the tables for the moment T′ the co-ordinates must be corrected by subtracting the motion during the interval $\tau_2 - \tau_1$, which we may take as the interval required for the light to pass from the planet to the earth.

Value of $\tau_2 - \tau_1$. If we put

$$\tau = \tau_2 - \tau_1 = \frac{497^s.8}{3600} (r' - r)$$

which may be considered as the time required for light to pass from the planet to the earth expressed in hours, the required corrections to the place of the planet will be

$$\delta l = - \tau \frac{dl}{dt}$$

$$\delta b = - \tau \frac{db}{dt}$$

$$\delta r = - \tau \frac{dr}{dt}$$

the differentials being the hourly motions of the several co-ordinates of Mercury.

In what follows these corrections will be supposed to be applied to the co-ordinates of the planet, and the corresponding ones to the positions of the earth as derived from the solar tables. That is, if we compute the tabular places of both the planet and earth for a moment T of absolute time, we must apply the preceding corrections to the heliocentric place of the planet, and add the aberration to the longitude of the earth in order that the quantities may be comparable for the moment T.

The co-ordinates x, y, and z of the center of the earth are given by the formulæ

$$x = \mp r' \sin c \cos \omega$$
$$y = r' \sin c \sin \omega$$
$$z = o$$

The angles c and ω are given by the equations

$$\begin{aligned} \sin c \cos \omega &= \cos b' \sin (l - l') \\ \sin c \sin \omega &= \cos b \sin b' - \sin b \cos b' \cos (l - l') \end{aligned} \tag{1}$$

For these equations we may put, owing to the minuteness of b', b, and $(l - l')$

$$\begin{aligned} \sin c \sin \omega &= \sin (b' - b) \\ \sin c \cos \omega &= \sin (l - l') \end{aligned} \tag{1$'$}$$

without an error exceeding $0''.01$ in a transit. Or, yet more simply, we may suppose

$$\begin{aligned} c \sin \omega &= b' - b \\ c \cos \omega &= l - l' \end{aligned} \tag{1$''$}$$

without introducing an error of which the average value will exceed $0'',01$.

By these formulæ the values of x and y, for the earth's center, may be computed for any required moment. Their derivatives with respect to the time will be given with sufficient approximation by the formulæ

$$\frac{dx}{dt} = r' \left(\frac{dl}{dt} - \frac{dl'}{dt} \right)$$
$$\frac{dy}{dt} = r' \left(\frac{db'}{dt} - \frac{db}{dt} \right)$$

the effect of the change of r' being unimportant. But the quantity $\frac{db'}{dt}$ may always be regarded as insensible.

For the radius of the shadow cone on the fundamental plane we have, by the theory of eclipses,

$$\rho_0 = r' \cos c \tan f \;\; \mathrm{R}' - \sec f$$

The value of f may be obtained by the consideration that the radius of the cone on a plane passing through the center of the sun is $-$ R′ sec f, while, on a plane passing through the center of the planet, it is $\pm$ R sec f, the positive sign holding for the cone whose vertex is between the sun and planet, and the negative sign for that

whose vertex is between the earth and planet. The former is the cone of external and the latter that of internal contact. The difference of these radii is $r \tan f$, that is,

$$r \tan f = (\mathrm{R}' \pm \mathrm{R}) \sec f \cdot$$

or

$$\sin f = \frac{\mathrm{R}' \pm \mathrm{R}}{r}$$

The value of the radius ρ_0 may now be expressed in the form

$$\rho_0 \cos f = \frac{r'}{r} (\mathrm{R}' \pm \mathrm{R}) \cos c - \mathrm{R}'$$

At the earth's center the condition of contact of the limb of the planet with that of the sun is

$$r' \sin c = \rho_0$$

or, dividing by r' and substituting the value of ρ_0,

$$\sin c = \frac{\rho_0}{r'} = \frac{\cos c}{\cos f}\left(\frac{\mathrm{R}'}{r} \pm \frac{\mathrm{R}}{r} - \frac{\mathrm{R}'}{r'} \sec c\right) \qquad (2)$$

The quantities c and f are so small, and vary so slightly for transits of the same planet at the same node, that their cosines may, in this formula, be supposed to have the same value for all such transits.

The moment at which the condition of geocentric contact is fulfilled may be found by BESSEL's method of eclipses. We select a moment near the time of contact and compute the values of c and ω for this moment from equations (1). Let us call T_0 the moment in question, and c_0 and ω_0 the special values of c and ω thus found. Let us also compute the hourly variations of $c \cos \omega$ and of $c \sin \omega$, which we call $n \cos \omega'$, and $n \sin \omega'$, from the formulæ,

$$n \sin \omega' = \frac{db'}{dt} - \frac{db}{dt} \cdot$$

$$n \cos \omega' = \frac{dl}{dt} - \frac{dl'}{dt}$$

Let us also put

$$\mathbf{r} = \frac{\cos c}{\cos f}\left(\frac{\mathrm{R}'}{r} \pm \frac{\mathrm{R}}{r} - \frac{\mathrm{R}'}{r'} \sec c\right) \qquad (3)$$

and

$$\sin \psi = \frac{c_0 \sin (\omega' - \omega_0)}{\mathbf{r}}$$

then

$$\tau = - \frac{c_0 \cos (\omega' - \omega_0) \mp \mathbf{r} \cos \psi}{n} \qquad (4)$$

Each value of $\mathrm{T}_0 + \tau$ will then be a time of true geocentric contact, the earlier being first, and the later second contact.

By using the two values of $\mathbf{r}$ we shall have four times of contact in all, the one

pair corresponding to external and the other to internal contact The condition to be fulfilled at each contact will be:

$$c = \mathbf{r} \tag{5}$$

By what precedes we have found a moment $T + \tau$ at which this condition is fulfilled at the center of the earth. In the case of an actual observation, the observer is not at the center of the earth, but at its surface, and we have next to find the time at which he is on the surface of the shadow cone, or the reduction from contact at center to contact at his station. If, now, we suppose c_1 to represent the angular distance of the observer from the planet as seen from the sun's center, and all the other quantities which refer to the earth's center to refer to the observer, the same equations will hold true. The difference of directions between the station and the earth's center, as seen from the sun, is simply the sun's parallax in longitude and latitude. If we put ρ, β, λ the observer's distance from the earth's center, in units of the equatorial radius of the earth, and the latitude and longitude of his zenith; π the sun's equatorial horizontal parallax at the date; b'_1, l'_1, r'_1, the latitude, longitude, and distance of the observer, as seen from the sun, we shall have with all necessary precision,

$$\begin{aligned} l'_1 &= l' + \rho \cos \beta \sin \pi \sin (\lambda - l') \\ b'_1 &= b' + \rho \sin \beta \sin \pi \\ r'_1 &= r' \{ 1 + \rho \cos \beta \sin \pi \cos (\lambda - l') \} \end{aligned} \tag{6}$$

or

$$\begin{aligned} l - l'_1 &= l - l' - \rho \cos \beta \sin \pi \sin (\lambda - l') \\ b'_1 - b &= b' - b + \rho \sin \beta \sin \pi \\ r_1' - r' &= r' \rho \cos \beta \sin \pi \cos (\lambda - l') \end{aligned}$$

Substituting these values in (1)″, noting that the parallax is so small that the change in the signs of $b' - b$ and $l - l'$ may be taken the same as in the arcs themselves, and putting c_1 and ω_1 for the values of c and ω which refer to the station, we shall have

$$\begin{aligned} c_1 \sin \omega_1 &= c \sin \omega + \rho \sin \beta \sin \pi \\ c_1 \cos \omega_1 &= c \cos \omega - \rho \cos \beta \sin \pi \sin (\lambda - l') \end{aligned} \tag{7}$$

From these equations the varying values of c and ω may be computed for any place at any moment. But what we really want is the time at which c_1 has the value $\mathbf{r}_1$, the latter being the value of $\mathbf{r}$ when r_1' is substituted for r' in (3). By this substitution we find, to quantities of the first order

$$\begin{aligned} \mathbf{r}_1 - \mathbf{r} &= \frac{R'}{r'} \rho \cos \beta \sin \pi \cos (\lambda - l') \\ &= \rho \sin S \cos \beta \sin \pi \cos (\lambda - l') \end{aligned}$$

where we put S for the sun's geocentric angular semi-diameter. At the moment of geocentric contact we have $c = \mathbf{r}$, while at the required moment of local contact we must have

$$c_1 = \mathbf{r}_1$$

and the problem is to find the interval after geocentric contact when this will occur. This interval will be so small that we may regard the quantities in the equations (7) as varying uniformly through its extent. If, then, we put

$$\begin{aligned} q &= + \rho \sin \beta \sin \pi \\ p &= - \rho \cos \beta \sin \pi \sin (\lambda - l') \end{aligned} \tag{8}$$

the quantities being taken for the moment of geocentric contact, the equation (7) gives, for the condition of local contact,

$$\left.\begin{aligned}\mathbf{r}_1 \sin \omega_1 &= \mathbf{r} \sin \omega + q + \left(\frac{dq}{dt} + n \sin \omega'\right) t \\ \mathbf{r}_1 \cos \omega_1 &= \mathbf{r} \cos \omega + q + \left(\frac{dp}{dt} + n \cos \omega'\right) t\end{aligned}\right\} \qquad (9)$$

and the problem is reduced to finding the values of ω_1 and t from these equations. The first will be the local position-angle of the two bodies at the moment of local contact, and t will be the correction to reduce geocentric to local contact. The latter quantity is, in fact, the only one that is wanted, since the position-angle ω cannot be observed.

The preceding equations can, if desired, be rigorously solved for any station by BESSEL'S eclipse formulæ, as follows:

$$\mathbf{r}_1 = \mathbf{r} + \rho \sin S \sin \pi \cos \beta \cos (\lambda - l')$$

Find m and M from the equations:

$$\begin{aligned} m \sin M &= \mathbf{r} \sin \omega + q \\ m \cos M &= \mathbf{r} \cos \omega + p \end{aligned} \qquad (10)$$

and m' and N, ψ and t, from the equations:

$$m' \sin N = n \sin \omega' + \frac{dq}{dt}$$

$$m' \cos N = n \cos \omega' + \frac{dp}{dt}$$

$$\sin \psi = \frac{m \sin (M - N)}{\mathbf{r}_1}$$

$$t = \frac{\mathbf{r}_1 \cos \psi - m \cos (M - N)}{m'}.$$

If T is the moment of geocentric contact, T + t will be that of local contact.

In most cases, however, an approximate method leading to the usual formulæ will be sufficient. The quantities t and q are, when at their maximum, smaller than $\mathbf{r}$ in the ratio of the radius of the earth to that of the shadow cone, or roughly in transits of Mercury, 1:150 in a May transit and 1:250 in a November transit. Also, p' and q' are smaller than n in about the same ratio. Therefore, the development in powers of these quantities will converge very rapidly.

Squaring the equations (9) and taking the sum, in order to eliminate ω' we have, omitting the second powers of the small quantities p, q, $p't$, $q't$,

$$\mathbf{r}_1^2 = \mathbf{r}^2 + 2\,\mathbf{r}\,(p \cos \omega + q \sin \omega) + 2\,\mathbf{r}\,nt \cos (\omega - \omega')$$

or

$$\mathbf{r}_1^2 - \mathbf{r}^2 = 2\,\mathbf{r}\,(p \cos \omega + q \sin \omega) + 2\,\mathbf{r}\,nt \cos (\omega - \omega') \qquad (11)$$

The quantity $\mathbf{r}_1^2 - \mathbf{r}^2 = (\mathbf{r}_1 + \mathbf{r})\,(\mathbf{r}_1 - \mathbf{r})$ is very small. $\mathbf{r}_1$ may be derived from

the value of $\mathbf{r}$ in (3) by substituting r_1' for r'. We thus find from the expression for r_1' in (6), neglecting the quotient of the radius of the planet divided by that of the sun,

$$\mathbf{r}_1 - \mathbf{r} = \frac{R'}{r'} \rho \sin \pi \cos \beta \cos (\lambda - l')$$

But, $\frac{R'}{r'}$ is the sine of the sun's angular semi-diameter as seen from the earth, which we have called S. Therefore,

$$\mathbf{r}_1 - \mathbf{r} = \rho \sin S \sin \pi \cos \beta \cos (\lambda - l')$$

Thus we have, with all necessary exactness,

$$\mathbf{r}_1^2 - \mathbf{r}^2 = 2\,\mathbf{r}\,(\mathbf{r}_1 - \mathbf{r}) = 2\,\mathbf{r} \sin S\, \rho \sin \pi \cos \beta \cos (\lambda - l')$$

It is scarcely possible, from any number of observations of a transit of Mercury, to obtain the time of contact without an uncertainty of more than a second of time. The error from neglecting all the powers of $\frac{p}{r}$ and $\frac{q}{r}$ will rarely amount to a second, except when the least geocentric distance of centers is nearly as great as the sun's semi-diameter. In this case it will be more convenient to use the rigorous formulæ (10). In all other cases we may take the approximate expression (11). So substituting the above value of $\mathbf{r}_1^2 - \mathbf{r}_2$ in (11), and solving with respect to t, we find

$$t = \frac{\rho \sin S \sin \pi \cos \beta \cos (\lambda - l') - p \cos \omega - q \sin w}{n \cos (\omega - \omega')}$$

Substituting for p and q their values, this expression reduces to

$$t = \frac{\rho \sin \pi \{\sin S \cos \beta \cos (\lambda - l') + \cos \omega \cos \beta \sin (\lambda - l') - \sin \omega \sin \beta\}}{n \cos (\omega - \omega')}$$

The quantities β and λ, which represent the latitude and longitude of the observer's geocentric zenith, are given by equations:

$$\begin{aligned} \cos \beta \cos \lambda &= \cos \varphi' \cos \tau \\ \cos \beta \sin \lambda &= \cos \varphi' \cos \varepsilon \sin \tau + \sin \varphi' \sin \varepsilon \\ \sin \beta &= \sin \varphi' \cos \varepsilon - \cos \varphi' \sin \varepsilon \sin \tau \end{aligned}$$

where ε is the obliquity of the ecliptic, φ' the observer's geocentric latitude and τ the local sidereal time. If we substitute these values in the expression for t, it may be expressed in the form

$$t = A\rho \sin \varphi' + B\rho \cos \varphi' \cos \tau + C\rho \cos \varphi' \sin \tau \qquad (12)$$

where

$$A = \frac{\sin \pi}{n \cos (\omega - \omega')} \{- \cos \varepsilon \sin \omega + \sin \varepsilon (\cos \omega \cos l' + \sin S \sin l')\}$$

$$B = \frac{\sin \pi}{n \cos (\omega - \omega')} \{- \cos \omega \sin l' + \sin S \cos l'\}$$

$$C = \frac{\sin \pi}{n \cos (\omega - \omega')} \{\sin \varepsilon \sin \omega + \cos \varepsilon (\cos \omega \cos l' + \sin S \sin l')\}$$

In these equations the coefficient sin S is only 0.0046; it is, therefore, of the same order of magnitude with quantities which we have neglected. It may, however, be readily taken into account by substituting for l' the quantity $l' - \sin S \sec \omega = l' - 16' \sec \omega$, which corrected value of l' we may call l''. Developing to quantities of the first order with respect to the small quantity S, we have

$$\cos \omega \sin l'' = \cos \omega \sin l' - \sin S \cos l'$$
$$\cos \omega \cos l'' = \cos \omega \cos l' + \sin S \sin l'$$

Making these substitutions, and putting π instead of $\sin \pi$, the values of A, B, and C will become

$$A = \frac{\pi}{n \cos (\omega - \omega')} \{\sin \varepsilon \cos \omega \cos l'' - \cos \varepsilon \sin \omega\}$$

$$B = - \frac{\pi}{n \cos (\omega - \omega')} - \cos \omega \sin l''$$

$$C = \frac{\pi}{n \cos (\omega - \omega')} \{\sin \varepsilon \sin \omega + \cos \varepsilon \cos \omega \cos l''\}$$

The computations of these quantities and of the final values of t in (12) may be most readily effected as follows. From

$$p \sin P = \sin \omega$$
$$p \cos P = \cos \omega \cos l''$$

find p and P Then,

$$A = \frac{p\pi \sin (\varepsilon - P)}{n \cos (\omega - \omega')}.$$

From

$$k \sin K = p \cos (\varepsilon - P)$$
$$k \cos K = - \cos \omega \sin l''$$

find k and K. Then,

$$D = \frac{\pi k}{n \cos (\omega - \omega')}$$

and

$$t = A\rho \sin \varphi' + D\rho \cos \varphi' \cos (K - \tau).$$

Here we may take for τ the local sidereal time of geocentric contact. If we put L, the west longitude of the place, τ_0, the Greenwich sidereal time of contact, expressed in arc, we shall have

$$\tau = \tau_0 - L$$
$$t = A\rho \sin \varphi' + D\rho \cos \varphi' \cos (K - \tau_0 + L) \qquad (13)$$

which is equivalent to the usual formula.

§ 2.

Reductions of the constants in the preceding formula to numbers.

Having a number of transits to reduce, the work will be facilitated by reducing the preceding formulæ, so far as possible, to numbers. The following are the adopted values of the semi-diameters of the sun and Mercury.

$$\text{Sun:} \quad R' = \sin 959''.78 = .00465316$$
$$\text{Mercury:} \quad R = \sin \quad 3''.30 = .00001600$$

In forming $\mathbf{r}$ from (3) the cosines of the small angles c and f vary so slightly that they may be assumed to have the same values for all May transits and for all November transits. The principal quantities to be used are as follows:

	May transits.	November transits.
r', (approximate) - - - - - -	1.0096	0.9904
r, (approximate) - - - - - -	0.4516	0.3139
$\sin f$ (external contact) - - -	0.01033	0.01487
$\sin f$ (internal contact) - - -	0.01026	0.01477
$\sin c$ (external contact) - - -	0.00572	0.01018
$\sin c$ (internal contact) - - -	0.00565	0.01008
$\log \cos c \sec f\,(R' + R)$ (in sec). - -	2.983678	2.983688
$\log \cos c \sec f\,(R' - R)$ (in sec). - -	2.980693	2.980702
$\log R' \sec f$ (in sec). - - - - - - -	2.982196	2.982220

We therefore have from (3) the following numerical formulæ for r in seconds:

$$\text{May transit, external contact,} \quad \mathbf{r} = \frac{[2.983678]}{r} - \frac{[2.982196]}{r'}$$

$$\text{May transit, internal contact,} \quad \mathbf{r} = \frac{[2.980693]}{r} - \frac{[2.982196]}{r'}$$

$$\text{November transit, ext. contact,} \quad \mathbf{r} = \frac{[2.983688]}{r} - \frac{[2.982220]}{r'}$$

$$\text{November transit, int. contact,} \quad \mathbf{r} = \frac{[2.980702]}{r} - \frac{[2.982220]}{r'}$$

The following are the extreme values of the aberration time, from which intermediate values can be found as with the argument $\log r$:

$$\text{May transits} - - \begin{cases} \log r = 9.6522; & \tau_2 - \tau_1 = 279^{s}.1 \\ \log r = 9.6580; & \tau_2 - \tau_1 = 276.1 \end{cases}$$

$$\text{November transits} \begin{cases} \log r = 9.4942; & \tau_2 - \tau_1 = 337.8 \\ \log r = 9.4997; & \tau_2 - \tau_1 = 335.6 \end{cases}$$

Taking $8''.848$ as the mean solar parallax, we have

For a May transit: $\log \pi = 0.9427$.
For a November transit: $\log \pi = 0.9510$.

Reductions to external contact.

For reasons already given, only last external contacts have been used. Instead of computing them independently, they have been obtained from those of last internal contact by computing the difference of times of the two phases. Taking the difference between the two values of r for the two classes of contact we find them to be:

November transits: $\Delta \mathbf{r} = 21''.02$.
May transits: $\Delta \mathbf{r} = 14''.61$.

If we represent by Δ the changes of the quantities to reduce from internal to external contact, we have

$$c\Delta c = (l - l')(\Delta l - \Delta l') + (b' - b)(\Delta b' - \Delta b)$$

By substituting

$$\Delta l = \frac{dl}{dt}\Delta t, \text{ etc.,}$$

we find

$$\Delta c = \left\{ \frac{l - l'}{c}\left(\frac{dl}{dt} - \frac{dl'}{dt}\right) + \frac{b' - b}{c}\left(\frac{db'}{dt} - \frac{db}{dt}\right) \right\} \Delta t = n \cos(\omega - \omega')\,\Delta t$$

The condition of contact being $\Delta c = \Delta \mathbf{r}$, we have, for the values of Δt

$$\text{For November transits, } \Delta t = \frac{21''.02}{n \cos(\omega - \omega')}$$

$$\text{For May transits, } \Delta t = \frac{14''.61}{n \cos(\omega - \omega')}$$

These corrections are to be applied to the times of internal contact to reduce them to those of external contact.

§ 3.

Tabular heliocentric positions of Mercury and the earth.

The tabular heliocentric positions of Mercury and the earth are derived from LEVERRIER'S tables in Vols. IV and V of his *Annales de l'Observatoire.* They are exhibited in the following table.

The first two columns give the dates and the Paris and Greenwich mean times of computation. In the second column the first number is the Paris time which was used unchanged in the computations. It was originally intended to choose this time so as to correspond to the nearest hour and sometimes the nearest simple fraction of an hour to the observed contact. But as the work was performed before the contacts were carefully examined this condition is frequently not fulfilled.

The second number of this column gives the Greenwich time corresponding to the Paris time.

The third number gives the Greenwich time as corrected for aberration, and is less than the time of computation by the interval required for light to pass from the planet to the earth.

The aberration time given in the last column is the time required for light to pass from Mercury to the earth. Along with it is given the motion of Mercury in longitude during this interval. As already stated, and as will be presently shown more fully, the longitude of Mercury is to be ultimately referred to a moment at which the ray of light, indicating contact to the observer, passed it.

The third column gives in the first line of each set the longitude of Mercury referred to the mean equinox of the date for a moment earlier than the first mean time in column two by the aberration time. The longitude was first computed for the given Paris time, and the motion in longitude during the aberration time was then subtracted. Hence, to find the longitude for the given Paris time, the correction given in the last column must be applied positively to the longitude of Mercury in the second column.

Under each longitude of Mercury is given the longitude of the earth, which is also referred to the mean equinox, for the moment first indicated in the second column. The tabular longitude of the sun is freed from the effect of aberration in order to have the longitude of the earth for the given moment.

Under each pair of longitudes is given their difference found by simple subtraction.

The latitudes of Mercury and the earth in the fifth column are given in the same way as the longitudes; that is, the latitude of Mercury is that which corresponds to the moment found by subtracting aberration time from the time of computation in the second column, and which is the third of the given times. The hourly motions in longitude and latitude are computed for each date independently by means of the differences in the tables. Hence, when two places are given for an interval of a few hours the hourly motions may be used to check the computations.

It may, however, be remarked that these hourly motions are those obtained for the original moment of computation without respect to the aberration time. Hence, to correspond strictly to the longitude, they should be reduced for aberration time. The necessary reduction is so small that no account has been taken of it.

In the column "Perturbations by Venus" are given the perturbations of Mercury and of the earth, and also their differences. Since the phenomena of contact depend entirely upon relative position, only the difference of perturbations need be considered.

The column log r gives the logarithm of the radii vectores of Mercury and the earth for the times given in the second column, uncorrected for aberration. In strictness they should be corrected for the aberration times like the longitudes and latitudes. This correction has been made in the subsequent work, but not in the tabular exhibit.

Theory of the correction for aberration when heliocentric elements are used.—The general basis of this theory has already been given. But the following more careful and rigorous examination of it may be desirable.

Let us put

$$l_0,\ b_0,\ r_0,$$

the absolute longitude, latitude, and radius vector of Mercury for a certain moment of computation from the tables, which moment we take as the zero of time:

l', b', r', the hourly variations of these quantities;

L_0, B_0, R_0, the absolute longitude, latitude, and radius vector of the earth for the same moment of time.

L', B', R', their hourly variations;

τ_1 the time required for light to pass from the sun to the planet;

τ_2 the time required to pass from the sun to the earth.

We shall then have, with all necessary approximation,

$$\tau_3 = \tau_2 - \tau_1$$

for the time required for light to pass from the planet to the earth.

We shall then have for any time t after the moment of computation values of the absolute co-ordinates of the two bodies given by equations of the form

$$l = l_0 + l't$$
$$L = L_0 + L't \text{ etc., etc.}$$

The phenomena of contact are determined by the condition that a ray of light leaving the limb of the sun at a certain moment shall graze the surface of the planet at a moment τ_1 later, and reach the eye of the observer yet later by the time τ_3. Hence, if we put t_0 for the interval after the zero of time when the required ray of light left the sun, it will reach the planet at the moment $t_0 + \tau_1$, and the observer at the moment $t_0 + \tau_2$. Hence the time t_0 is to be determined by the condition that the position of some point on the planet at the moment $t_0 + \tau_1$ and of some point on the earth at the moment $t_0 + t_2$ shall be in the same straight line, or be in some definite relative position not differing much from a straight line.

The co-ordinates of the two bodies at these moments will be,

For the planet,

$$\begin{aligned} &l_0 + l' \ (t_0 + \tau_1) \\ &\beta_0 + \beta' \ (t_0 + \tau_1) \\ &r_0 + r' \ (t_0 + \tau_1) \end{aligned}$$

And for the earth,

$$\begin{aligned} &L_0 + L' \ (t_0 + \tau_2) \\ &B_0 + B' \ (t_0 + \tau_2) \\ &R_0 + R' \ (t_0 + \tau_2) \end{aligned}$$

Now the condition of contact is that the position of the three bodies at the times thus indicated shall be such that the observer shall be on the cone surrounding the sun and planet. Assuming t_0 to be determined by this condition the actual moment of contact as seen by the observer will be $t_0 + \tau_2$.

If then we put

$$t_1 = t_0 + \tau_2$$

If also we put

$$\begin{aligned} l_1 &= l_0 - l' \ (\tau_2 - \tau_1) \\ \beta_1 &= \beta_0 - \beta' \ (\tau_2 - \tau_1) \\ r_1 &= r_0 - r' \ (\tau_2 - \tau_1) \end{aligned}$$

The condition of contact will be that at the time t_1 the co-ordinates determined by the equations

$$
\begin{aligned}
l &= l_1 + l' \, t_1 \\
\beta &= \beta_1 + \beta' \, t_1 \\
r &= r_1 + r' \, t_1 \\
L &= L_0 + L' \, t_1 \\
B &= \beta_0 + B' \, t_1 \\
R &= R_0 + R' \, t_1
\end{aligned}
$$

shall fulfill the required condition.

We may now describe the third and fifth columns of the table as giving the values of the quantities l_1, L_0, b_1, B_0 and their differences. The quantities l, b, r, L, B, R, are now of the same general form with that assumed in § 1 of this part as the basis of the investigation. The time t_1 takes the place of t, and the co-ordinates l_1, β_1, etc., take the place of the co-ordinates at the zero of time. Hence, the equations of § 1 may be applied unchanged, merely taking t_1 as the unknown quantity instead of t. Moreover, the time t_1 will express the local time of actual contact at the earth's surface.

The computation of the several quantities in this table were all made in duplicate, and in case where the discrepancy approximated to the tenth of a second the computations were re-examined and reconciled. Where the difference did not exceed two or three hundredths of a second, and might, therefore, be attributed to the accumulation of accidental errors in taking out numbers from the tables, the mean of the two results was adopted. The form in whioh the work is presented is such as to render very easy the discovery of any accidental error. The aberration time in the last coumn is equal to $\tau_1 - \tau_2$.

Positions of Mercury and the Earth, from LEVERRIER'S *Tables.*

Date.	Paris M. T. / Greenwich M. T.	Longitude Mercury, Earth. / Diff.	Hourly Motions.	Latitude Mercury, Earth. / Diff.	Hourly Motions.	Pert. by Venus.	Log r Mercury, Earth.	Aberration time.
	h. m. s.	° ′ ″	″	′ ″	″	″		
1677. Nov. 6	21 9 21	44 54 29.63	908.75	+ 2 54.49	+ 111.58	− 16.51	9.4963396	337^{s}.0
	21 0 0	45 36 0.34	150.99	− 0.40	+0.01	− 3.85	9.9953667	− 85″.07
	20 54 23.0	− 41 30.71	757.76	+ 2 54.89	+ 111.57	− 12.66		
1677. Nov. 7	3 9 21	46 25 32.62	912.55	+ 14 4.87	+ 112.05	− 16.78	9.4954334	337^{s}.4
	3 0 0	45 51 6.07	151.01	− 0.35	+0.01	− 3.81	9.9953441	− 85″.50
	22 54 22.6	+ 34 26.55	761.54	+ 14 5.22	+ 112.04	− 12.97		
1690. Nov. 9	19 54 21	48 47 11.93	917.59	+ 30 18.95	+ 112.70	− 1.70	9.4942272	337^{s}.8
	19 45 0	48 25 4.94	151.17	+ 0.24	0.00	− 1.40	9.9951279	− 86″.10
	19 39 22.2	+ 22 6.90	766.42	+ 30 18.71	+ 112.70	− 0.39		
1697. Nov. 2	19 53 0	42 9 21.16	900.38	− 19 6.65	+ 110.55	+ 4.92	9.4983092	336^{s}.2
	19 43 39	41 40 31.77	150.69	+ 0.59	0.00	+ 2.17	9.9957905	− 84″.06
	19 38 2.8	+ 28 49.39	749.69	− 19 [illegible].24	+ 110.55	+ 2.75		
1723. Nov. 9	2 36 0	46 6 49.01	910.00	+ 7 46.12	+ 111.73	− 9.79	9.4960269	337^{s}.1
	2 26 39	46 40 41.89	151.01	+ 0 72	0.00	+ 1.13	9.9953517	− 85″.18
	2 21 1.9	− 33 52.88	758.99	+ 7 45.40	+ 111.73	− 10.92		
1736. Nov. 10	19 9 21	48 24 57.80	915.12	+ 23 35.71	+ 112.36	− 4.39	9.4948091	337^{s}.6
	19 0 0	49 13 54.98	151.17	− 0.03	0.00	− 6.59	9.9951362	− 85″.84
	18 54 22.4	− 48 57.18	763.95	+ 23 35.74	+ 112.36	+ 2.20		
1736. Nov. 10	21 9 21	48 55 29.01	916.29	+ 27 19.70	+ 112 54	− 4.41	9.4945279	337^{s}.7
	21 0 0	49 18 57.42	151.17	− 0.03	0.00	− 6.59	9.9951279	− 85″.95
	20 54 22.3	− 23 28 41	765.12	+ 27 19.73	+ 112.54	+ 2.18		
1736. Nov. 11	0 9 21	49 41 20.79	918.02	+ 32 57.03	+ 112.72	− 4.26	9.4941174	337^{s}.8
10	0 0 0	49 26 30.89	151.18	0.00	0.00	− 6.59	9.9951152	− 86″.11
	23 54 22.2	+ 14 49.90	766.84	+ 32 57.03	+ 112.72	+ 2.33		
1740. May 2	9 59 21	222 39 7.89	440.71	+ 19 9 63	− 54.11	− 2.87	9.6534729	278^{s}.5
	9 50 0	222 41 41.26	145.13	− 0.[illegible]2	0.00	+ 1.00	0.0039904	− 34″.11
	9 45 21.5	− 2 33.37	295.58	+ 19 9.[illegible]5	− 54.11	− 3.87		
1743. Nov. 4	20 31 21	42 9 28.75	898.46	− 23 5.59	+ 110.20	+ 15.72	9.4987932	335^{s}.6
	20 22 0	42 33 11.44	150.69	− 0.66	−0.01	+ 3.66	9.9958021	− 83″.74
	20 16 24.4	− 23 42.69	747.77	− 23 4.93	+ 110.21	+ 12.06		
1743. Nov. 5	0 9 21	43 3 57.35	901.04	− 16 24.89	+ 110.63	+ 15.72	9.4981719	336^{s}.3
4	0 0 0	42 42 18.97	150.70	− 0.69	−0.01	+ 3.66	9.9957866	− 84″.16
	23 54 23.7	+ 21 38.38	750.24	− 16 24.20	+ 110.64	+ 12.06		
1753. May 5	22 15 0	226 15 15.51	434.34	− 6 13.85	− 53.33	− 4.46	9.6566[illegible]37	276^{s}.7
	22 5 39	225 57 0.04	144.03	+ 0.16	0.00	+ 2.80	0.0043[illegible]22	− 33″.40
	22 1 2.3	+ 18 15.47	289.41	− 6 14.01	− 53.33	− 7.35		
1756. Nov. 6	13 39 21	44 34 13.06	904.63	− 6 28.48	+ 111.11	− 10.56	9.4973108	336^{s}.6
	13 30 0	45 7 37.97	150.87	+ 0.17	0.00	− 1.91	9.9955457	− 84″.58
	13 24 23.4	− 33 24.91	753.76	− 6 28.65	+ 111.11	− 8.65		
1756. Nov. 6	19 3 21	45 55 47.14	908.22	+ 3 32.45	+ 111.55	− 10.62	9.4964505	33[illegible]^{s}.9
	18 54 0	45 21 12.47	150.88	+ 0.17	0.00	− 1.90	9.995522[illegible]	− 84″.99
	18 48 23.1	+ 34 34.67	757.34	+ 3 32.28	+ 111.55	− 8.72		
1769. Nov. 9	7 31 21	47 11 20.44	910.00	+ 11 40.75	+ 111.84	− 9.35	9.4958061	337^{s}.1
	7 22 0	47 44 11.23	151.02	− 0.45	0.00	+ 2.71	9.9953103	− 85″.26
	7 16 22.9	− [illegible]2 50.70	759.88	+ 11 41.20	+ 111.84	− 12.06		

Positions of Mercury and the Earth, etc.—Continued.

Date.	Paris M. T. Greenwich M. T. h.	m.	s.	Longitude Mercury, Earth. Diff. °	′	″	Hourly Motions. ″	Latitude Mercury, Earth. Diff. ′	″	Hourly Motions. ″	Pert. by Venus. ″	Log r Mercury, Earth.	Aberration time.
1769. Nov. 9	12	18	21	48	24	4.37	913.87	+ 20	36.02	+ 112.25	− 9.57	9.4951008	337^s.4
	12	9	0	47	56	13.64	151.05		− 0.44	0.00	+ 2.76	9.9952902	− 85″.64
	12	3	22.6	+	27	50.73	762.82	+ 20	36.46	+ 112.25	− 12.33		
1782. Nov. 12	3	9	21	50	16	51.05	917.74	+ 33	17.15	+ 112.34	+ 16.80	9.4941700	337^s.8
	3	0	0	50	25	11.56	151.19		+ 0.31	0.00	+ 6.65	9.9950936	− 86″.08
	2	54	22.2	−	8	20.51	766.55	+ 33	16.84	+ 112.34	+ 10.15		
1782. Nov. 12	4	9	21	50	32	9.12	918.31	+ 35	9.65	+ 112.38	+ 16.80	9.4940354	337^s.8
	4	0	0	50	27	42.75	151.19		+ 0.31	0.00	+ 6.65	9.9950900	− 86″.08
	3	54	22.2	+	4	26.37	767.12	+ 35	9.34	+ 112.38	+ 10.15		
1786. May 3	15	9	21	223	34	7.91	440.28	+ 16	25.22	− 54.06	− 5.09	9.6536747	278^s.2
	15	0	0	223	44	46.17	145.12		− 0.01	+0.01	− 10.28	0.0040243	− 34″.03
	14	55	21.8	−	10	38.26	295.16	+ 16	25.23	− 54.07	+ 5.19		
1786. May 3	20	9	21	224	10	47.77	439.10	+ 11	55.51	− 53.92	− 5.04	9.6542616	278^s.2
	20	0	0	223	56	51.75	145.10		+ 0.02	+0.01	− 10.31	0.0040451	− 33″.94
	19	55	21 8	+	13	56.02	294.00	+ 11	55.49	− 53.93	+ 5.27		
1789. Nov. 5	1	4	21	43	7	45.06	899.21	− 19	58.02	+ 110.41	+ 4.40	9.4986204	336^s.0
	0	55	0	43	35	20.93	150.71		+ 0.36	0.00	− 7.51	9.9957774	− 83″.90
	0	49	24.0	−	27	35.87	748.50	− 19	58.38	+ 110.41	+ 11.91		
1789. Nov. 5	5	51	21	44	19	33.74	902.56	− 11	9.07	+ 110.86	+ 4.28	9.4978112	336^s.3
	5	42	0	43	47	21.62	150.73		+ 0.39	0.00	− 7.49	9.9957570	− 84″.33
	5	36	23.7	+	32	12.12	751.83	− 11	9.46	+ 110.86	+ 11.77		
1799. May 6	21	9	21	226	24	56.07	435.30	− 3	24.56	− 53.45	+ 3.95	9.6561498	276^s.7
	21	0	0	226	44	44.38	144.92		− 0.12	0.00	− 8.84	0.0043103	− 33″.47
	20	55	23.3	−	19	48.31	290.38	− 3	24.44	− 53.45	+ 12.79		
1799. May 7	4	29	21	227	18	2.55	433.75	− 9	55.88	− 53.26	+ 3.91	9.6569333	276^s.7
	4	20	0	227	2	27.46	144.90		− 0.15	0.00	− 8.80	0.0043400	− 33″.36
	4	15	23.3	+	15	35.09	288.85	− 9	55.73	− 53.26	+ 12.71		
1802. Nov. 8	23	54	21	46	59	9.22	909 10	+ 7	18.71	+ 111.62	− 3.77	9.4962414	337^s.1
	23	45	0	46	24	24.60	151.32		− 0.31	+0.01	− 1.48	9.9955122	− 85″.09
	23	39	22.9	+	34	44.62	757.78	+ 7	19.02	+ 111.61	− 2.29		
1802. Nov. 9	0	54	21	47	14	18.47	909.74	+ 9	10.36	+ 111.74	− 3.81	9.4960877	337^s.1
	0	45	0	46	26	55.55	151.32		− 0.30	+0.01	− 1.47	9.9955081	− 85″.15
	0	39	22.9	+	47	22.92	758.42	+ 9	10.66	+ 111.73	− 2.34		
1822. Nov. 4	13	12	21	41	52	47.41	894.05	− 32	1.98	+ 109.77	+ 18.53	9.4998451	335^s.6
	13	3	0	42	5	14.76	150.56		− 0.41	0.00	+ 6.29	9.9960157	− 84″.33
	12	57	24.4	−	12	27.35	743.49	− 32	1.57	+ 109.77	+ 12.24		
1822. Nov. 4	15	54	21	42	33	5.10	896.02	− 27	5.89	+ 110.02	+ 18.62	9.4993629	335^s.6
	15	45	0	42	12	1.25	150.56		− 0.41	0.00	+ 6.26	9 9960042	− 83″.51
	15	39	24.4	+	21	3.85	745.46	− 27	5.48	+ 110.02	+ 12.37		
1832. May 4	21	9	21	224	35	22.48	439.70	+ 12	55.23	− 53.99	− 3.11	9.6539706	278^s.1
	21	0	0	224	50	21.96	145.10		+ 0.35	0.00	+ 8.89	0.0040566	− 33″.99
	20	55	21.9	−	14	59.48	294.60	+ 12	54.88	− 53.99	− 12.00		
1832. May 5	3	51	21	225	24	23.24	438.12	+ 6	54.02	− 53.79	− 3.12	9.6547443	277^s.6
	3	42	0	225	6	34.12	145.07		+ 0.32	0.00	+ 8.89	0 0040844	− 33″.78
	3	37	22.4	+	17	49.12	293.05	+ 6	53.70	− 53.79	− 12.01		

Positions of Mercury and the Earth, etc.—Continued.

Date.	Paris M. T. Greenwich M. T.	Longitude Mercury, Earth. Diff.	Hourly Motions.	Latitude Mercury, Earth. Diff.	Hourly Motions.	Pert. by Venus.	Log r Mercury. Earth.	Aberration time.
	h. m. s.	° ′ ″	″	′ ″	″	″		
1845. May 8	4 29 21	227 35 31.26	434.49	− 8 4.20	− 53.35	+ 3.14	9.6565614	277°.1
	4 20 0	227 53 22.92	144.90	+ 0.19	0.00	− 1.39	0.0043575	− 33″.46
	4 15 22.9	− 17 51.66	289.59	− 8 4.39	− 53.35	+ 4.53		
1845. May 8	10 51 21	228 21 33.40	433.15	− 13 43.26	− 53.18	+ 3.20	9.6572317	276°.5
	10 42 0	228 8 45.59	144.88	+ 0.19	0.00	− 1.43	0.0043833	− 33″.27
	10 37 23.5	+ 12 47.81	288.27	− 13 43.45	− 53.18	+ 4.63		
1848. Nov. 8	23 16 21	46 39 5.25	906.39	+ 0 49.79	+ 111.29	− 20.42	9.4968828	336°.7
	23 7 0	47 13 37.74	150.89	+ 0.30	0.00	− 6.79	9.9955213	− 84″.75
	23 1 23.3	− 34 32.49	755.50	+ 0 49.49	+ 111.29	− 13.63		
1848. Nov. 9	4 39 21	48 0 33.91	909.91	+ 10 50.18	+ 111.76	− 20.49	9.4960434	337°.1
	4 30 0	47 27 9.84	150.92	+ 0.35	+0.01	− 6.74	9.9954983	− 85″.20
	4 24 22.9	+ 33 24.07	758.99	+ 10 49.83	+ 111.75	− 13.75		
1861. Nov. 11	17 24 21	49 21 10.60	912.72	+ 19 35.88	+ 112.00	+ 9.01	9.4953715	337°.4
	17 15 0	49 50 54.92	151.06	− 0.29	0.00	− 4.60	9.9952807	− 85″.52
	17 9 22.6	− 29 44.32	761.66	+ 19 36.17	+ 112.00	+ 13.61		
1861. Nov. 11	21 24 21	50 22 6.25	915.15	+ 27 4.03	+ 112.17	+ 9.03	9.4947957	337°.6
	21 15 0	50 0 59.14	151.07	− 0.30	0.00	− 4.57	9.9952642	− 85″.84
	21 9 22.4	+ 21 7.11	764.08	+ 27 4.33	+ 112.17	+ 13.60		
1868. Nov. 4	17 33 21	42 48 9.87	894.68	− 29 14.93	+ 109.85	+ 5.24	9.4997015	335°.6
	17 24 0	43 7 2.04	150.58	+ 0.14	0.00	− 0.98	9.9959840	− 83″.38
	17 18 24.4	− 18 52.17	744.10	− 29 15.07	+ 109.85	+ 6.22		
1868. Nov. 4	21 12 21	43 42 40.19	897.35	− 22 34.03	+ 110.18	+ 5.14	9.4990547	335°.9
	21 3 0	43 16 11.63	150.59	+ 0.13	0.00	− 0.94	9.9959683	− 83″.72
	20 57 24.1	+ 26 28.56	746.76	− 22 34.16	+ 110.18	+ 6.08		
1878. May 6	0 9 21	225 14 45.63	439.81	+ 12 6.29	− 54.02	+ 4.16	9.6539212	278°.1
	0 0 0	225 47 59.46	145.09	− 0.33	0.00	− 3.72	0.0040746	− 33″.98
5	23 55 21.9	− 33 13.83	294.72	+ 12 6.62	− 54.02	+ 7.88		
1878. May 6	3 24 21	225 38 33.78	439.06	+ 9 10.95	− 53.92	+ 4.17	9.6543012	278°.0
	3 15 0	225 55 51.13	145.04	− 0.33	0.00	− 3.70	0.0040878	− 33″.90
	3 10 22.0	− 17 17.35	294.02	+ 9 11.28	− 53.92	+ 7.87		
1878. May 6	10 54 21	226 33 20.02	437.30	+ 2 27.31	− 53.71	+ 4.14	9.6551599	277°.6
	10 45 0	226 13 59.33	145.05	− 0.29	0.00	− 3.69	0.0041188	− 33″.72
	10 40 22.4	+ 19 20.69	292.25	+ 2 27.60	− 53.71	+ 7.83		
1881. Nov. 7	10 27 21	45 9 7.39	900.91	− 13 5.83	+ 110.65	− 3.20	9.4981949	336°.2
	10 18 0	45 41 3.73	150.76	− 0.28	0.00	+ 9.23	9.9957346	− 84″.13
	10 12 23.8	− 31 56.34	750.15	− 13 5.55	+ 110.65	− 12.43		
1881. Nov. 7	12 9 21	45 34 39.92	902.05	− 9 57.68	+ 110.79	− 3.13	9.4979094	336°.3
	12 0 0	45 45 19.96	150.70	− 0.28	+0.01	+ 9.27	9.9957273	− 84″.25
	11 54 23.7	− 10 40.04	751.35	− 9 57.40	+ 110.78	− 12.40		
1881. Nov. 7	15 45 21	46 28 51.87	904.56	− 3 18.38	+ 111.10	− 3.18	9.4973155	336°.6
	15 36 0	45 54 22.52	150.77	− 0.25	0.00	+ 9.26	9.9957120	− 84″.57
	15 30 23.4	+ 34 29.35	753.79	− 3 18.13	+ 111.10	− 12.44		

§ 4.

Computation of tabular times of contact and other quantities.

In ~~order~~ to afford the most convenient method of introducing any necessary correction, the principal numbers which occur in computing the tabular times of contact and other elements from the preceding formulæ are shown in the following table. Only those lines in which the results are completely carried out are to be regarded as definitive; the others were provisional computations made for the purpose of approximately determining the tabular times. They are inserted to make more easy the discovery of accidental errors, or the introduction of any changes in the elements.

The second column of the table gives the assumed times of computation.

Column Δt gives the computed correction to the assumed time.

Column t gives the definitive Greenwich mean times of geocentric internal contact thus obtained.

Date.	G. M. T.	r	c	ω			Δt	t			A	D	K		
	h	"	"	°	'	"	h	h	m	s	s	s	°	'	"
1677. Nov. 6	21.55	2080.73	2087.26												
	21.55868	2080.75	2080.83	186	32	51.9	+ .00011	21	33	31.6	+ 7.482	− 42.784	316	7	35.0
	21.70	2080.89	1976.39												
Nov. 7	2.80	2086.29	2083.60												
	2.80371	2086.27	2086.36	336	45	37.1	− .00012	2	48	12.9	+ 26.673	+ 34.025	146	53	30.0
	3.0	2086.48	2232.72												
1690. Nov. 9	19.44813	2094.16	2094.25	301	32	44.8	+ 00018	19	26	52.6	+ 59.531	+ 25.352	183	4	15.8
	19.45	2094.16	2095.18												
1697. Nov. 2	19.71345	2067.27	2067.42	33	45	23.6	− .00027	19	42	47.4	− 15.010	+ 55.233	124	56	57.8
	19.7275	2067.30	2075.32												
1723. Nov. 9	2.44417	2082.33	2085.50												
	2.44846	2082.32	2082.43	192	55	49.3	+ .00015	2	26	55.0	+ 2.750	− 44.910	315	16	1.8
1736. Nov. 10	21.00	2092.37	2161.56												
	21.17416	2092.55	2092.69	232	27	32.1	+ .00037	21	10	28.3	− 48.541	− 70.136	304	11	37.3
	21.20	2092.56	2083.03												
	23.80	2095.05	2088.66												
	23.81727	2095.10	2095.19	290	58	7.2	− .00024	23	49	1.3	+ 80.410	+ 26.657	210	14	28.0
	24.00	2095.24	2168.08												
1740. May 2	9.69773	1173.38	1173.35	260	30	37.0	− .00029	9	41	50.8	− 264.643	− 159.533	257	29	42.4
	9.73333	1173.36	1169.75												
1743. Nov. 4	20.23333	2063.79	2068.00												
	20.23990	2063.80	2063.90	137	19	42.0	+ .00016	20	14	24.2	+ 43.131	− 28.145	335	28	18.5
	20.36667	2063.95	1985.47												
Nov. 5	0.75210	2069.14	2069.34	25	48	35.4	− .00032	0	45	6.4	− 6.966	+ 50.749	128	4	20.5
	0.8	2069.20	2099.52												
1753. May 5	22.09417	1158.64	1157.56												
	22.09775	1158.64	1158.60	18	50	35.2	+ .00014	22	5	52.4	− 60.511	+ 89.911	325	4	38.0
1756. Nov. 6	13.45828	2073.71	2073.85	169	4	1.0	+ .00019	13	27	30.5	+ 19.008	− 37.733	321	10	35.8
	13.5	2073.75	2042.09												
	18.89248	2079.72	2079.76	354	9	52.7	− .00006	18	53	32.7	+ 15.630	+ 39.040	139	40	30.0
	18.9	2079.73	2085.58												

Computation of Tabular Times of Contact, etc.—Continued.

Date.	G. M. T.	r	c	ω	Π	t	A	D	K
	h.	"	"	° ′ ″	h	h m s	s	s	° ′ ″
1769. Nov. 9	7.36667	2083.78	2091.81						
	7.37836	2083.80	2083.88	199 42 4.4	+ .00012	7 22 42.5	− 2.708	− 47.375	314 15 26.4
	12.15	2088.70	2078.58						
	12.16505	2088.73	2088.80	323 38 41.7	0	12 9 54.2	+ 35.832	+ 30.774	156 35 36.8
1782. Nov. 12	2.69933	2094.55	2094.64	249 35 2.5	+ .00056	2 41 59.6	− 153.342	− 126.609	294 58 26.8
	2.7	2094.54	2094.53						
	3.83665	2095.63	2095.74	273 51 35.8	− .00067	3 50 9.5	+ 183.090	+ 71.046	261 44 59.8
	3.85	2095.62	2097.94						
	4.0	2005.77	2126.10						
1786. May 3	15.0	1172.39	1173.90						
	15.00755	1172.39	1172.36	237 8 38.0	− .00015	15 0 26.6	− 142.233	− 57.709	183 48 35.3
	20	1169.57	1100.39						
	20.35	1169.37	1169.11						
	20.35106	1169.37	1169.33	323 26 16.9	+ .00020	20 21 0.5	+ 48.909	+ 146.747	305 59 46.2
1789. Nov. 5	0.88513	2065.05	2065.22	144 24 43.0	+ .00025	0 53 7.4	+ 36.719	− 30.583	331 10 25.5
	0.91667	2065.10	2044.02						
	5.7	2070.71	2044.75						
	5.73863	2070.75	2070.83	18 43 56.1	− .00059	5 44 16.9	− 1.062	+ 47.533	130 57 35.8
1799. May 6	21.16260	1160.86	1160.82	169 25 13.2	− .00014	21 9 44.9	− 11.421	− 113.907	136 1 17.7
	21.16667	1160.86	1159.70						
May 7	4.33333	1157.21	1108.73						
	4.5	1157.13	1154.25						
	4.5106	1157.12	1157.08	31 32 2.6	+ .00014	4 30 37.8	− 81.868	+ 80.965	332 31 13.2
1802. Nov. 8	23.68581	2081.16	2081.27	348 1 27.1	− .00014	23 41 8.4	+ 19.307	+ 37.377	142 52 56.2
	23.75	2081.22	2130.34						
1822. Nov. 4	13.05	2057.08	2061.79						
	13.06240	2057.09	2057.19	111 1 36.4	+ .00027	13 3 45.6	+ 83.288	− 23.555	27 54 24.8
	15.75	2060.42	2059.01						
	15.75407	2060.40	2060.51	52 3 34.6	− .00030	15 45 13.6	− 46.918	+ 72.728	119 26 20.0
1832. May 4	21.0	1171.02	1187.22						
	21.06294	1170.99	1171.00	221 12 36.6	+ .00004	21 3 46.7	− 100.187	− 70.782	156 49 50.0
May 5	3.75	1167.30	1146.37						
	3.83183	1167.26	1167.22	339 28 19.6	+ .00016	3 46 55.2	+ 7.228	+ 123.242	311 39 19.2
	3.85	1167.25	1171.86						
1845. May 8	4.33333	1159.06	1176.05						
	4.40385	1159.02	1159.06	155 5 31.7	+ .00021	4 24 14.6	+ 18.948	− 129.984	132 50 37.3
	10.82597	1155.74	1155.75	45 54 45.2	+ .00017	10 49 34.1	− 111.496	+ 70.941	344 53 45.1
	10.86667	1155.78	1165.48						
1848. Nov. 8	23.11174	2076.68	2076.80	181 21 1.1	+ .00016	23 6 42.8	+ 10.624	− 41.384	319 15 26.6
	23.11667	2076.68	2073.08						
Nov. 9	4.46797	2082.52	2082.64	341 55 25.0	− .00016	4 28 4.1	+ 23.041	+ 35.778	146 20 57.9
	4.5	2082.53	2106.87						
1861. Nov. 4	17.25	2086.77	2137.10						
	17.33913	2086.87	2086.98	214 40 10.0	+ .00019	17 20 21.6	− 17.759	− 51.320	11 24 25.7
	17.35	2086.87	2080.87						

Computation of Tabular Times of Contact, etc.—Continued.

Date.	G. M. T.	r	c	ω	Δt	t	A	D	K
	h.	″	″	° ′ ″	h	h m s	s.	s	° ′ ″
186[illegible] Nov. 4	21.3	2090.85	2090.71						
	21.30042	2090.85	2090.96	308 46 49.7	− .00019	21 18 0.8	+ 49.718	+ 27.464	173 0 20.0
1868. Nov. 4	17.46188	2058.08	2058.19	121 51 3.2	+ .00023	17 27 43.6	+ 61.705	− 23.889	357 34 42.6
	17.46667	2058.09	2055.86						
	20.99953	2062.41	2062.53	41 14 33.4	− .00025	20 59 57.5	− 25.448	+ 60.664	123 58 43.5
	21.0	2062.43	2062.76						
1878. May 6	3.25	1169.48	1174.68	207 58.6	. . . . .	. . . . . .	− 74.94	− 81.96	229 30
	3.26833	1169.47	1169.52	. . . . . .	+ .00018	3 16 6.6			
	10.7[illegible]233	1165.36	1165.04	. . . . . .	+ .00114	. . . . . .	. . . . .	. . . . .	
	10.75	1165.35	1170.02	352 45.2	− .01649	10 44 0.5	− 17.26	+ 110.13	219 11
1881. Nov. 7	10.3	2067.97	2071.12						
	10.30411	2067.98	2068.10	157 41 16.2	+ .00021	10 18 15.54	+ 26.458	− 34.773	326 16 50.0
	15.59374	2074.06	2074.13	5 30 0.3	− .00005	15 35 37.3	+ 8.163	+ 42.705	136 42 17.4
	15.6	2074.08	2078.81						

Special Computation of Transit of 1782 *for Paris and Cambridge.*

PARIS.

Date.	G. M. T.	r + Δr	c	ω	Δt	t
	h	″	″	° ′ ″	h	h m s
1782. Nov. 12	2.64928	2094.50	2094.55	248 30 39	+ .00032	2 38 58.6
	3.88312	2095.67	2095.63	274 55 32	+ .00025	3 53 0.1
	3.99366	2116.92	2116.98	277 11 8	− .00030	3 59 36.1

CAMBRIDGE.

Date.	G. M. T.	r + Δr	c	ω	Δt	t
Nov. 12	2.67479	2094.49	2094.56	248 52 53	+ .00014	2 40 30.8
	3.87940	2095.66	2095.71	274 40 29	+ .00032	3 52 47.0
	3.99210	2116.92	2116.94	276 59 6	− .00010	3 59 31.2

§ 5.

Symbolic corrections to the tabular relative positions of Mercury and the earth in terms of corrections to elements.

We next require the change in the time of geocentric contact produced by changes in the elements. If we first take as the co-ordinates for the position of Mercury,

θ, the longitude of its node;
u, its argument of latitude;
i, the inclination of its orbit;

we shall have the following values of l and b:

$$
\begin{aligned}
l &= \theta + u_1 \\
\sin b &= \sin i \sin u \qquad (1) \\
\text{where } \tan u_1 &= \cos i \tan u
\end{aligned}
$$

Hence differentiating and substituting for $\frac{\cos^2 u_1}{\cos^2 u}$ its value, $\sec^2 b$,

$$\delta l = \delta\vartheta + \cos i \sec^2 b \, \delta u$$
$$\delta b = \sin i \sec l \cos u \, \delta u$$

Owing to the minuteness of the latitude during a transit, we may put $\sec b = 1$. We then have, with sufficient approximation,

$$\delta l = \delta\theta + \cos i \, \delta u$$
$$\delta b = \sin i \cos u \, \delta u$$

We need not vary i because it cannot be corrected from transits.

From the equations (1)″ of § 1 we have

$$c = (l - l') \cos \omega + (b' - b) \sin \omega.$$

The earth's latitude b' may be assumed as so well known as not to need correction. So, differentiating this last equation, we have

$$\delta c = \cos \omega \, (\delta l - \delta l') - \sin \omega \, \delta b$$

Substituting for δl and δb their values just given

$$\delta c = \cos \omega \, (\delta\theta - \delta l') + (\cos \omega \cos i - \sin \omega \sin i \cos u) \, \delta u$$

During a transit at the ascending node (a November transit) the value of u must be contained within the limits $\pm 5^\circ$, and during one at the descending node (a May transit) within the limits $180^\circ \pm 3^\circ$. We may therefore suppose $\cos u$ equal to $+1$ during a November transit, and to -1 during a May transit.

The preceding expression will thus become:

$$\delta c = \cos \omega \, (\delta\vartheta - \delta l') + \cos (\omega + i) \, \delta u \text{ for November.}$$
$$\delta c = \cos \omega \, (\delta\vartheta - \delta l') + \cos (\omega - i) \, \delta u \text{ for May.} \qquad (2)$$

We have next to express δu in terms of the corrections of such of the elements of the orbit of Mercury as admit of correction from observed transits. We shall however first transform the equations, so that the longitude in orbit shall enter instead of u, putting

v, the longitude in orbit, counted from a departure point in its moving plane.
We shall then have

$$\delta u = \delta v - \cos i \delta\vartheta$$

Substituting this value of δu in (2)

$$\delta c = \{\cos \omega - \cos i \cos (\omega + i\} \, \delta\theta - \cos \omega \, \delta l' + \cos (\omega + i) \, \delta v$$
$$= \sin (\omega + i) \sin i \, \delta\theta - \cos \omega \, \delta l' + \cos (\omega + i) \, \delta v$$

in which i is to be taken positive in a November and negative in a May transit.

The coefficients of δv and $\delta l'$ are so nearly identical that separate values of these quantities cannot be obtained. Indeed, it is evident that, since the phenomena depend only upon the relative positions of the earth and Mercury, it is not possible to obtain the absolute position of either. We may, in fact, express the last equation in the form

$$\delta c = \sin (\omega + i) \sin i \, (\delta\theta - \delta l') + \cos (\omega + i) \, (\delta v - \cos i \, \delta l') \qquad (3)$$

Supposing the corrections $\delta\theta$, $\delta l'$, and v constant, we could, from a system of equations of this form, obtain values of the two expressions $\delta\theta - \delta l'$ and $\delta v - \cos i\,\delta l'$. When, from other data, the value of $\delta l'$ is found, its substitution will give the required values of $\delta\theta$ and δv.

Since observations of contact give only the time when the relative position of the bodies have a certain relation to their semi-diameters, namely, the moment at which $c - \mathbf{r} = 0$, it is necessary to include $\delta\mathbf{r}$ as well as δc in the equations. An approximate value of $\mathbf{r}$ from the equation (3) of § 4, is, for internal contact,

$$\mathbf{r} = \frac{R' - R}{r} - \frac{R'}{r'}$$

R and R′ being the angular semi-diameters of the sun and Mercury at distance unity.

We have by differentiating this expression

$$\delta\mathbf{r} = \left(\frac{1}{r} - \frac{1}{r'}\right)\delta R' - \frac{1}{r}\,\delta R,$$

or reduced to numbers

$$\left.\begin{array}{c}\text{FOR A NOVEMBER TRANSIT,}\\ \delta\mathbf{r} = 2.18\,\delta R' - 3.19\,\delta R\\ \text{FOR A MAY TRANSIT,}\\ \delta\mathbf{r} = 1.22\,\delta R' - 2.21\,\delta R\end{array}\right\} \quad (4)$$

For external contacts we have only to change the sign of δR, obtaining

$$\begin{array}{c}\delta\mathbf{r} = 2.18\,\delta R' + 3.19\,\delta R\\ \delta\mathbf{r} = 1.22\,\delta R' + 2.21\,\delta R\end{array} \quad (5)$$

In the equation (3) the absolute residual is δc, or the correction to the tabular distance of centers. But, in practice, it may be more convenient to make use of times of contact. The equations (1)″ give, by differentiation, and omission of the change in b', which is insensible,

$$\frac{dc}{dt} = \cos\omega\left(\frac{dl}{dt} - \frac{dl'}{dt}\right) - \sin\omega\,\frac{db}{dt}$$

So near the node as a transit of Mercury can be observed we may put

$$\frac{dl}{dt} = \frac{dv}{dt}\cos i$$

$$\frac{db}{dt} = \frac{dv}{dt}\sin i$$

which will give

$$\frac{dc}{dt} = \cos(\omega + i)\,\frac{dv}{dt} - \cos\omega\,\frac{dl'}{dt}$$

i being, as before, positive at the ascending node (November) and negative at the descending node (May). Since

$$\delta c = \frac{dc}{dt}\delta t$$

the correction to the tabular time would be expressed by the equation

$$\left(\cos(\omega + i)\frac{dv}{dt} - \cos\omega\frac{dl'}{dt}\right)\delta t = \sin(\omega + i)\sin i\,(\delta\theta - \delta l') + \cos(\omega + i)(\delta v - \cos i\,\delta l')$$

A somewhat more elegant form might be given this equation by dividing it throughout by $\cos(\omega + i)$, but since the probable errors of observations should be referred to the distance of centers rather than to the time we shall retain it in its present form.

From the tables of heliocentric positions to be given hereafter it will be seen that the values of $\frac{dv}{dt}$ for a November transit range between 901″ and 925″ per hour, the mean value being 913″. By adopting this mean value as applicable to all the November transits we shall nearly always have the correct value of the co-efficient within one hundredth, and as the errors of LEVERRIER's tables can scarcely ever exceed 20 seconds, and the necessary probable error of all contact observations is an entire second or more, we may use this mean value. For the same reason we may use 441″ as the value of $\frac{dv}{dt}$ for all May transits. Using also the mean values for the sun's change of longitude, and reducing the unit of time to seconds we shall have

FOR A NOVEMBER TRANSIT,

$$(0''.253\cos(\omega+i) - 0''.042\cos\omega)\,\delta t = \sin(\omega+i)(\delta\theta - \delta l')\sin i + \cos(\omega+i)(\delta v - \cos i\,\delta l')$$

FOR A MAY TRANSIT,

$$(0''.122\cos(\omega-i) - 0''.040\cos\omega)\,\delta t = \sin(\omega-i)(\delta\theta - \delta l')\sin i + \cos(\omega-i)(\delta v - \cos i\,\delta l')$$

When, instead, of δc, we use $\delta c - \delta\mathbf{r}$, as we should, we add $\delta\mathbf{r}$ (4) and (4′) to the second member of the equations, obtaining

FOR A NOVEMBER TRANSIT, INTERNAL CONTACT,

$$\begin{aligned}(0''.253\cos(\omega+i) - 0''.042\cos\omega)\,\delta t = {} & \sin(\omega+i)(\delta\theta - \delta l')\sin i \\ & + \cos(\omega+i)(\delta v - \cos i\,\delta l') \\ & + 2.18\,\delta R' - 3.19\,\delta R\end{aligned} \tag{6}$$

$$(\delta\theta - \delta l')\sin i$$

FOR A MAY TRANSIT, INTERNAL CONTACT,

$$(0''.122 \cos(\omega - i) - 0''.040 \cos \omega)\, \delta t = \sin(\omega - i)(\delta\theta - \delta l') \sin i \\ + \cos(\omega - i)(\delta v - \cos i\, \delta l') \qquad (7) \\ + 1.22\, \delta R' - 2.21\, \delta R$$

For external contacts we use the same equations, changing the sign of the coefficient δR.

For the value of the inclination i to be used we may take 7° 0′ throughout. Moreover, since $\cos i$ differs from unity by less than .01, we may suppose $\cos i\, \delta l' = \delta l'$, as $\delta l'$ itself can never exceed 2″.

The next step in order is the substitution of the elements of the earth and Mercury and the mass of Venus for the indeterminate quantities in the second members of (6) and (7).

We put

g, the mean anomaly;
π, the longitude of the perihelion on the orbit;
e, the eccentricity;
λ, the mean longitude at any epoch;
$\delta\mu$, the correction to the mass of Venus.

Then, in the case of Mercury,

$$\delta v = \delta\lambda \begin{Bmatrix} 1 + 0.409 \cos g + 0.104 \cos 2g \\ + 0.027 \cos 3g + 0.007 \cos 4g \end{Bmatrix}$$

$$+ \delta\pi \begin{Bmatrix} - 0.409 \cos g - 0.104 \cos 2g \\ - 0.027 \cos 3g - 0.007 \cos 4g \end{Bmatrix}$$

$$+ \delta e \begin{Bmatrix} 1.97 \sin g + 0.50 \sin 2g \\ + 0.13 \sin 3g + 0.104 \sin 4g \end{Bmatrix}$$

$$+ \delta\mu \text{ (perturbations by Venus).}$$

Also, for the earth,

$$\begin{aligned} \delta l' = {} & \delta\lambda' \times (1 + 0.033 \cos g') \\ & - e\delta\pi' \times 2 \cos g' \\ & + \delta e' \times 2 \sin g \\ & + \delta\mu \times \text{(perturbations by Venus).} \end{aligned}$$

We may put, for brevity,

$$h = 0.409 \cos g + 0.104 \cos 2g + 0.027 \cos 3g + 0.007 \cos 4g$$
$$k = 1.97 \sin g + 0.50 \sin 2g + 0.13 \sin 3g + 0.04 \sin 4g$$

and then find h and k from the following table, which includes all the values which g can have at the time of a transit.

g	h	k
°		
135	− 0.277	+ 0.985
136	− 0.279	+ 0.958
137	− 0.282	+ 0.931
138	− 0.284	+ 0.904
139	− 0.286	+ 0.877
140	− 0.288	+ 0.851
338	+ 0.465	− 1.308
339	+ 0.473	− 1.260
340	+ 0.480	− 1.211
341	+ 0.486	− 1.160
342	+ 0.492	− 1.107
343	+ 0.498	− 1.054
344	+ 0.503	− 0.999
345	+ 0.508	− 0.942

If we also put
P_1, the periodic perturbations of the longitude of Mercury by Venus,
P_2, the same for the earth,
h', k', the quantities corresponding to h and k in the sun's longitude,
we shall have

$$\delta v = (1 + h)\,\delta\lambda - h\ \delta\pi + k\ \delta e + P_1\ \delta\mu$$
$$\delta l' = (1 + h')\,\delta\lambda' - h'\ \delta\pi' + k'\,\delta e' + P_2\ \delta\mu$$

If we also put

FOR A NOVEMBER TRANSIT,

$$n = 0''.253 \cos\,(\omega + i) - 0''.042 \cos\,\omega$$

FOR A MAY TRANSIT,

$$n = 0''.122 \cos\,(\omega - i) - 0''.040 \cos\,\omega$$

and suppose $\cos i = 1$, the general equations of condition (6) and (7) will reduce to

$$n\delta t = \sin\,(\omega \pm i)\ (\delta\theta - \delta l')\ \sin i$$
$$+ \cos\,(\omega \pm i)\ \{(1 + h)\,\delta\lambda - (1 + h')\,\delta\lambda' - k\delta\pi + h'\,\pi' + k\delta e - k'\,\delta e' + (P_1 - P_2)\,\delta\mu\}$$
$$+ \begin{Bmatrix}2.18\\1.22\end{Bmatrix}\delta R' - \begin{Bmatrix}3.19\\2.21\end{Bmatrix}\delta R$$

If all the unknown quantities in this general equation could be independently determined from transits of Mercury, they might all appear with their secular variations in the equations of condition. But, owing to the fact that transits of Mercury can be observed only at or near two opposite points of the orbit, only certain linear functions of these corrections to the elements can be actually determined from the

observed transits. In fact, the coefficients h, k, h', and k' have each nearly the same value for all the transits occurring at the same point of the orbits. It is therefore necessary to find what linear functions of the elements the transits actually observed are best adapted to give, and to determine these functions alone, leaving the elements themselves to be subsequently determined from meridian observations. The following are the expressions of δv in terms of the corrections to the elements of Mercury at the times of the several transits. An approximate weight is assigned to each, expressing the suitability of the observations for determining the value of δv.

NOVEMBER TRANSITS.

1677,	$\delta v = 1.495\delta\lambda$	$-0.495\delta\pi$	$-1.081\delta e$	Wt.	$= 0$
1690,	$= 1.503$	-0.503	-1.002		$= 0$
1697,	$= 1.479$	-0.479	-1.217		$= 0$
1723,	$= 1.500$	-0.500	-1.032		$= 1$
1736,	$= 1.505$	-0.505	-0.977		$= 1$
1743,	$= 1.480$	-0.480	-1.221		$= 1$
1756,	$= 1.488$	-0.488	-1.139		$= 1$
1769,	$= 1.498$	-0.498	-1.056		$= 1$
1782,	$= 1.508$	-0.508	-0.940		$= 0$
1789,	$= 1.477$	-0.477	-1.206		$= 1$
1802,	$= 1.493$	-0.493	-1.102		$= 1$
1822,	$= 1.470$	-0.470	-1.277		$= 1$
1848,	$= 1.491$	-0.491	-1.112		$= 2$
1861,	$= 1.500$	-0.500	-1.027		$= 2$
1868,	$= 1.472$	-0.472	-1.265		$= 2$
1881,	$= 1.483$	-0.483	-1.185		$= 2$

MAY TRANSITS.

1740,	$\delta v = 0.722\delta\lambda$	$+0.278\delta\pi$	$+0.975\delta e$	Wt.	$= 0$
1753,	$= 0.712$	$+0.288$	$+0.851$		$= 1$
1786,	$= 0.721$	$+0.279$	$+0.958$		$= 1$
1799,	$= 0.712$	$+0.288$	$+0.851$		$= 2$
1832,	$= 0.719$	$+0.281$	$+0.942$		$= 3$
1845,	$= 0.711$	$+0.289$	$+0.836$		$= 3$
1878,	$= 0.718$	$+0.282$	$+0.926$		$= 4$

Corrections to the solar elements are to be included in each of the equations, but as their coefficients may be assumed constant for all the transits at one node, they are omitted in the above table. They are, however, included in the following mean values of $\delta v - \cos i\ \delta l'$ derived from the tables above.

In the equations we shall put

V, the mean value of $\delta v - \cos i\ \delta l'$ for the November transits;

W, the mean value of $\delta v - \cos i\ \delta l'$ for the May transits;

V′, W′, the secular variations of V and W.

We then have

$$V = 1.487\delta\lambda - 0.487\delta\pi - 1.137\delta e - 1.01\delta\lambda' + 1.19e'\delta\pi' + 1.58\delta e',$$
$$W = 0.716\delta\lambda + 0.284\delta\pi + 0.896\delta e - 0.97\delta\lambda' - 1.11e'\delta\pi' - 1.62\delta e',$$
$$V' = 1.487D_t\,\delta\lambda - 0.487D_t\,\delta\pi - 1.137D_t\,\delta e - 1.01D_t\,\delta\lambda' + 1.19D_t\,e'\delta\pi' + 1.58D_t\,\delta e',$$
$$W' = 0.716D_t\,\delta\lambda + 0.284D_t\,\delta\pi + 0.896D_t\,\delta e - 0.97D_t\,\delta\lambda' - 1.11D_t\,e'\delta\pi' - 1.62D_t\,\delta e'.$$

From these values of V and W we have the following expressions for $\delta v - \cos i\,\delta l'$ in the several transits:

NOVEMBER TRANSITS.

1677,	$\delta v - \cos i\,\delta l' = V$	$+ 0.008\delta\lambda$	$- 0.008\delta\pi$	$+ 0.056\delta e$
1690,	$= V$	$+ 0.016$	$- 0.016$	$+ 0.135$
1697,	$= V$	$- 0.008$	$+ 0.008$	$- 0.080$
1723,	$= V$	$+ 0.013$	$- 0.013$	$+ 0.105$
1736,	$= V$	$+ 0.018$	$- 0.018$	$+ 0.160$
1743,	$= V$	$- 0.007$	$+ 0.007$	$- 0.084$
1756,	$= V$	$+ 0.001$	$- 0.001$	$- 0.002$
1769,	$= V$	$+ 0.011$	$- 0.011$	$+ 0.081$
1782,	$= V$	$+ 0.021$	$- 0.021$	$+ 0.197$
1789,	$= V$	$- 0.010$	$+ 0.010$	$- 0.069$
1802,	$= V$	$+ 0.006$	$- 0.006$	$+ 0.035$
1822,	$= V$	$- 0.017$	$+ 0.017$	$- 0.140$
1848,	$= V$	$+ 0.004$	$- 0.004$	$+ 0.025$
1861,	$= V$	$+ 0.013$	$- 0.013$	$+ 0.110$
1868,	$= V$	$- 0.015$	$+ 0.015$	$- 0.128$
1881,	$= V$	$- 0.004$	$+ 0.004$	$- 0.048$

MAY TRANSITS.

1740,	$\delta v - \cos i\,\delta l' = W$	$+ 0.006\delta\lambda$	$- 0.006\delta\pi$	$+ 0.079\delta e$
1753,	$= W$	$- 0.004$	$+ 0.004$	$- 0.045$
1786,	$= W$	$+ 0.005$	$- 0.005$	$+ 0.062$
1799,	$= W$	$- 0.004$	$+ 0.004$	$- 0.045$
1832,	$= W$	$+ 0.003$	$- 0.003$	$+ 0.046$
1845,	$= W$	$- 0.005$	$+ 0.005$	$- 0.060$
1878,	$= W$	$+ 0.002$	$- 0.002$	$+ 0.030$

The rigorous course would now be, in forming the equations of condition, to transfer the small terms in $\delta\lambda$, $\delta\pi$, and δe to the second members of the equations, and retain $\delta\lambda$, $\delta\pi$, and δe in a symbolic form in the solution But the corrections to the elements of Mercury are so small that it can hardly be practicable to determine them without an uncertainty equal to their tenth part. Their coefficients in the equations are always much less than 0.1, and it is probable that in the final values of the unknown quantities these corrections would not exceed 0.01. We may, therefore, in the solution, neglect these small terms entirely.

The semi-diameters of the two bodies are also two quantities which cannot be separately determined. If we put

$$S = \delta R' - 1.60\ \delta R$$

and replace $\delta R'$ by S in the general equations (6) and (7) we shall have

$$2.18\ \delta R' - 3.19\ \delta R = 2.18\ S + 0.31\ \delta R$$
$$1.22\ \delta R' - 2.21\ \delta R = 1.22\ S - 0.26\ \delta R$$

δR, the correction to the semi-diameter of Mercury at distance unity, cannot exceed a small fraction of a second, the terms in δR may therefore be regarded as insensible, and we may consider the equations as determining S alone.

Finally, we shall put, for convenience in solving the equations,

$$N = \delta\theta$$
$$M = 10\ \delta\mu$$

The equations for correcting the tabular times of contact will then be:

FOR A NOVEMBER TRANSIT.

$$\begin{aligned} n\delta t = {} & \sin(\omega + i)\ N \\ & + \cos(\omega + i)\ V \\ & + \cos(\omega + i)\ \frac{P}{10}\ M \\ & - 2.2\ S \end{aligned} \qquad (3)$$

FOR A MAY TRANSIT.

$$\begin{aligned} n\delta t = {} & \sin(\omega - i)\ N \\ & + \cos(\omega - i)\ W \\ & + \cos(\omega - i)\ \frac{P}{10}\ M \\ & - 1.2\ S \end{aligned} \qquad (3)'$$

Here δt is the difference between the observed and tabular times of internal contact. This difference gives rise to another question for consideration.

§ 6.

Introduction of a term depending on hypothetical variations of the earth's rotation.

In several papers published in the *American Journal of Science and Arts* during the past twelve years the author has called attention to the fact that the mean motion of the moon is apparently subject to certain inequalities of long period which are not accounted for by any existing theory. He therefore suggested that this apparent inequality might really be due, not to the moon's motion, but to inequalities in the axial rotation of the earth on which our astronomical reckoning of time necessarily depends. It was pointed out that this question could best be settled by observations on other rapidly moving bodies, with a view of determining whether they also show apparent inequalities which could be accounted for in the same way. Eclipses of Jupiter's satellites and transits of Mercury were especially suggested as suitable for this object.

The results of the *Researches on the Motion of the Moon*, published in 1878, were such as to encourage the belief that the observed inequality was really in the moon's motion. It was in fact found that the moon's mean motion for about 250 years could

be represented with approximate accuracy by the addition of a single term with a period not differing greatly from 300 years. Since it seemed quite improbable that the inequality in the earth's rotation should be periodic, the balance of probability seemed in favor of the inequality being in the moon itself. But since theory had entirely failed to show any such inequality in the moon's motion the question had still to be regarded as unsettled.

Transits of Mercury have now been observed for two centuries, and for a century and a half the times of contact may be considered as determined within a very few seconds. Taking as the standard of time the earth's axial rotation between 1750 and 1850, and assuming that the observed inequalities in the moon's mean motion are to be accounted for by actual inequalities in the earth's rotation, then our measurement of time would be in error by amounts ranging from 17 seconds in one direction to 17 seconds in the other direction between 1723 and 1881. Inequalities of this amount could not fail to be indicated by the preceding observations on the transits of Mercury.

At the same time, considering the imperfections of the older observations, these assumed inequalities are not so many times greater than the possible errors of observation as to make them evident without careful treatment. Let us then consider what method is best adapted to decide the question.

On page 266 of the *Researches on the Motion of the Moon* is given the errors with which the astronomical determinations of time must be supposed affected, in order that the apparent inequalities in the moon's mean motion not yet accounted for by theory may be represented.

The following table shows the amount of these errors when interpolated to the times of the several observed transits of Mercury.

Year of transit.	Δt	Year of transit.	Δt
	s.		s.
1677	+ 33	1789	− 18
1690	+ 29	1799	− 17
1697	+ 26	1802	− 16
1723	+ 17	1822	− 9
1736	+ 9	1832	− 6
1740	+ 6	1845	− 2
1743	+ 4	1848	0
1753	− 2	1861	+ 2
1756	− 4	1868	+ 10
1769	− 12	1878	+ 15
1782	− 17	1881	+ 16
1786	− 18		

It is to be remarked that these times are subject to a probable uncertainty of two or three seconds, arising from the fact that HANSEN's tables have not been directly compared with observations of the moon between the years 1750 and 1840. It is, however, known that the errors during this period must be small, and they have necessarily been assumed to vanish in determining the value of the hypothetical error of time.

The most natural method of making the required investigation would be to form two solutions of the equations of condition afforded by transits of Mercury, one with and the other without this hypothetical correction, and to find which solution gives the smallest residuals. But, in a case like the present, in which we must not except a striking difference in the magnitude of the residuals, the result of the solutions would not necessarily be conclusive. What we shall do, therefore, will be to assume that the values of Δt should all be multiplied by a constant factor k, and to determine from the equations of condition that value of k which best satisfies the observations.

If the hypothesis of perfect uniformity in the earth's rotation is the true one, the value of k should vanish.

If the observed inequalities in the moon's mean motion arise from the cause supposed, the value of k should come out nearly equal to unity.

If k should come out different from either zero or unity by an amount greater than its possible error it would tend to show that both causes might be in operation.

Closely associated with the value of k is another constant which it would be desirable to determine. That a tidal retardation of the earth's rotation must exist can scarcely be doubted, although no reliable estimate of its amount has yet been made. We must, therefore, suppose that our astronomical measures of time need a correction of the form cT^2, in which T is the time reckoned from any standard epoch and c is a minute constant. If we seek to determine the possible value of c from transits of Mercury we should introduce it into the equations of condition. But it will be noticed that during the period within which transits of Mercury have been observed with any accuracy the coefficient k will be of the same general kind with c, so that k and c can not be separately determined. In fact, we shall find the values of Δt to be closely represented by the formula

$$-18^s + 60^s\, T^2$$

If, therefore, we should introduce c as an additional unknown quantity it could not be determined independently of k, but our equations would give the value only of a linear function of c and k. The relation between the two quantities is such that, supposing the true value of k to be zero, the existence of a regular tidal retardation would be indicated by a small negative value of k.

§ 7.

Numerical comparison of observed and tabular quantities, with the resulting equations of condition.

In the first of the following tables is given the comparison of the observed and tabular times of contacts, to be subsequently used as a check upon the equations of condition. The following columns are the ones which seem to need explanation.

The third column gives the Greenwich mean times of geocentric contact derived from the observations already given in Part I. Since, however, the time itself, as determined from astronomical observations, hypothetically needs a correction $-k\Delta t$, this correction is added symbolically to the observed time to render it strictly comparable with the tabular time.

The next column gives the adopted weight of the observation, which refers, not merely to the time, but to the distance of centers. No precise formula has been applied in determining these weights because of the extremely heterogenous character of the data. As a rule, the result of five fairly accordant and satisfactory observations of internal contact is considered entitled to weight 1 But the weight is not proportional to the number of observations, but varies in a less degree, so that 6 is the maximum weight for any one transit. Moreover, account is taken of the skill of the observers and the general accordance and certainty of the observations.

Next we have the tabular times, the computation of which has already been given, followed by the symbolic corrections produced by corrections to the elements.

Comparison of Observed and Tabular Geocentric Contacts.

I.—NOVEMBER TRANSITS, INTERIOR CONTACTS.

Contact.	Date.	Observed G. M. T. of geoc. contact.				Wt.	Tabular times, with symbolic corrections.							
		h.	m.	s.	s.		h.	m.	s.					
II	1677, Nov. 6	21	34	1	− 33k	0.1	21	33	31.6	+ 1.1N	+ 4.8V	+ 6.3M	− 10.7R′	+ 15.6R
III	1677, Nov. 7	2	47	27	− 33k	0.1	2	48	12.9	− 1.4	+ 4.7	− 6.4	+ 10.7	− 15.6
III	1697, Nov. 2	19	42	53	− 26k	0.3	19	42	47.4	+ 4.2	+ 4.8	+ 1.7	+ 13.9	− 20.3
II	1723, Nov. 9	2	26	52	− 17k	2.0	2	26	55.0	+ 1.7	+ 4.8	+ 5.5	− 11.1	+ 16.2
II	1736, Nov. 10	21	10	30	− 9k	1.0	21	10	28.3	+ 8.5	+ 5.0	− 2.2	− 21.4	+ 31.3
III	1736, Nov. 10	23	48	51	− 9k	1.0	23	49	1.3	− 8.5	+ 4.5	+ 2.3	+ 21.0	− 30.7
II	1743, Nov. 4	20	14	21	− 4k	1.0	20	14	24.2	− 3.4	+ 4.7	− 7.0	− 12.5	+ 18.3
III	1743, Nov. 5	0	45	5	− 4k	1.5	0	45	6.4	+ 3.1	+ 4.8	+ 7.0	+ 12.5	− 18.2
II	1769, Nov. 9	7	22	47	+ 12k	1.0	7	22	42.5	+ 2.4	+ 4.8	+ 6.5	− 11.7	+ 17.2
III	1769, Nov. 9	12	9	51	+ 12k	0.2	12	9	54.2	− 2.6	+ 4.7	− 6.6	+ 11.7	− 17.2
III	1782, Nov. 12	2	42	6	+ 17k	3.0	2	41	59.5	+ 22.1	+ 5.3	− 23.1	− 49.5	+ 72.5
III	1782, Nov. 12	3	49	37	+ 17k	3.0	3	50	9.5	− 21.8	+ 4.2	+ 22.6	+ 48.4	− 70.9
II	1789, Nov. 5	0	53	2	+ 18k	2.0	0	53	7.4	− 2.5	+ 4.7	− 6.3	− 11.6	+ 17.0
III	1789, Nov. 5	5	44	12	+ 18k	1.0	5	44	16.9	+ 2.2	+ 4.8	+ 6.2	+ 11.5	− 16.9
III	1802, Nov. 8	23	41	5	+ 16k	3.0	23	41	8.4	− 0.4	+ 4.7	− 1.1	+ 10.3	− 15.1
II	1822, Nov. 4	13	3	42	+ 9k	0.5	13	3	45.6	− 8.5	+ 4.5	− 11.8	− 21.0	+ 30.7
III	1822, Nov. 4	15	45	18	+ 9k	1.0	15	45	13.6	+ 8.3	+ 4.9	+ 11.9	+ 21.0	− 30.7
II	1848, Nov. 8	23	6	47	− 0k	5.0	23	6	42.8	+ 0.7	+ 4.8	+ 6.6	− 10.5	+ 15.3
III	1848, Nov. 9	4	28	8	− 0k	0.3	4	28	4.1	− 0.9	+ 4.7	− 6.6	+ 10.5	− 15.3
II	1861, Nov. 11	17	20	16	− 2k	0.7	17	20	17.9	+ 4.3	+ 4.8	− 8.8	− 14.2	+ 20.7
III	1861, Nov. 11	21	18	20	− 2k	5.0	21	18	16.8	− 4.5	+ 4.6	+ 8.8	+ 14.1	− 20.6
II	1868, Nov. 4	17	27	0	− 10k	0.5	17	27	43.6	− 5.7	+ 4.6	− 4.5	− 15.9	+ 23.3
III	1868, Nov. 4	21	0	9.8	− 10k	6.0	20	59	57.5	+ 5.4	+ 4.9	+ 4.4	+ 15.9	− 23.3
II	1881, Nov. 7	10	18	38	− 16k	3.0	10	18	15.5	− 1.3	+ 4.7	+ 6.1	− 10.6	+ 15.6
III	1881, Nov. 7	15	35	54	− 16k	3.0	15	35	37.3	+ 1.1	+ 4.8	− 6.1	+ 10.6	− 15.6

II.—MAY TRANSITS, INTERIOR CONTACTS.

Contact.	Date.	h.	m.	s.	s.	Wt.	h.	m.	s.					
II	1740, May 2	9	43	9	− 6k	0.1	9	41	50.8	+ 34.2N	+ 10.4V	+ 13.8M	− 43.6R′	− 78.9R
III	1753, May 5	22	6	0.5	+ 2k	1.5	22	5	52.4	+ 2.7	+ 12.9	− 9.7	+ 16.1	− 29.1
II	1786, May 3	14 / 15	59 / 0	25 / 8	+ 18k	0.3	15	0	26.6	+ 14.0	+ 11.7	− 9.4	− 22.2	+ 40.2
III	1786, May 3	20	21	27	+ 18k	2.0	20	21	0.5	− 12.8	+ 13.4	+ 9.6	+ 22.6	− 40.9
II	1799, May 6	21	9	42	+ 17k	1.5	21	9	44.9	− 4.0	+ 12.7	− 17.1	− 16.3	+ 29.5
III	1799, May 7	4	30	32	+ 17k	2.0	4	30	37.8	+ 5.5	+ 12.1	+ 16.9	+ 16.3	− 29.5
II	1832, May 4	21	3	30	+ 6k	3.0	21	3	46.7	+ 8.2	+ 12.0	+ 17.4	− 17.7	+ 32.0
III	1832, May 5	3	46	40	+ 6k	3.0	3	46	55.2	− 6.7	+ 12.9	− 17.4	+ 17.7	− 32.0
II	1845, May 8	4	23	50	+ 2k	4.0	4	24	14.6	− 8.0	+ 12.9	− 6.9	− 18.5	+ 33.5
III	1845, May 8	10	49	7	+ 2k	4.0	10	49	34.1	+ 9.5	+ 11.8	+ 7.0	+ 18.5	− 33.5
II	1878, May 6	3	15	49.2	− 15k	6.0	3	16	6.6	+ 4.6	+ 12.1	− 10.2	− 15.8	+ 28.7
III	1878, May 6	10	43	41.2	− 15k	4.0	10	44	0.5	− 3.2	+ 12.8	+ 10.3	+ 16.1	− 29.1

Comparison of Observed and Tabular Contacts.

NOVEMBER TRANSITS, EXTERIOR CONTACTS.

Date.	Observed G. M. T. of geoc. contact.	Wt.	Tabular time.
	h. m. s. s.		h. m. s.
1677, Nov. 7	2 49 33 − 33k	0.1	2 49 55
1690, Nov. 9	19 28 54 − 29k	0.2	19 29 25
1697, Nov. 2	19 44 39 − 26k	0.3	19 45 2
1736, Nov. 10	23 51 50 − 9k	0.6	23 52 21
1743, Nov. 5	0 46 54 − 4k	0.7	0 47 7
1756, Nov. 6	18 54 34 + 4k	0.3	18 55 12
1769, Nov. 9	12 11 26 + 12k	0.2	12 11 45
1782, Nov. 12	3 56 6 + 17k	1.0	3 57 17
1789, Nov. 5	5 46 8 + 18k	0.2	5 46 9
1802, Nov. 8	23 42 34 + 16k	1.2	23 42 37
1822, Nov. 4	15 48 13 + 9k	0.2	15 48 37
1848, Nov. 8	4 29 40 0k	0.2	4 29 42
1861, Nov. 11	21 20 27 − 2k	1.5	21 20 14
1868, Nov. 4	21 2 33 − 10k	2.0	21 2 32
1881, Nov. 7	15 37 33 − 16k	1.5	15 37 20

MAY TRANSITS, EXTERIOR CONTACTS.

Date.	Observed G. M. T. of geoc. contact.	Wt.	Tabular time.
1753, May 5	22 8 45 + 2k	1.0	22 8 53
1786, May 3	20 25 3 + 18k	1.0	20 25 18
1799, May 7	4 33 16 + 17k	1.0	4 33 50
1832, May 5	3 49 52 + 6k	1.2	3 50 21
1845, May 8	10 52 35 + 2k	1.0	10 53 14
1878, May 6	10 46 23 − 15k	1.5	10 47 2

The equations of condition might now be formed directly from this comparison. But, in order to secure the greatest amount of certainty in the results, the absolute terms of the equations have been independently determined by computing the values of $c - r$ for the concluded observed moments. The following table shows the results of the two methods of determining these terms:

Interior Contacts.

NOVEMBER TRANSITS.

Date.	Contact.	n	$n\delta t$	c_0	r_0	$c_0 - r_0$
		$''$	$''$	$''$	$''$	$''$
1677	II	− 0.204	− 6.0	2074.61	2080.77	− 6.16
	III	+ 0.204	− 9.4	2076.78	2086.26	− 9.48
1697	III	+ 0.157	+ 0.87	2068.10	2067.27	+ 0.83
1723	II	− 0.197	+ 0.59	2082.89	2082.32	+ 0.57
1736	II	− 0.102	− 0.17	2092.39	2092.55	− 0.16
	III	+ 0.104	− 1.07	2094.03	2095.10	− 1.07
1743	II	− 0.174	+ 0.56	2064.33	2063.80	+ 0.53
	III	+ 0.175	− 0.24	2068.90	2069.14	− 0.24
1769	II	− 0.186	− 0.84	2082.96	2083.80	− 0.84
	III	+ 0.186	− 0.60	2088.15	2088.80	− 0.65
1782	II	− 0.044	− 0.29	2094.32	2094.55	− 0.23
	III	+ 0.045	− 1.46	2094.17	2095.62	− 1.45
1789	II	− 0.188	+ 1.01	2066.06	2065.05	+ 1.01
	III	+ 0.189	− 0.92	2069.58	2070.75	− 1.17
1802	III	+ 0.211	− 0.72	2080.45	2081.16	− 0.71
1822	II	− 0.104	+ 0.37	2057.46	2057.09	+ 0.37
	III	+ 0.104	+ 0.46	2060.86	2060.40	+ 0.46
1848	II	− 0.208	− 0.87	2075.80	2076.68	− 0.88
	III	+ 0.208	+ 0.81	2083.26	2082.52	+ 0.74
1861	II	− 0.154	+ 0.29	2087.17	2086.87	+ 0.30
	III	+ 0.155	+ 0.50	2091.34	2090.85	+ 0.49
1868	II	− 0.137	+ 6.0	2063.97	2058.07	+ 5.90
	III	+ 0.137	+ 1.63	2064.04	2062.41	+ 1.63
1881	II	− 0.205	− 4.61	2063.34	2067.98	− 4.64
	III	+ 0.205	+ 3.42	2077.58	2074.06	+ 3.52
			MAY TRANSITS.			
1740	II	− 0.028	− 2.19	1171.17	1173.37	− 2.20
1753	III	+ 0.076	+ 0.62	1159.31	1158.64	+ 0.67
1786	II	− 0.055	+ 3.39 + 1.02	1175.91 1173.45	1172.39	+ 3.52 + 1.06
	III	+ 0.054	+ 1.43	1170.64	1169.37	+ 1.27
1799	II	− 0.075	+ 0.22	1161.09	1160.86	+ 0.23
	III	+ 0.075	− 0.44	1156.69	1157.12	− 0.43
1832	II	− 0.069	+ 1.15	1172.19	1170.99	+ 1.18
	III	+ 0.069	− 1.05	1166.18	1167.26	− 1.08
1845	II	− 0.066	+ 1.62	1160.64	1159.02	+ 1.62
	III	+ 0.066	− 1.79	1153.99	1155.75	− 1.76
1878	II	− 0.077	+ 1.34	1170.84	1169.47	+ 1.37
	III	+ 0.076	− 1.47	1163.84	1165.36	− 1.52

Exterior Contacts.

NOVEMBER TRANSITS.

	$''$		$''$
1677,	$n\delta t = -4.5$	1789,	$n\delta t = -0.2$
1690,	-4.2	1802,	-0.6
1697,	-3.6	1822,	-2.5
1736,	-3.2	1848,	-0.4
1743,	-2.3	1861,	$+2.0$
1756,	-8.0:	1868,	$+0.1$
1769,	-3.5	1881,	$+2.7$
1782,	-3.2		

MAY TRANSITS.

1753,	$n\delta t = -0.6$	1832,	$n\delta t = -2.0$
1786,	-0.8	1845,	-2.6
1799,	-2.6	1878,	-3.0

The quantities thus obtained under the heads $n\delta t$ and $c_0 - r_0$ are the absolute terms of the equation of condition which are next given. The unknown quantities which enter into these equations and the expressions for the coefficients have already been given in part in § 5.

The method of forming the equations is as follows:

The datum supposed to be given by each time of contact derived from observation is that, at a certain moment of apparent astronomical time, the heliocentric distance of centers of the earth and Mercury was equal to the sum or difference of their semi-diameters. The requirement of the equation thence derived is that, for this same moment when reduced to absolute time, the tabular quantities, when affected by the proper symbolic corrections, shall give the same equality. We now have—

Moment of observation, in absolute time, $t_0 - k\Delta t$.

At this moment, $c = r$.

If we put c_0 and r_0 for the values of c and r given on page 454 we have, for the theoretical terms:

Moment of computation, in absolute time, t_0.

At this moment—

$$\begin{aligned} c &= c_0 \\ &+ \sin(\omega \pm i)\,\mathrm{N} \\ &+ \cos(\omega \pm i)\,(\mathrm{V \text{ or } W}) \\ &+ \cos(\omega \pm i)\,\frac{\mathrm{P}}{10}\,\mathrm{M} \end{aligned}$$

$$r = r_0 + 2.18\ \mathrm{S} \text{ for November}$$
$$r = r_0 + 1.22\ \mathrm{S} \text{ for May.}$$

To reduce the tabular value of c to the moment $c - k_0\Delta t$ it is necessary to apply the farther correction

$$-\,nk\Delta t$$

The quantities N, V, and W are not constants, but are subjected to a secular variation. We must therefore suppose

$$\begin{aligned} N &= N_0 + N't \\ V &= V_0 + V't \\ W &= W_0 + W't \end{aligned}$$

t being the time from an arbitrary mean epoch.

The equation $c - r = 0$ now becomes,

FOR NOVEMBER,

$$\begin{aligned} 0 = &\sin (\omega + i) (N_0 + N't) \\ &+ \cos (\omega + i) (V_0 + V't) \\ &+ \cos (\omega + i) \frac{P}{10} M \\ &- 2.18\, S - nk\Delta t \\ &+ c_0 - r_0 \end{aligned}$$

FOR MAY,

$$\begin{aligned} 0 = &\sin (\omega - i) (N_0 + N't) \\ &+ \cos (\omega - i) (W_0 + W't) \\ &+ \cos (\omega - i) \frac{P}{10} M \\ &- 1.22\, S - nk\Delta t \\ &+ c_0 - r_0 \end{aligned}$$

The following explanations on special points are, however, necessary:

Exterior contacts.—In combining exterior contacts with interior ones, it is necessary to avoid as far as possible the introduction of any possible systematic error arising from the different methods of observing the two classes of phenomena. Such conditions may be expected to arise from the fact that the external tangency of the limbs cannot be really observed. The time noted by the observer is that at which the notch made by Mercury in the sun's limb became so small that he could no longer see it. This magnitude is an unknown quantity, to be determined from the observations, and the functions of the semi-diameters which enter into the expression for external contact must be considered as entirely independent of that for internal contact.

Again, the magnitude of the notch when the observer loses sight of it will depend upon the optical power of his telescope and the condition of the atmosphere. Now the optical power of the telescope has gradually improved from the time of observation of the first transit until the present. The magnitude of the last visible notch must therefore be considered as subject to a gradual variation during the period of observations of the transit. We may without danger of serious error suppose this change to have been proportional to the time. The function of the semi-diameters which enters

into the equations must therefore be supposed affected by a secular variation. Since the time of observation depends upon the optical power of the telescope, which varies with different observers, the question arises whether we are to apply corrections depending on the telescope. This is impracticable in the greater number of cases from want of the necessary data. Variations arising from differences of telescopic power must therefore be regarded as merged with the accidental errors. The question how far the accidental errors of observation will thus be increased is a serious one, to be settled only by a comparison of results.

It is an observed fact that if we reject those observations in which the telescopic power was insufficient, or in which the observer evidently could not have seen the smallest visible notch, it is found that the discordance among the observations of external contact are not enormously greater than among those of internal contact. Now, as it cannot be supposed that the observations at one transit are made with instruments systematically different from those at another transit, the result is that the probable error arising from differences of telescopic power cannot be regarded as many times greater than the regular errors of internal contact.

It is however proper to remark that the weights assigned to the observations of internal contacts in this discussion have been below rather than above that to which the author would consider them fairly entitled.

It may be questioned whether there may not be a similar progression in the observations of internal contact arising from differences of telescopic power. That such a systematic change could be found among an infinity of observations cannot be doubted. But the observations actually made do not seem to afford any sufficient data for its investigation. As a general rule, there appears to be no marked difference between observations at the same transit made with instruments of different powers. The same thing may therefore be supposed true of the earlier and later observations of transits. The fact that eleven unknown quantities are already introduced into the equations of condition affords another reason for laying the discussion of this question aside.

But there are two cases in which a difference of this kind is evident, the one the transit of 1677, the other that of 1756. The time of duration observed by HALLEY seems to indicate that his observed time of ingress was too late, and that of egress too early, a circumstance which we may attribute to deficiency of optical power. This difference is so much more striking than in the case of the following transits that the equations of condition given by HALLEY's observations have been combined into one in such a way as to eliminate the semi-diameter.

The observations of 1756, which are also exceptional from the same apparent cause, have been rejected entirely.

Owing to the small weight assigned to the observations of external contact, it has not been deemed necessary to form separate equations for them. The coefficients of the unknown quantities have therefore been assumed to be the same as those corresponding to internal contact.

If the errors of LEVERRIER's tables were of considerable magnitude this course would not be advisable, but since they must be regarded as almost vanishing quanti-

ties it does not appear that any serious error will result from using the same coefficients in the two cases. In fact, the preceding identity of coefficients supposes merely that the tabular interval between external and internal contact is absolutely correct, a function of the semi-diameter alone excepted.

Two modifications have been made in the equations as thus derived.

I. It was long a subject of embarrassment what to do with HALLEY'S observations on the transit of 1677. As already remarked, the two phases are discordant by more than a minute. To express the result in another form the semi-diameter of the sun, as it would result from his observations, is some $3''.5$ less than its true value. After much consideration the conclusion was finally reached that this discordance was due not so much to an error in time as to a personality in HALLEY'S method of observing the contact. Accepting this hypothesis the mean result of his observations of ingress and egress would be correct. The difference of the equations resulting from his two observations was therefore taken as entitled to a small weight, and the semi-diameter was thus eliminated from the result.

II. In assigning the relative weights given in the preceding section no account was taken of the fact that Mercury is nearer the earth in a May transit than in a November one. An error of $1''$ in the heliocentric place would, in November, cause an error of $0''.46$ in the geocentric place, and in May an error of $0''.80$. Hence the heliocentric place can be determined with more accuracy by a May observation than by a November one. The weights of the May transits, as given in Part I, were therefore all multiplied by 2

III. In discussing the observations the question what to do with the second contacts observed in 1740 and 1786 was laid aside. These observations were, therefore, omitted with the view of seeing how they would be represented by the concluded theory. It would seem from this that internal contact must have actually passed some time before the moment at which WINTHROP noted it as not having occurred, and, therefore, that his observation is affected with some undiscoverable error. It would also appear that the second hypothesis respecting the internal contact of 1786 is the one to be accepted, but it was not thought worth while to re-solve the equations of condition.

The observations of 1756 have been entirely dropped, as the weight to which they could be considered entitled is too small to have any influence on the result.

Equations of Condition.

INTERNAL CONTACTS IN NOVEMBER.

											Wt.
1677,	II	$0 = -0.23N_0$	$+0.33N'$	$-0.98V_0$	$+1.40V'$	$+1.23M$	$-2.2S$	$+6.7k$	$-$	$6''.1$	Rej.
	III	$0 = -0.28$	$+0.40$	$+0.96$	-1.38	-1.24	-2.2	-6.7	$-$	9.5	Rej.
1677,	II and III	$0 = +0.02$	-0.03	-0.97	$+1.39$	$+1.24$	0.0	$+6.7$	$+$	1.7	0.3
1697,	III	$0 = +0.65$	-0.80	$+0.75$	-0.93	$+0.21$	-2.2	-4.1	$+$	0.9	0.3
1723,	II	$0 = -0.34$	$+0.33$	-0.95	$+0.92$	$+1.02$	-2.2	$+3.4$	$+$	0.6	2.0
1736,	II	$0 = -0.86$	$+0.72$	-0.51	$+0.43$	-0.11	-2.2	$+0.9$	$-$	0.2	1.0
	III	$0 = -0.88$	$+0.74$	$+0.47$	-0.40	$+0.11$	-2.2	-0.9	$-$	1.1	1.0
1743,	II	$0 = +0.58$	-0.45	-0.81	$+0.62$	-0.98	-2.2	$+0.7$	$+$	0.5	1.0
	III	$0 = +0.54$	-0.42	$+0.84$	-0.64	$+1.01$	-2.2	-0.7	$-$	0.2	1.5
1769,	II	$0 = -0.45$	$+0.23$	-0.90	$+0.46$	$+1.08$	-2.2	-2.2	$-$	0.8	1.0
	III	$0 = -0.49$	$+0.25$	$+0.88$	-0.45	-1.05	-2.2	$+2.2$	$-$	0.6	0.2

Equations of Condition—Continued.

INTERNAL CONTACTS IN NOVEMBER.

										″	Wt.
1782,	II	o =	− 0.97N_0	+ 0.37N'	− 0.23V_0	+ 0.09V'	− 0.23M	− 2.2S	− 0.8k	− 0.2	3.0
	III	o =	− 0.98	+ 0.37	+ 0.19	− 0.07	+ 0.19	− 2.2	+ 0.8	− 1.5	3.0
1789,	II	o =	+ 0.48	− 0.15	− 0.87	+ 0.27	− 1.05	− 2.2	− 3.4	+ 1.0	2.0
	III	o =	+ 0.43	− 0.13	+ 0.90	− 0.28	+ 1.07	− 2.2	+ 3.4	− 1.2	1.0
1802,	III	o =	− 0.09	+ 0.02	+ 1.00	− 0.17	− 0.23	− 2.2	+ 3.4	− 0.7	3.0
1822,	II	o =	+ 0.88	+ 0.02	− 0.46	− 0.01	− 0.57	− 2.2	− 0.9	+ 0.4	0.5
	III	o =	+ 0.86	+ 0.02	+ 0.51	+ 0.01	+ 0.64	− 2.2	+ 0.9	+ 0.5	1.0
1848,	II	o =	− 0.14	− 0.04	− 0.99	− 0.28	+ 1.35	− 2.2	0.0	− 0.9	5.0
	III	o =	− 0.19	− 0.05	+ 0.98	+ 0.28	− 1.34	− 2.2	0.0	+ 0.8	0.3
1861,	II	o =	− 0.66	− 0.27	− 0.75	− 0.31	− 1.02	− 2.2	+ 0.3	+ 0.3	0.7
	III	o =	− 0.70	− 0.29	+ 0.72	+ 0.30	+ 0.97	− 2.2	− 0.3	+ 0.5	5.0
1868,	II	o =	+ 0.78	+ 0.37	0.62	− 0.30	− 0.39	− 2.2	+ 1.4	+ 5.9	0.5
	III	o =	+ 0.75	+ 0.36	+ 0.66	+ 0.32	+ 0.41	− 2.2	− 1.4	+ 1.6	6.0
1881,	II	o =	+ 0.26	+ 0.16	− 0.96	− 0.59	+ 1.20	− 2.2	+ 3.3	− 4.6	3.0
	III	o =	+ 0.22	+ 0.13	+ 0.97	+ 0.59	− 1.21	− 2.2	− 3.3	+ 3.5	3.0

INTERNAL CONTACTS IN MAY.

1740,	II	o =	− 0.96N_0	+ 0.77N'	− 0.29W_0	+ 0.23W'	+ 0.11M	− 1.2S	+ 0.2k	− 2.2	Rej.
1753,	III	o =	+ 0.20	− 0.14	+ 0.97	− 0.65	− 0.72	− 1.2	+ 0.2	+ 0.6	3.0
1786,	II	o =	− 0.77	+ 0.26	− 0.64	+ 0.22	− 0.33	− 1.2	− 1.0	+ 3.4 + 1.0	Rej.
	III	o =	− 0.69	+ 0.27	+ 0.73	− 0.23	+ 0.38	− 1.2	+ 1.0	+ 1.3	4.0
1799,	II	o =	+ 0.30	− 0.06	− 0.95	+ 0.20	− 1.22	− 1.2	− 1.3	+ 0.2	3.0
	III	o =	+ 0.41	− 0.09	+ 0.90	− 0.19	+ 1.16	− 1.2	+ 1.3	− 0.4	4.0
1832,	II	o =	− 0.56	− 0.07	− 0.83	− 0.10	+ 0.99	− 1.2	− 0.4	+ 1.2	6.0
	III	o =	− 0.46	− 0.06	+ 0.89	+ 0.11	− 1.06	− 1.2	+ 0.4	− 1.1	6.0
1845,	II	o =	+ 0.53	+ 0.13	− 0.84	− 0.21	− 0.38	− 1.2	− 0.1	+ 1.6	8.0
	III	o =	+ 0.63	+ 0.16	+ 0.77	+ 0.19	+ 0.36	− 1.2	+ 0.1	− 1.8	8.0
1878,	II	o =	− 0.36	− 0.21	− 0.94	− 0.54	− 0.73	− 1.2	+ 1.1	+ 1.3	12.0
	III	o =	− 0.25	− 0.14	+ 0.97	+ 0.56	+ 0.76	− 1.2	− 1.1	− 1.5	8.0

EXTERNAL CONTACTS IN NOVEMBER.

										″	Wt.
1677,	o =	− 0.28N_0	+ 0.40N'	+ 0.96V_0	− 1.38V'	− 1.24M	− 2.2S_1	+ 3.1S_1'	− 6.7k	− 4.5	0.1
1690,	o =	− 0.78	+ 1.02	+ 0.63	− 0.82	− 0.02	− 2.2	+ 2.9	− 3.9	− 4.3	0.2
1697,	o =	+ 0.65	− 0.80	+ 0.75	− 0.93	+ 0.21	− 2.2	+ 2.7	− 4.1	− 3.6	0.3
1736,	o =	− 0.88	+ 0.74	+ 0.47	− 0.40	+ 0.11	− 2.2	+ 1.8	− 0.9	− 3.2	0.6
1743,	o =	+ 0.54	− 0.42	+ 0.84	− 0.64	+ 1.01	− 2.2	+ 1.7	− 0.7	− 2.3	0.7
1769,	o =	− 0.49	+ 0.25	+ 0.88	− 0.45	− 1.05	− 2.2	+ 1.1	+ 2.2	− 3.5	0.2
1782,	o =	− 0.98	+ 0.37	− 0.19	− 0.07	+ 0.19	− 2.2	+ 0.8	+ 0.8	− 3.2	1.0
1789,	o =	+ 0.43	− 0.13	+ 0.90	− 0.28	+ 1.07	− 2.2	+ 0.7	+ 3.4	− 0.2	0.2
1802,	o =	− 0.09	+ 0.02	+ 1.00	− 0.17	− 0.23	− 2.2	+ 0.4	+ 3.4	− 0.6	1.2
1822,	o =	+ 0.86	+ 0.02	+ 0.51	+ 0.01	+ 0.64	− 2.2	− 0.0	+ 0.9	− 2.5	0.2
1848,	o =	− 0.19	− 0.05	+ 0.98	+ 0.28	− 1.34	− 2.2	− 0.6	0.0	− 0.4	0.2
1861,	o =	− 0.70	− 0.29	+ 0.72	+ 0.30	+ 0.97	− 2.2	− 0.9	− 0.3	+ 2.0	1.5
1868,	o =	+ 0.75	+ 0.36	+ 0.66	+ 0.32	+ 0.41	− 2.2	− 1.1	− 1.4	+ 0.1	2.0
1881,	o =	+ 0.22	+ 0.13	+ 0.97	+ 0.59	− 1.21	− 2.2	− 1.3	− 3.3	+ 2.7	1.5

EXTERNAL CONTACTS IN MAY.

1753,	o =	+ 0.20N_0	− 0.14N'	+ 0.97W_0	− 0.65W'	− 0.72M	− 1.2S_1	+ 0.8S_1'	+ 0.2k	− 0.6	2.0
1786,	o =	− 0.69	+ 0.27	+ 0.73	− 0.23	+ 0.38	− 1.2	+ 0.4	+ 1.0	− 0.8	2.0
1799,	o =	+ 0.41	− 0.09	+ 0.90	− 0.19	+ 1.16	− 1.2	+ 0.3	+ 1.3	− 2.6	2.0
1832,	o =	− 0.46	− 0.06	+ 0.89	+ 0.11	− 1.06	− 1.2	− 0.1	+ 0.4	− 2.0	2.4
1845,	o =	+ 0.63	+ 0.16	+ 0.77	+ 0.19	+ 0.36	− 1.2	− 0.3	+ 0.1	− 2.6	2.0
1878,	o =	− 0.25	− 0.14	+ 0.97	+ 0.56	+ 0.76	− 1.2	− 0.7	− 1.1	− 3.0	3.0

These equations of condition, when treated by the method of least squares, lead to the following normal equations:

$$\begin{aligned}
(1)\ &+ 37.809N_0 - 0.177N' + 2.817V_0 - 0.068V' + 1.339W_0 + 1.025W' \\
&\qquad - 1.072M + 10.380S + 1.620S_1 - 2.035S_1' - 16.549k + 7.892 = 0
\end{aligned}$$

$$\begin{aligned}
(2)\ &- 0.177N_0 + 7.571N' - 0.204V_0 + 1.443V' + 1.322W_0 + 0.720W' \\
&\qquad + 1.160M - 7.130S - 2.104S_1 + 0.212S_1' + 0.066k - 2.822 = 0
\end{aligned}$$

$$\begin{aligned}
(3)\ &+ 2.817N_0 - 0.204N' + 33.765V_0 + 1.996V' + 0.000W_0 + 0\ \ 00W' \\
&\qquad - 6.813M - 5.346S - 15.039S_1 - 1.008S_1' - 19.446k + 27.994 = 0
\end{aligned}$$

$$\begin{aligned}
(4)\ &- 0.068N_0 + 1.443N' + 1.996V_0 + 10.099V' + 0.000W_0 + 0.000W' \\
&\qquad - 4.072M - 6.981S - 0.757S_1 - 5.419S_1' - 11.000k + 30.263 = 0
\end{aligned}$$

$$\begin{aligned}
(5)\ &+ 1.339N_0 + 1.322N' + 0.000V_0 + 0.000V' + 58.783W_0 + 10.505W' \\
&\qquad + 16.728M - 3.432S - 14.143S_1 - 0.037S_1' - 2\ 457k - 80.480 = 0
\end{aligned}$$

$$\begin{aligned}
(6)\ &+ 1.025N_0 + 0.720N' + 0.000V_0 + 0.000V' + 10.505W_0 + 10.591W' \\
&\qquad + 8\ 922M + 6.156S - 0.221S_1 - 2.654S_1' - 17.457k - 28.376 = 0
\end{aligned}$$

$$\begin{aligned}
(7)\ &- 1.072N_0 + 1.160N' - 6.813V_0 - 4.072V' + 16.728W_0 + 8.922W' \\
&\qquad + 88.021M - 28.928S - 4.599S_1 - 0.342S_1' + 27.607k - 64.899 = 0
\end{aligned}$$

$$\begin{aligned}
(8)\ &+ 10.380N_0 - 7.130N' - 5.346V_0 - 6.981V' - 3.432W_0 - 6.156W' \\
&\qquad - 28.928M + 307.080S - 0.000S_1 - 0.000S_1' - 16.120k - 12.510 = 0
\end{aligned}$$

$$\begin{aligned}
(9)\ &+ 1.620N_0 - 2.104N' - 15.039V_0 - 0.757V' - 14.143W_0 - 0.221W' \\
&\qquad - 4.599M + 0.000S + 67.212S_1 - 0.050S_1' + 9.174k + 41.002 = 0
\end{aligned}$$

$$\begin{aligned}
(10)\ &- 2\ 035N_0 + 0.212N' - 1.008V_0 - 5.419V' - 0.037W_0 - 2.654W' \\
&\qquad - 0.342M + 0.000S - 0.050S_1 + 19.665S_1' + 7.741k - 19.601 = 0
\end{aligned}$$

$$\begin{aligned}
(11)\ &- 16.549N_0 + 0.066N' - 19.446V_0 - 11.000V' - 2.457W_0 - 17.457W' \\
&\qquad + 27.607M - 16.120S + 9.174S_1 + 7.741S_1' + 308.208k - 86.022 = 0
\end{aligned}$$

The solution of these equations gives the following values of the unknown quantities in terms of k:

Values.	W ($k = 0$)	W′ (k indeterminate.)	Probable errors. $k = 0$	Probable errors. $k = 0.295$
$''$ $''$			$''$	$''$
$N_0 = -0.16 + 0.38k$	36.7	35.9	± 0.18	± 0.17
$V_0 = -0.90 + 0.33k$	28.6	28.3	± 0.21	± 0.19
$W_0 = +0.84 - 0.30k$	44.0	43.3	± 0.17	± 0.15
$N' = +0.28 - 0.37k$	7.0	7.0	± 0.42	± 0.38
$V' = -2.63 + 1.01k$	7.8	7.6	± 0.40	± 0.36
$W' = +1.84 + 2.38k$	7.5	6.3	± 0.41	± 0.40
$M = +0.15 - 0.43k$	71.5	67.6	± 0.13	± 0.12
$S = -0.04 - 0.03k$	270.3	270.3	± 0.07	± 0.06
$S_1 = -0.64 - 0.16k$	55.0	54.7	± 0.15	± 0.13
$S_1' = +0.46 + 0.26k$	15.7	15.6	± 0.28	± 0.25
$k = +0.295$	. .	. .	. .	± 0.065

The solution has been so conducted as to give separate results on two distinct hypotheses:

I. That the rotation of the earth is really uniform, and, therefore, that the true value of k is zero, and that this quantity is to be omitted from the equations.

II. That k has a certain definite value to be derived from the equations themselves.

On the first supposition there will be ten unknown quantities, and on the second eleven.

The required result has been reached by solving the equations so as to express each of the other ten quantities in terms of k. The result of omitting k is then obtained by putting k equal to zero in these results, as above given.

The solution was then continued so as to obtain the most probable value of k itself. The weights were then obtained separately on the two hypotheses, and, irrespective of the probable errors, should be a little larger for the less number of unknown quantities. On the other hand, the probable error by supposing k to have the value of 0.295 is decidedly less, because the residuals are smaller. Hence, on the whole, the probable errors are less when we assign to k the value given by the equations than when we suppose it to vanish.

The epoch for the variable quantities N, V, W, and S_1 is 1820. For any other year Y, we have

$$W = W_0 + W' \frac{Y - 1820}{100}$$

$$V = V_0 + V' \frac{Y - 1820}{100}$$

$$N = N_0 + N' \frac{Y - 1820}{100}$$

PART III.

DISCUSSION OF RESULTS.

Of results to be derived from transits of Mercury, there are two which outweigh all others in importance: One is the possible variation of the sidereal day, which has been already described, and the other the discordance between the theoretical and the observed motions of the perihelion of Mercury. The two questions thus arising have to be considered separately, and it will be convenient to take up first the question of the variability of the earth's axial rotation.

§ 1.

Do the transits of Mercury prove or disprove the hypothesis of the variability of the earth's axial rotation?

We have made this question depend upon the value of the constant k, deduced in the preceding sections. The evidence that we have hitherto obtained of the supposed variability is found in the discordance between the observed and theoretical mean motions of the moon. As already explained, we have so arranged the equations of condition that the hypothesis of perfect uniformity in the earth's rotation will be represented by $k = 0$, and that of such variability in the rotation as will account for the inequalities of the motion of the moon by $k = 1$. A value of k differing from either 0 or 1 must either arise from the unavoidable errors of observation or from a combination of both hypotheses.

As a matter of fact we have found $k = +0.295$. This result does not correspond to either hypothesis.

To facilitate the judgment how far we are to consider this value of k as indicating a general change in the earth's rotation, we present the following values of the residuals corresponding to the several cases, $k = 0$, $k = 0.295$, and $k = +1$. The residuals are presented in two forms—those of heliocentric arc between the positions of Mercury and the earth and those of times of contact.

We begin with the former, and express them as functions of k, so that those for $k = 0$, $k = 0.295$, and $k = 1$ can be readily formed. We thus find the following values:

Residuals of Equations of Condition in terms of k.

NOVEMBER TRANSITS, INTERNAL CONTACTS.

Year.	Contact.	Residuals.	$k=0$	$k=0.295$	$k=1$	Wt.
		$''$ $''$	$''$	$''$	$''$	
1677	II & III	$-0.90+7.25k$	-0.90	$+1.24$	$+6.35$	0.3
1697	III	$+2.45-4.26k$	$+2.45$	$+1.19$	-1.81	0.3
1723	II	$-0.58+3.38k$	-0.58	$+0.42$	$+2.80$	2.0
1736	II	$-0.47+0.68k$	-0.47	-0.27	$+0.21$	1.0
1736	III	$-0.03-1.74k$	-0.03	-0.54	-1.77	1.0
1743	II	$-0.68+1.94k$	-0.68	-0.11	$+1.26$	1.0
1743	III	$+0.75-1.07k$	$+0.75$	$+0.43$	-0.32	1.5
1769	II	$-0.82-2.70k$	-0.82	-1.62	-3.52	1.0
1769	III	$-0.14+2.28k$	-0.14	$+0.53$	$+2.14$	0.2
1782	II	$+0.07-1.13k$	$+0.07$	-0.26	-1.06	3.0
1782	III	$-1.12+0.27k$	-1.12	-1.04	-0.85	3.0
1789	II	$+0.88-2.66k$	$+0.88$	$+0.10$	-1.78	2.0
1789	III	$-1.14+3.23k$	-1.14	-0.19	$+2.09$	1.0
1802	III	$-1.09+3.68k$	-1.09	0.00	$+3.59$	3.0
1822	II	$+0.70-0.42k$	$+0.70$	$+0.58$	$+0.28$	0.5
1822	III	$+0.06+1.18k$	$+0.06$	$+0.41$	$+1.24$	1.0
1848	II	$+1.02-1.16k$	$+1.02$	$+0.68$	-0.14	5.0
1848	III	$-0.92+1.20k$	-0.92	-0.57	$+0.28$	0.3
1861	II	$+1.75+0.11k$	$+1.75$	$+1.78$	$+1.86$	0.7
1861	III	$-0.68-0.27k$	-0.68	-0.76	-0.95	5.0
1868	II	$+7.25+1.28k$	$+7.25$	$+7.63$	$+8.53$	0.5
1868	III	$+0.29-0.83k$	$+0.29$	$+0.05$	-0.54	6.0
1881	II	$-1.92+1.98k$	-1.92	-1.34	$+0.06$	3.0
1881	III	$+0.97-1.77k$	$+0.97$	$+0.45$	-0.80	3.0

MAY TRANSITS, INTERNAL CONTACTS.

Year.	Contact.	Residuals.	$k=0$	$k=0.295$	$k=1$	Wt.
1753	III	$+0.08-1.15k$	$+0.08$	-0.26	-1.07	3.0
1786	III	$+1.77-0.25k$	$+1.77$	$+1.70$	$+1.52$	4.0
1799	II	$-0.43+0.15k$	-0.43	-0.39	-0.28	3.0
1799	III	$+0.14+0.31k$	$+0.14$	$+0.23$	$+0.45$	4.0
1832	II	$+0.58-0.97k$	$+0.58$	$+0.29$	-0.39	6.0
1832	III	$-0.21+0.74k$	-0.21	$+0.01$	$+0.53$	6.0
1845	II	$+0.45+0.00k$	$+0.45$	$+0.45$	$+0.45$	8.0
1845	III	$-0.76+0.38k$	-0.76	-0.65	-0.38	8.0
1878	II	$-0.55+0.49k$	-0.55	-0.41	-0.06	12.0
1878	III	$+0.50-0.40k$	$+0.50$	$+0.38$	$+0.10$	8.0

Residuals of Equations of Condition in terms of k—Continued.

NOVEMBER TRANSITS, EXTERNAL CONTACTS.

Year.	Residuals.	$k=0$	$k=0.295$	$k=1$	Wt.
	″ ″	″	″	″	
1677	$+1.04 - 6.35k$	$+1.04$	-0.83	-5.31	0.1
1690	$+0.47 - 4.09k$	$+0.47$	-0.74	-3.62	0.2
1697	$+0.51 - 3.28k$	$+0.51$	-0.46	-2.77	0.3
1736	$+0.01 - 1.00k$	$+0.01$	-0.29	-0.99	0.6
1743	$+0.75 - 0.36k$	$+0.75$	$+0.64$	$+0.39$	0.7
1756	$-3.62 + 0.92k$	-3.62	-3.35	-2.70	0.0
1769	$-1.22 + 2.84k$	-1.22	-0.38	$+1.62$	0.2
1782	$-0.79 + 0.63k$	-0.79	-0.60	-0.16	1.0
1789	$+1.50 + 3.69k$	$+1.50$	$+2.59$	$+5.19$	0.2
1802	$+0.51 + 4.07k$	$+0.51$	$+1.71$	$+4.58$	1.2
1822	$-1.62 + 1.46k$	-1.62	-1.19	-0.16	0.2
1848	$-1.09 + 1.32k$	-1.09	-0.70	$+0.23$	0.2
1861	$+1.72 - 0.22k$	$+1.72$	$+1.65$	$+1.50$	1.5
1868	$-0.40 - 0.84k$	-0.40	-0.65	-1.24	2.0
1881	$+0.90 - 1.82k$	$+0.90$	$+0.36$	-1.92	1.5
	MAY TRANSITS, EXTERNAL CONTACTS.				
1753	$-0.03 - 0.81k$	-0.03	-0.27	-0.84	2.0
1786	$+0.58 \quad 0.00k$	$+0.58$	$+0.58$	$+0.58$	2.0
1799	$-1.21 + 0.54k$	-1.21	-1.05	-0.67	2.0
1832	$-0.44 + 0.87k$	-0.44	-0.18	$+0.43$	2.4
1845	$-0.98 + 0.45k$	-0.98	-0.85	-0.53	2.0
1878	$-0.60 - 0.42k$	-0.60	-0.72	-1.02	3.0

We thus derive, by direct computation,

$$\Sigma W_\epsilon^2 = 109.4 - 136.4k + 237.8k^2$$

while the result from the solution of the normal equations is

$$\Sigma W_\epsilon^2 = 109.5 - 137.2k + 232.8k^2.$$

Hence, for

$$k = 0; \quad \Sigma W_\epsilon^2 = 109.4$$
$$k = 0.295; \quad \Sigma W_\epsilon^2 = 89.9$$
$$k = 1; \quad \Sigma W_\epsilon^2 = 210.8$$

Since what we are now aiming at is the determination of a hypothetical error of the astronomical time, a conclusion will be facilitated by presenting the mean error of time for each transit. We remark that, continuing the notation already employed, to the residual $\varDelta c$ in arc will correspond the residual $\varDelta t = n\varDelta c$ in time. Hence, to a weight W of $\varDelta c$ will correspond a weight proportional to Wn^2 of $\varDelta t$. Hence we shall have, as the mean by weights of any number of results:

$$\varDelta t = \frac{\Sigma Wn\varDelta c}{\Sigma Wn^2}$$

We thus have the following results from the one, two, or three contacts observed at each transit. The probable error corresponding to the unit of weight is assumed to be $1^s.5$.

Year.	$k = 0$	$k = 0.295$	$k = 1$	Wt.	ε
	s.	s.	s		
1677	+ 4.6	− 5.6	− 30.0	.0165	12
1690	+ 3.4	− 5.4	− 26.3	.0039	24
1697	+ 9.4	+ 2.3	− 14.6	.0143	12
1723	+ 2.9	− 2.2	− 14.3	.0776	5
1736	+ 1.6	− 1.7	− 9.7	.0277	9
1743	+ 4.2	+ 2.2	− 2.6	.0975	4.8
1753	+ 0.5	− 3.5	− 12.9	.0288	9
1769	+ 2.1	+ 6.3	+ 16.4	.0483	7
1782	− 14.1	− 9.6	+ 1.2	.0139	13
1786	+ 25.5	+ 24.6	+ 22.4	.0175	11
1789	− 4.3	+ 0.2	+ 11.1	.1136	4.5
1799	− 0.8	0.0	+ 2.0	.0506	7
1802	− 3.0	+ 2.3	+ 15.0	.1869	3.5
1822	− 3.5	− 0.7	+ 6.0	.0184	11
1832	− 5.8	− 2.1	+ 6.6	.0686	6
1845	− 9.8	− 8.8	− 6.5	.0784	5
1848	− 4.9	− 3.3	+ 0.7	.2380	3.1
1861	− 1.8	− 2.3	− 3.4	.1727	3.6
1868	− 2.3	− 4.1	− 8.5	.1595	3.8
1878	+ 5.0	+ 3.3	− 0.9	.1347	4.1
1881	+ 6.5	+ 3.8	− 2.6	.3153	2.7

In order still farther to trace the course of the changes of long period, we take the mean results from groups of transits with the following results:

Limits of dates.	Mean year.	$k = 0$	$k = 0.295$	$k = 1$	Wt.	ε
		s.	s.	s.		s.
1677—1697	1690	+ 6.5	− 2.3	− 23.1	.0352	8
1723—1753	1740	+ 3.0	− 0.4	− 8.6	.2316	3.2
1769—1802	1787	− 1.7	+ 2.5	+ 12.5	.4308	2.3
1822—1832	1822	− 5.3	− 1.8	+ 6.5	.0870	5 1
1845—1848	1847	− 6.1	− 4.7	− 1.1	.3164	2.7
1861—1868	1865	− 2.0	− 3.2	− 5.8	.3322	2.6
1878—1881	1879	+ 6.1	+ 3.7	− 2.1	.4500	2.2

If we are compelled to choose between the two limiting values of ko, and unity the value zero is far the more probable. The probable error of k being the probability that the true value of k can be as great as 0.8 is only . This would be the probability if no systematic errors entered into the observations. But the possibility of systematic differences between observations of different transits is such that we should regard this probable error as quite illusory. Still it must be admitted that the probability that k can be nearly unity is so small that we must regard it as quite improbable that the inequalities in the mean motion of the moon are entirely to be

accounted for by changes in the earth's rotation. One of the conclusions of the present discussion is therefore this:

Inequalities in the motion of the moon not accounted for by the theory of gravitation really exist, and exist in such a way that the mean motion of the moon between 1800 *and* 1875 *was really less than it was between* 1720 *and* 1800.

If on the other hand we adopt the hypothesis $k = 0$, the systematic character of the residuals is such that this hypothesis must also appear quite improbable though not wholly impossible. The question then arises, can we admit the actual existence of inequalities of both classes? The most remarkable circumstance in this connection is that a value of k equal to about $\frac{1}{3}$ should so closely satisfy the whole series of observations. That there could be any such relation between variations in the earth's rotation and in the moon's mean motion as would be implied by supposing this value of k to be real would be a result which cannot be accounted for by known physical laws. But it is a singular circumstance that the whole series of observed transits through two centuries should so closely follow this law. It is also singular that the changes during the last 40 years should be so closely represented. It is to be remarked that the apparent retardation of the moon's mean motion during the present century has not been uniform, but that during a few years preceding 1860 there was a temporary acceleration which continued until perhaps 1862. A rapid retardation then commenced, which has gradually brought the moon back into its regular position as given by the hypothetical inequalities of long period. Now it is most remarkable that the observations of transits of Mercury agree with those of the moon, and those of the first satellite of Jupiter, in indicating that this apparent inequality was in part at least due to the earth's rotation. If we should accept this result it would lead to the conclusion that the motions of the earth and moon are so connected that one is retarded when the other is accelerated. But, it is difficult to see how such a conection could result from the mutual action of the two bodies. If these motions were connected in a way which could be accounted for by the action of a couple of forces between the two bodies they would be accelerated and retarded together. *This relation would be indicated by a negative value of k.*

On the whole it would seem premature to reach any positive conclusion upon these results, though they seem to suggest the desirableness of further physical investigation to ascertain the possibility of any such relation

At present the best course would seem to be to suppose $k = 0$ in our subsequent investigations. The effect of k is so small that our general conclusions respecting the motion of Mercury will not be materially altered should it subsequently be found to have a value different from zero. By constructing theories and tables on the simpler hypothesis, the existence of any real deviation will be made more evident by the results of future transits.

§ 2.

Concluded corrections to LEVERRIER'S *elements.*

Since we cannot derive separate and independent values of all the elements from observations of transits, the corrections which we obtain must be regarded as those

applicable to certain functions of the elements, as shown in Chap. II, § 5. Transferring the epoch from 1820 to 1850, and putting $k = o$, we have the following values of the corrections to the tabular quantities. T is here the time after 1850.0, the unit being a century.

$$\begin{aligned}
N &= (\delta\theta - \delta l') \sin i = -0''.07 + 0''.28T \\
V &= 1.487\delta\lambda - 0.487\delta\pi - 1.137\delta e \\
&\qquad - 1.01\delta\lambda' + 1.19e'\delta\pi' + 1.58\delta e' = -1''.69 - 2''.63T \\
W &= 0.716\delta\lambda + 0.284\delta\pi + 0.896\delta e \\
&\qquad - 0.97\delta\lambda' - 1.11e'\delta\pi' - 1.62\delta e' = +1''.39 + 1''.84T \\
M &= +0.15
\end{aligned}$$

Hence, the mass of Venus derived from the periodic perturbations at the times of transits is

$$\frac{1.015}{401847} = \frac{1}{396000}$$

For the corrections of semi-diameters we have

$$S = \delta R' - 1.60\delta R = -0''.04$$

Hence, for the sun's semi-diameter at distance unity we have

$$959''.75 - 1.60\,\delta R$$

δR being the correction to the semi-diameter of Mercury at distance unity.

The value of S_1

$$-0''.50 + 0''.46T$$

expresses the extent to which Mercury impinged upon the sun at the time of an average external contact. The term $0''.46T$ represents the diminution of this quantity in consequence of the gradual improvement of the telescope.

§ 3.

Comparison of observed and theoretical secular variations and of results for the mass of Venus.

The observed secular variation of the perihelion of Mercury, as derived from observation, can, without difficulty, be accounted for by suitably increasing the adopted mass of Venus. The only argument against such an increase is that the variations of other elements will not then be represented. But in the absence of any reason for preferring one determination to another, the true form in which we should put the result is that the variations of different elements give different values of the mass of Venus. We can reject one result only when we have found that all the methods but one give accordant results and that this one alone is discordant. The first step toward a satisfactory solution of the question is, therefore, to find what values of the mass of Venus are given by different data and discuss the discordances among them.

Five methods are available for the determination of the mass of Venus.

I. *The secular motion of the perihelion of Mercury.*—More exactly we should say the secular motions of V and W, which arise from variations both in the eccentricities and in the perihelion of Mercury and the earth.

II. *The secular motion of the node of Mercury.*—Any uncertainty that may exist in the theoretical motion of this node arises almost entirely from the uncertainty in the mass of Venus, since the influence of all the other planets can be accurately determined.

III. *The secular motion of the node of Venus on the ecliptic.*—Properly speaking we should say the secular motion of the ecliptic itself, because that portion of the motion of the node of Venus which depends on the mass of that planet arises solely from the motion of the ecliptic.

IV. *The secular diminution of the obliquity of the ecliptic.*—This, like the first, is a motion of the ecliptic due to the action of Venus. Hence these two determinations cannot be considered as wholly independent, though each would strengthen the other.

V. *The periodic perturbations of Mercury and the earth produced by the action of Venus.*

Since a discordance of the kind in question indicates the continuous action of some unknown cause, we cannot say that any one of the first four methods is necessarily free from the effects of such action. Hence, if the results are discordant, we have no right to deduce with certainty any mass of Venus from them. It is different with the last method. It is beyond all moral probability that any unknown cause should produce periodic inequalities in the planetary motions corresponding to those produced by the action of the planets on each other. We may therefore consider the mass of Venus derived from periodic perturbations to be that which is to be accepted as the real mass to be used in comparing the other results. Unfortunately, the best mass that can be derived from transits is very uncertain, while that of discussing the meridian observations will be very laborious.

Mass of Venus from the motion of the perihelion of Mercury.

I. To determine what mass of Venus will best represent the secular variations of the eccentricity and perihelion, let us consider the values of V′ and W′, which depend upon the corrections to the secular variations. If we put

$$\begin{aligned} \delta H_1 &= 0.487\delta\pi + 1.137\delta e - 1.19e'\delta\pi' - 1.58\delta e' \\ \delta H_2 &= 0.284\delta\pi + 0.896\delta e - 1.11e'\delta\pi' - 1.62\delta e' \end{aligned} \qquad (a)$$

The values of V′ and W′, which we have found, give the equations—

$$\begin{aligned} 1.487\delta n - 1.01\delta n' - \frac{d\delta H_1}{dt} &= -2''.63 \\ 0.716\delta n - 0.97\delta n' + \frac{d\delta H_2}{dt} &= +1''.84 \end{aligned} \qquad (b)$$

Where δn and $\delta n'$ are the corrections to the centennial mean motions of Mercury and the earth, respectively. There being four unknown quantities in these two equations, we cannot determine them all from the data afforded by the transits. We shall

therefore take the tabular mean motion of the sun as correct, which amounts to supposing $\delta n' = 0$, and express $\frac{dH_1}{dt}$ and $\frac{dH_2}{dt}$ in terms of the mass of Venus as the single unknown quantity in addition to δn

The following values of the secular variations of π, e, π' and e' are given by LEVERRIER,* and will be accepted with the single change of substituting, for the action of Venus, the value found by Mr. HILL by GAUSS's method. (Ante, p. 342.)

$$D_t\pi = 527''.00 + 280''.51\nu' + 83''.64\nu'' + 2''.85\nu''' + 152''.59\nu^{iv} + 7''.25\nu^{v} + 0''.14\nu^{vi} + 0''.06\nu^{vii}$$

$$D_te = +4''.18 + 2''.82\nu' + 1''.06\nu'' - 0''.07\nu''' + 0''.32\nu^{iv} + 0''.05\nu^{v}$$

$$e'D\pi' = +19''.30 - 0''.46\nu + 5.89\nu' + 1.89\nu''' + 11''.66\nu^{iv} + 0.31\nu^{v}$$
$$D_te' = -8''.95 - 0''.29\nu + 1''.36\nu' - 1''.82\nu''' - 8''.16\nu^{iv} - 0''.04\nu^{v}$$

The coefficients ν, ν', etc., are determined by the condition that $1 + \nu$, $1 + \nu'$, etc., are the factors by which we must multiply the provisional masses adopted by LEVERRIER to obtain the true masses Since the time when LEVERRIER wrote the masses of most of the planets have been determined with a certainty far exceeding any then attainable. The following seem at present to be the most reliable values of the planetary masses:

Mercury.—VON ASTEN's investigations on ENCKE's comet indicate a large diminution of the mass of Mercury generally assumed. The different results for this mass are so discordant that the choice among them must be a matter of judgment rather than of calculation. Analogy would lead us to suppose that the density of this planet is probably less than that of the earth. It is the opinion of the writer, from a consideration of all the data, that we may adopt the value

$$\text{Mass of Mercury} = \frac{1}{7\,500\,000}$$

as being at present the most probable value.

The Earth—The most recent determinations of the solar parallax appear to group themselves around the value 8''.91, which we may regard as the most probable value now obtainable. To this corresponds

$$\text{Combined mass of the Earth and Moon} = \frac{1}{327\,000}$$

Mass of Mars.—Professor HALL's discussion, from the motions of the satellites, gives

$$\text{Mass of Mars} = \frac{1}{3\,093\,500}$$

which does not seem to need any further discussion or correction.

Mass of Jupiter.—There does not seem to be any reason for changing BESSEL's mass, which we shall therefore adopt.

* Annales de l'Observatoire, tome ii, p. 100.

Saturn, Uranus, and Neptune.—The action of these planets on Mercury and the earth is so small that there is no need of changing the masses employed by LEVERRIER.

We now have the following comparison of the masses here adopted with those adopted by LEVERRIER, with the resulting values of the coefficients ν.

Planet.	Mass adopted by LEVERRIER.	Corrected mass.	Value of ν.
Mercury . .	$\frac{1}{3,000,000}$	$\frac{1}{7,500,000}$	-0.6
Venus . . .	$\frac{1}{401,847}$	Indeterm.	Unknown.
Earth . . .	$\frac{1}{354,936}$	$\frac{1}{327,000}$	$+0.0854$
Mars	$\frac{1}{2,680,337}$	$\frac{1}{3,093,500}$	-0.134
Jupiter . .	$\frac{1}{1050}$	$\frac{1}{1047.88}$	$+0.00202$

Substituting these values of ν, ν'', etc., the preceding expressions for the secular variations in terms of the mass of Venus become

$$\begin{aligned} D_t\pi &= 534''.07 + 280''.5\nu' \\ D_te &= 4.28 + 2.8\nu' \\ e'D_t\pi &= 19.35 + 5.9\nu' \\ D_te' &= -8.55 + 1.4\nu' \end{aligned}$$

In LEVERRIER'S tables of Mercury and the sun the adopted secular variations, assuming the precession for 1850 to be $50''.2357$, are

$$\begin{aligned} D_t\pi &= 567''.81 \\ D_te &= 4.20 \\ e'D_t\pi' &= 19.23 \\ D_te' &= -8.76 \end{aligned}$$

The corrections to the tabular secular variations are therefore expressed in the form

$$\begin{aligned} D_t\delta\pi &= -33''.74 + 280''.5\nu \\ D_t\delta e &= +0.08 + 2.8\nu' \\ e'D_t\delta\pi' &= +0.12 + 5.9\nu' \\ D_t\delta e' &= +0.21 + 1.4\nu' \end{aligned}$$

Substituting these values in the derivatives (a) of δH_1 and δH_2 with respect to the time, we have the following expressions for the theoretical corrections to the tabular secular variations of H_1 and H_2

$$\begin{aligned} D_t\delta H_1 &= -16''.81 + 130''.6\nu' \\ D_t\delta H_2 &= -9.98 + 73.4\nu' \end{aligned}$$

Substituting these theoretical expressions in the formulæ (*b*), and putting $\delta n' = 0$, the equations derived from observation become

$$
\begin{aligned}
1.487\delta n + 16''.81 - 130''.6\nu' &= - 2''.63 \\
0.716\delta n - 9.98 + 73.4\nu' &= + 1.84
\end{aligned}
$$

The solution of these equations gives

$$
\begin{aligned}
\delta n &= + 0''.58 \\
\nu' &= + 0.1554
\end{aligned}
$$

This value of ν' gives

$$\text{Mass of Venus} = \frac{1}{347800}$$

II. *Motion of the node of Mercury.*—Our next inquiry is, what mass of Venus results from the observed motion of the node of Mercury upon the ecliptic? If we put, with LEVERRIER,

$$
\begin{aligned}
p &= \tan i \sin \theta, \\
q &= \tan i \cos \theta,
\end{aligned}
$$

we have the following theoretical values of the secular variations of the planes of the orbit, derived and expressed as in the case of the perihelion of Mercury.

FOR MERCURY.

$$
\begin{aligned}
D_t p &= - 53''.69 - 27''.71\nu' - 8''.76\nu'' - 0''.21\nu''' - 16''.08\nu^{iv} \\
D_t q &= + 24.65 + 7.06\nu' + 7.32\nu'' + 0.17\nu''' + 9.75\nu^{iv}
\end{aligned}
$$

FOR THE EARTH.

$$
\begin{aligned}
D_t p'' &= + 5''.89 + 0''.62\nu + 7''.57\nu' + 0''.73\nu''' - 2''.50\nu^{iv} \\
D_t q'' &= - 47.59 - 0.52\nu - 28.90\nu' - 0.83\nu''' - 16.01\nu^{iv}
\end{aligned}
$$

FOR MERCURY RELATIVE TO EARTH.

$$
\begin{aligned}
D_t(p - p'') &= - 59''.58 - 0''.62\nu - 35''.28\nu' - 8''.76\nu'' - 0''.94\nu''' - 13''.58\nu^{iv} \\
D_t(q - q'') &= + 72.24 + 0.52\nu + 35.97\nu' + 7.32\nu'' + 1.00\nu''' + 25.76\nu^{iv}
\end{aligned}
$$

Substituting the values of ν, ν'', ν''', and ν^{iv}, already given, these last equations become

$$
\begin{aligned}
D_t(p - p'') &= - 59''.86 - 35''.28\nu' \\
D_t(q - q'') &= + 72.48 + 35.97\nu'
\end{aligned}
$$

The secular motion of the inclination and node of Mercury relative to the moving ecliptic is found by substituting $p - p''$ and $q - q''$ for p and q in the expressions for the latter quantities and then differentiating. We thus find

$$\sin i \, D_t\theta = \cos i \cos \theta \, D_t(p - p'') - \cos i \sin \theta \, D_t(q - q'')$$

For the epoch 1850 we have

$$
\begin{aligned}
i &= 7^\circ\ 0'\ 7''.7 \\
\theta &= 46^\circ\ 33'\ 8''.8
\end{aligned}
$$

whence

$$\sin i\, D_t\theta = -\ 93''.09 - 50''.00\nu'$$

The observed value of this same quantity is found by applying to LEVERRIER's tabular value the corrections already derived. We thus have

$$\text{Observed } \sin i\, D_t\theta = -\ 92''.56 + 0''.28 = -\ 92''.28$$

Equating the values we find

$$\nu' = -\ .016$$

Hence for the mass of Venus derived from the motion of the node of Mercury, we have

$$m' = \frac{1}{408\,400}$$

III. *Motion of the node of Venus.*—The most recent determination of the motion of the node of Venus, and of the consequent mass of that planet, is that of Mr. G. W. HILL, who finds

$$
\begin{aligned}
\text{Annual motion of node} &= \ 32''.515 - \text{precession} \\
&= -\ 17.737
\end{aligned}
$$

$$\text{Mass of Venus*} = \frac{1}{427\,240}$$

The motion adopted in LEVERRIER's tables of Venus (*Annales de l'Obs.*, vol. vi) corresponds to a yet smaller mass of Venus not far from $\frac{1}{450\,000}$, so that there is, apparently, an extraordinary discrepancy between the mass of Venus derived from this source and from the others. But the observations of the transit of Venus in 1874 showed that LEVERRIER's position of the node needed a correction about twice that found by Mr. HILL. From this it would seem probable that the geocentric latitude of Venus derived from the transits of 1761 and 1769 was several seconds in error. It must, therefore, be deemed probable that the actual motion of the node corresponds to a mass of Venus decidedly greater than that found by Mr. HILL, and not differing greatly from that found by the motion of the node of Mercury. But in the absence of a definitive investigation of the subject, no value of the mass in question can at present be derived from this source.

IV. *Obliquity of the ecliptic.*—The secular diminution of the obliquity of the ecliptic, as found from observation by LEVERRIER, indicates a diminution of the provisional

* Tables of Venus, Introduction, p. 36.

mass of Venus. But this is another constant of which a definitive value is yet to be investigated, and no certain result can be laid down until this is done.

V. *Results of periodic perturbations.*—We have found from the equations of condition

$$M = 10\nu' = +0.15 - 0.43k \pm 0.13.$$

The large value of the coefficient of k shows that the concluded mass of Venus from the periodic perturbations will be materially affected by any inequalities in the earth's rotation. We can, therefore, only attribute small weight to the result, which is

$$\frac{1}{396\,000}.$$

Should the true value of k be that given by the equations, the denominator would be increased to 401 000.

§ 4.

Concluded mass of Venus and excess of motion of perihelion of Mercury.

We have now the following results for the mass of Venus:

From perihelion of Mercury $1 \div 1000m' =$	347.8
From node of Mercury - - - - - -	408.4
From periodic inequalities - - - - -	396.

while the results from the other two sources will probably not differ much from the second of the above values.

The third value is too uncertain to permit of any conclusion being drawn from its deviation from the second. By merely supposing the constant k to have the value 0.295 not only will the last value be increased to 401, but the value 408.4 from the motion of the node will be diminished. The two values will, therefore, be made more accordant.

There is, therefore, a decided preponderance of evidence that the true value of the mass of Venus does not differ much from $\frac{1}{405\,000}$, and is probably contained between the limits $\frac{1}{400\,000}$ and $\frac{1}{410\,000}$. The value $\frac{1}{347\,800}$ is entirely inconsistent with all the others. We must, therefore, conclude that *the discordance between the observed and theoretical motions of the perihelion of Mercury, first pointed out by* LEVERRIER, *really exists, and is indeed larger than he supposed.*

Determination of excess of motion of perihelion.—In investigating the actual amount of the discordance we call to mind that we have no certain evidence as to how the discordance is to be divided among the several elements which enter into the expressions for V′ and W′. But, so far as has yet been noticed, it does not appear that any other element than the perihelion of Mercury is affected by this abnormal variation. We, therefore, put the inquiry into this form: assuming that the variations of e, e', and π'

correspond to theory, how much is the variation of π in excess of the value given by theory? In considering this question we shall assume $\nu' = -.008$ and hence $m' = \frac{1}{405\,000}$. We shall also put,

p, the excess in the centennial motion of π.

With this adopted value of the mass of Venus the motions of the elements which are to be reconciled with observation will become

$$\begin{aligned} D_t\pi &= 531''.83 - p \\ D_te &= 4.26 \\ e'D_t\pi' &= 19.30 \\ D_te' &= -\ 8.56 \end{aligned}$$

The excess of the values adopted in the tables over these values are

$$\begin{aligned} \varDelta D_t\pi &= 35''.98 - \pi' \\ \varDelta D_te &= -\ 0.06 \\ \varDelta e'D_t\pi' &= -\ 0.07 \\ \varDelta D_te' &= -\ 0.20 \end{aligned}$$

We thence derive from the equations (a) the following values of the excess of the tabular values of D_tH_1 and D_tH_2 over the modified theory

$$\begin{aligned} \varDelta D_tH_1 &= 17''.85 - 0.487\,p \\ \varDelta D_tH_2 &= 10.57 - 0.284\,p \end{aligned}$$

Next, the equations (b) give for the excess of observation over the tables

$$\begin{aligned} \delta D_tH_1 &= +\ 2''.63 + 1\ 487\delta n \\ \delta D_tH_2 &= +\ 1.84 - 0.716\delta n \end{aligned}$$

the terms in $\delta n'$ being omitted as before.

Hence, the excesses of observation over theory, which is to be reduced to zero by attributing suitable values to p and δn, are

$$\begin{aligned} &20''.48 - 0.487p + 1.487\delta n \\ &12.41 - 0.284p - 0.716\delta n \end{aligned}$$

Equating these expressions to zero we find

$$\begin{aligned} \delta n &= +\ 0''.37 \\ p &= +\ 42.95 \end{aligned}$$

It follows that the observed centennial motion of the perihelion of Mercury is greater by 43″ than the theoretical motion computed from the best attainable values of the masses of the planets.

§ 5.

Speculation on possible causes of the excess of motion of the perihelion of Mercury.

Should physicists succeed in discovering some modification of the law of attraction between different bodies, which would closely represent the phenomenon in question, further investigation of the subject from an astronomical standpoint would be greatly limited. But, in the absence of any such modification, no satisfactory conclusion can be reached without more certain data than we now possess as to the exact character of the excess of motion of the perihelion of Mercury, and of the other phenomena which may be associated with it. It is therefore difficult, in discussing the possible cause of such a motion, to speak with the confidence of certainty on every point that may come up. What we have to say must be to a considerable extent provisional, and must be founded on the supposition that the character of the phenomena with which we are concerned is that which appears most probable from the preceding discussion.

Of course the first thing to be sure of before basing any theory upon the observed discordance is, that the latter does not arise from any imperfection either in the theory or in the discussion of the observations. The close agreement of the secular variations produced by Venus, as computed by Mr. Hill in the preceding paper of this series, and as computed by Leverrier, seem to prove conclusively the correctness of the latter's results. For any other planet than Venus the uncertainty must be much smaller. We cannot, therefore, look with any probability for an error in the computed secular variations. The question may, however, be raised, whether there is a possibility of any term of very long period. This question also must, it would seem, be answered in the negative. Any term having a period of a number of centuries would depend upon multiples of the mean motion so high that there is no possibility of their being sensible.

To show this let us develop the ratio of the mean motions of Venus and Mercury as a continued fraction. It will be—

$$\cfrac{1}{2+\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{4+\cfrac{1}{7+\text{etc.}}}}}}$$

The convergents will be

$$\frac{1}{2}, \quad \frac{1}{3}, \quad \frac{2}{5}, \quad \frac{9}{23}, \quad \frac{65}{166}.$$

The period of the term $23\,V - 9\,M$ will be little more than 50 years. The term $166\,V - 65\,M$ could not be sensible in the motion of the perihelion.

The most simple hypothesis is the well-known one of Leverrier, which presupposes the existence of a planet or group of planets between Mercury and the sun. That any such body or bodies of sufficient mass to produce the motion in question can really exist seems to be out of the question, for a number of reasons.

In the first place, on any probable hypothesis of the relation of mass to reflecting power, it is impossible that a planet or group of planets of sufficient mass to produce the observed motion of the perihelion of Mercury could exist without being very conspicuous objects during total eclipses of the sun, if at no other time. We cannot, indeed, assign an exact value to the mass unless we know the mean distance. But the less we suppose the mean distance, and therefore the greater we suppose the liability that the planet should be lost in the sun's rays, the greater the mass required and the more brilliant the planet or planets would shine during a total eclipse. In fact the more distant from the sun the required planet, the less readily it would be detected during an eclipse; but, on the other hand, it would be more readily detected at other times. In a paper published in Gould's Astronomical Journal, volume vi, the writer showed that if a group of sufficient magnitude existed, the transits over the sun would be too frequent to escape detection.

In the next place, no such group could exist and produce the observed effect without also disturbing the secular motions of the node of Mercury and Venus. It was shown in the paper just referred to that, supposing the group to lie in the ecliptic, the excess of motion of the node would be as great as that of the perihelion. But observations do not indicate any such excess. If, therefore, the group exists its plane must be very nearly coincident with the orbit of Mercury. But here we meet with two difficulties:

If the mean plane of the group were at any epoch coincident with that of Mercury, it could not remain so permanently, but the planes of the different orbits would, in time, group themselves near the invariable plane of the planetary system. Again, if the coincidence had place with the orbit of Mercury it could not have place with reference to the plane of Venus, and the plane of motion of that planet would be subject to a secular variation.

Now it is quite true, as already pointed out, that these several secular motions of the planes have not been investigated with such thoroughness that we can speak positively on this question. At the same time it appears extremely improbable that any disturbing action can exist of such magnitude as the hypothesis would imply.

The hypotheses just considered are those of a single planet or a group of planets. It may be asked to what limit we must suppose the subdivision carried in order that the individual bodies may escape detection. The reply is that they must be so small as to be invisible either in transit across the sun or by reflected light during a total eclipse, or in the evening after sunset. Their diameters at the distance unity cannot, therefore, exceed a very small fraction of a second.

The limit of mean diameter may be roughly placed at $\frac{1}{50}$ that of the earth, and the limit of individual volume at $\frac{1}{100000}$ that of the earth. Since the total mass must be an appreciable fraction of the mass of the earth the number of the hypothetical planets must be thousands and probably tens of thousands.

It may be suggested that in the zodiacal light we have evidence of at least the possibility that a group of many thousand bodies, too minute to be visible to the naked

eye, circulate between the earth and the sun. It would be an interesting photometrical investigation to ascertain the limit of volume of these bodies of the supposition that they are of ordinary whiteness. The extreme softness of the zodiacal light is such that the minimum number of separate bodies would have to be estimated at hundreds of thousands. The writer thinks it probable that the result would be that a collection of 100,000 bodies with a combined volume one-tenth that of the earth would glow with a much brighter light than the zodiacal light actually does. The hypothesis of the zodiacal light is subject to the same difficulties with respect to motions of the nodes as have already been pointed out with respect to the group of planets. But we have at present no way of positively disproving it.

We may next inquire whether either a possible ellipticity of the sun or of his atmosphere, or of the matter in his interior, can produce the observed effect. The reply to this would be that the most exact measures have failed to show any ellipticity of the body of the sun at all approaching that required. Indeed, if we suppose the elliptic disturbance of matter, if I may use the expression, to be within the sun, it would probably be found that the consequent deviation of the level surfaces at the photosphere from a spherical form would lead to a sensible ellipticity of the sun's disk.

There is a field for investigation in the question what the mass of a ring round the sun must be to produce the observed effect, and what influence that mass would have upon the motion of the nodes of Mercury and Venus. This is a question which can be more profitably discussed when the character of the phenomena is more accurately ascertained. But, as the question now stands, all hypotheses that the observed phenomenon is produced by the attraction of unknown matter in the neighborhood of the sun or Mercury must be dismissed as at least highly improbable.

We may next inquire whether any deviation from or modification of the law of gravitation which would produce the observed effect is admissible. The most natural modification of this kind would be the addition of a term varying as the inverse third or fourth power of the distance. This hypothesis can, however, be refuted very readily. A term of the inverse third power which, at the distance of Mercury, should have a value even the millionth part of the total gravitative force of the sun would, at the distance of a foot, have a value two hundred thousand times that of the term depending on the inverse square. If higher powers than the cube were added the discrepancy would be yet more enormous. The existence of a term of such magnitude is out of the question.

Another hypothesis which has been considered in this connection is that of WEBER'S electro-dynamic theory. According to this theory the gravitative force between two bodies is expressed by an equation of the form

$$\frac{m}{r^2}\left(1 - \frac{1}{h^2}\left(\frac{dr}{dt}\right)^2 + \frac{2r}{h^2}\frac{d^2r}{dt^2}\right)$$

in which the constant h, as is evident from the formula, must be a velocity. This velocity WEBER has sought to determine experimentally; his value is 439,450 kilometers per second. From this datum TISSERAND has computed the secular variations of the planets.*

* *Compte Rendus*, vol. lxxv. p. 760.

His results are that the only element affected with a sensible inequality is the perihelion, and that the secular motions of the perihelia of Mercury and Venus would have the following values:

Mercury,	$6.''28$
Venus,	1.32

If h be the velocity of light his result is,

Mercury,	$13.''65$
Venus,	2.86

But the actual motion has been found to be three times this. To produce this motion the value of h must be reduced to about 174,000 kilometers per second.

Objections have been raised to WEBER's whole theory on the part of physicists, to whom the discussion of its possibility must be left.

Assuming that we are still to look to a more exact determination of the astronomical character of the phenemena for a solution of the question, the necessary steps are an exact determination of the mass of Venus from the periodic perturbations of the inner planets, an investigation of the secular motions of the planes of the orbits of these planets, and the comparison of the theoretical and observed secular motion of the perihelion of Venus.

The latter research would be of especial interest in this connection. Unfortunately, however, owing to the very small eccentricity of Venus, a motion of its perihelion amounting to only a few seconds in a century would escape the observations hitherto made. Moreover, the very imperfect way in which observations of Venus were made during the last century precludes our obtaining a satisfactory result. The question whether this element is effected by a motion corresponding to that of Mercury can, therefore, hardly be settled until after 20 or 30 years more of careful meridian observations of Venus. But a general investigation of the secular variations of all four of the inner planets might result in showing discordances which would throw some light on the question. This investigation is one for which the material is being prepared under the writer's direction.

§ 6.

Law of recurrence of transits of Mercury.

The conception of conjunction points, developed in Part I of the present series of papers, pages 8 to 10, enables us to lay down the law of recurrence of transits of Mercury in such a way that the times and circumstances of all possible transits during several centuries past and future may be determined with great ease. Since, however, we have to consider only those conjunctions which take place near the node, it will not be necessary to consider the arrangement and motion of the whole series of conjunction points. Moreover, it is only when we neglect the eccentricities that the conjunction points are uniformly distributed and move uniformly. The eccentricity of the orbit of Mercury is so great that the positions of the mean conjunction points give

us no index to the actual circumstances of transits. What we therefore have to do is to treat the relations of the sun and Mercury at each node separately, and consider their motions at this point as if they were mean motions. Notwithstanding the eccentricity and the secular motion of the perigee, the intervals between consecutive passages of each planet through either of the common nodes will be nearly the same for many centuries. These intervals will not, however, be the same for each node, nor will they coincide with the periods corresponding to the mean motions. By a simple computation from LEVERRIER's tables we find the following intervals between consecutive passages of the earth and Mercury through the common nodes during the first half of the present century:

Interval between passages of Mercury through the ascending node in November	$87^d.969204$
Through descending node in May	$87^d.969046$
Interval between successive passages of the earth through the ascending node in November	$365^d.254268$
Interval for descending node in May	$365^d.254147$

If we develop the ratio of each of these pairs of periods as a continued fraction, we have the following results:

NOVEMBER.

$$\text{Ratio of Periods} = \cfrac{1}{4 + \cfrac{1}{6 + \cfrac{1}{1 + \cfrac{1}{1 + \cfrac{1}{2 + \cfrac{1}{1 + \cfrac{1}{3 \cdot \frac{9}{11}}}}}}}}$$

MAY.

$$\text{Ratio of Periods} = \cfrac{1}{4 + \cfrac{1}{6 + \cfrac{1}{1 + \cfrac{1}{1 + \cfrac{1}{2 + \cfrac{1}{1 + \cfrac{1}{4 \cdot \frac{16}{161}}}}}}}}$$

The converging fractions for each period, so far as it is necessary to carry them for our present purpose, are

$$\frac{1}{4},\quad \frac{6}{25},\quad \frac{7}{29},\quad \frac{13}{54},\quad \frac{33}{137},\quad \frac{46}{191},\quad \frac{171}{710},\quad \frac{217}{901}.$$

These fractions are common to the extent to which we have carried them, but beyond this point we should have different convergents for May and November.

The first ratio to be considered is 46 : 191. It shows that at the end of forty-six years Mercury will have made nearly 191 revolutions, so that the two bodies will

have returned nearly to their original positions. The number of conjunctions will have been 145, which is therefore the number of conjunction points when this system is adopted. In order that each conjunction may occur at one of these 145 points, we must attribute a suitable motion to the whole system. This motion may be best conceived by determining the intervals between consecutive passages of the conjunction points through the node, an interval which is given by the equation

$$I = \frac{T\,T'}{i\,T - i'\,T'}$$

T and T′ being the periodic times of Mercury and the earth, respectively, and i and i' the chosen coefficients; in the present case 191 and 46.

From the preceding values of T and T′ we have

		d.
191 T for November	- - - - - -	= 16802.1179
May	- - - - - - - - -	= 16802.0877
46 T′ for November	- - - - - -	= 16801.6963
May	- - - - - - - - -	= 16801.6908

We thus find:

For November transits - - - - I = 208.6 years.
For May transits - - - - - - - I = 221.6 years.

The value of I for November is gradually increasing, and that for May gradually diminishing, in consequence of the secular recession of the perihelion from the node.

The last passage of a November conjunction point through the ascending node occurred about the year 1776. The adjacent point will therefore pass the node about the year 1985. At the present time, 1882, the node is about half way between these points. The limits within which a November transit may occur are distant a little more than four intervals between conjunction points. Hence, at the present time, four transits occur during each 46-year period.

The last passage of a May conjunction point through the descending node occurred about 1725; the next will therefore occur about 1946. Only two May transits can occur during the 46-year period.

The conditions under which transits will recur for several centuries may be conceived by the following scheme. The horizontal lines are those along which Mercury may be supposed to pass as it crosses the several conjunction points. The planet must be supposed to pass along each of these lines in November of every 46th year, in an indefinite series. The dates of several passages are given at the right of the line, and the series may be continued at pleasure in either direction.

Thirty-three years after passing each line it passes along the next line below. Thirteen years after passing each line it passes along the line next above. Thus all the passages along these six lines may be indefinitely continued, and additional lines may be added above and below.

The sun must be supposed to move downward across the lines at such a rate that it passes over the space between two lines in 208.6 years. The line on the left represents the sun's vertical diameter, the position being that which it occupies in 1800. Its length is about $4\frac{1}{10}$ spaces between the horizontal lines. Its downward motion is

such that the north end passed the first conjunction point about 1794, and its south end crossed the fifth conjunction point about 1764. The passages of each end over the conjunction lines are given on the diagrams, the intervals being 208.6 years, with a minute increase in future centuries. These motions being supposed, we have the following rule for predicting transits. *Every time that the planet in its passage along a conjunction line strikes the sun's diameter there will be a transit across the disk.*

If it passes near the end of the diameter without striking it there will be a near approach to the sun. If the passage across the transit occurs near the center of the diameter, the transit will be a nearly central one. Thus one can, in a few minutes, map out all the transits and all the near approaches to the sun which are to occur for several thousand years, with a close approach to precision. It will be noticed that there is for each conjunction point a period of 864 years, during which the sun is in such a position that the planet will strike it at each passage. By continuing the series the limiting dates can thus be computed for each point. The dates of passage of Mercury along each line are found by adding to any one line the quantity $33 + 46\,i$ years to form the dates for the lines next below. Here i may be any integer. We thus have belonging to each line an indefinite system of numbers, congruous with respect to the modulus 46. Such of these numbers as fall within the interval of 865 years between the two dates above the line will correspond to transits of the planet. The first date of the series will be very near the south limb. A date corresponding exactly to that given on the line will indicate a case in which the planet grazes the sun's limb; dates outside of the interval will indicate approaches more or less near the limb. The successive transits will then occur nearer and nearer the sun's center for a period of four centuries, when the line will pass the center and the following ones of the series will occur near the north limb.

Scheme for November transits.

☉'s vertical diameter.	Transit of South Limb of ☉	Transit of North Limb of ☉	
N.	930	1794	1644, 1690, 1736, 1782, 1828, etc.
	1138	2003	1677, 1723, 1769, 1815, 1861, etc.
Node in 1800	1347	2211	1710, 1756, 1802, 1848, 1894, etc.
	1555	2420	1743, 1789, 1835, 1881, 1927, etc.
S.	1764	2628	1776, 1822, 1868, 1914, 1960, etc.
	1973	2837	1993, 2039, 2085, 2131, 2177, etc.
	2181	3046	2164, 2210, 2256, 2302, 2348, etc.

The corresponding scheme for May transits is given below. The two points in which the schemes differ is that the thirty-three years' interval is measured in the opposite direction from that of the November transits. The motion of the node being also reversed, the diagram itself is inverted, so that the motion shall be downward. The north end of the sun's diameter is the lower one. Moreover the length of the diameter line instead of being equal to four spaces between the horizontal lines, is equal to a little less than two.

The successive transits are now determined in the same way as the November ones, but owing to the diminished relative length of the sun's diameter there will be fewer transits along each line. Moreover the first transit of each series will occur near the sun's north limb.

Scheme for May transits.

Transit of North Limb of ☉	Transit of South Limb of ☉	
1285	1710	1628, 1674, 1720, etc.
1506	1931	1707, 1753, 1799, 1845, 1891, etc.
1728	2153	1740, 1786, 1832, 1878, 1924, etc.
1950	2374	1957, 2003, 2049, 2095, 2141, etc.
2171	2595	2174, 2220, 2266, 2312, 2358, etc.
2393	2817	2391, 2437, 2483, 2529, 2575, etc.
2614	3038	2608, 2654, 2700, 2746, 2792, etc.

Node in 1800

S. ☉'s vert. diam. N.

The next higher system of conjunction points which it is advantageous to consider is that corresponding to the ratio 217:901. This ratio is obtained by supposing the last denominator of each continued fraction to be 4. It expresses so nearly the relative motion of the earth and Mercury from their common node, that it is a little too great for the one node and a little too small for the other. The corresponding number of conjunction points is 684. We may therefore say that, as a rule, 217 years after each transit there will be another transit over the same part of the solar disk.

Two plates are appended hereto showing the apparent paths of Mercury over the disk of the sun during all the transits from 1667 to 1881 inclusive, which constitute one series of 217 years. At the end of this period the transits are repeated. To find the slight deviation of the new series from the old one, we note that the last denominator $\left(4-\frac{2}{11}\right)$ in the continued fraction expressing the ratio for the November tran-

sits, shows that at the end of the period the remaining transit will fall about $\frac{1}{5}$ of the 46 years' interval below the transit 217 years preceding. These recurring transits are shown by two dotted lines near the points of egress and ingress with the corresponding years.

In the case of May transits the fraction is about $\frac{1}{10}$, so that the coincidence will be relatively closer. The diagrams give all the transits within the interval, whether observed or not. Those which have not been observed are indicated by dotted lines. In cases where only one of the phases, egress or ingress, has been observed, one-half of the line is dotted and the other half is left entire.

In the case of the past and future series of May transits, namely, those before 1707 and those after 1881, it may be remarked that the dates are on the wrong side of the lines. For instance, in 1924 the path will be a little north of what it was in 1707. Neglecting inequalities, the change should be one-tenth the space between two consecutive paths.

The times given on each path are those of the middle of the transit. In the case of observed transits these times are the actual means, to the nearest minute, between internal contact at ingress and at egress. They are, therefore, affected by small inequalities arising from periodic perturbations by Venus and the other planets. In the case of November transits these perturbations rarely amount to a minute, so that they do not materially affect the progression of the given times. But in the case of a May transit the effect may amount to several minutes. In the case of transits which have not been observed, no computation of the times has been made, but the times as given are derived by induction from the transits preceding and following.

The times of beginning and ending may be obtained by subtracting and adding the semi-duration from or to the middle times given on the diagram. A scale at the bottom of each diagram will enable us to determine the duration of any transit within one or two minutes. To do this we take in a pair of dividers the length of the chord described by the planet on the diagram, and find the corresponding time on the scale. This time will be the duration from internal contact at ingress to internal contact at egress. It may be expected that the times of egress and ingress thus found will not, for several centuries, be more than three or four minutes in error for the November transits, nor more than five or six minutes for the May transits. Of course the errors may be greater when the chord is very short.

In the case of transits outside the period 1677–1881 the diagrams give only the years. But the times, within a few minutes, may be found by adding to each time during the given period:

$79260^{d}\ 6^{h}\ 24^{m}$ for November transits.
$79260^{d}\ 2^{h}\ 10^{m}$ for May transits.

We thus find the following approximate Greenwich mean times of middle of transit for the period beginning with 1891:

			h.	m.	
1891.	May	9,	14	20.	
1894.	Nov.	10,	6	36.	
1907.	Nov.	14,	0	7.	
1914.	Nov.	7,	0	5.	
1924.	May	7,	13	34.	
1927.	Nov.	9,	17	45.	
1937.	May	~~10~~ 11,	21	22.	(A near approach.)
1940.	Nov.	11,	11	22.	
1953.	Nov.	14,	4	53.	
1957.	May	~~5~~ 6,	13	12.	+12 hrs
1960.	Nov.	7,	4	54.	
1970.	May	~~8~~ 9,	20	22.	
1973.	Nov.	~~9~~ 10,	22	34.	
1986.	Nov.	~~12~~ 13,	16	9.	
1993.	Nov.	5,	15	59.	
1999.	Nov.	15,	9	40.	(Mercury grazes sun's limb.)
2003.	May	6,	19	51.	
2006.	Nov.	8,	9	43.	
2016.	May	9,	3	0.	
2019.	Nov.	11,	3	23.	
2032.	Nov.	12,	20	56.	
2039.	Nov.	6,	20	49.	
2049.	May	7,	2	35.	
2052.	Nov.	8,	14	32	
2062.	May	10,	9	46.	
2065.	Nov.	11,	8	11.	
2078.	Nov.	14,	1	44.	
2085.	Nov.	7,	1	39.	
2095.	May	8,	9	10.	
2098.	Nov.	9,	19	21.	
2108.	May	11,	16	30.	

Remarkable transits.—By the aid of the diagrams we are enabled on sight to select transits which are remarkable from any circumstance and to determine those which are visible in any longitude.

During the last two centuries the transits in which Mercury passed at the shortest distance within the sun's limb are those of 1776, November 2, and 1782, November 12. The two transits correspond closely in their general features, but they occurred near opposite limbs of the sun. That of 1782 was fully observed both in Europe and America. The other does not however seem to have been observed at all, although the ingress at least was visible throughout the United States, and the whole transit in the Middle and Southern States.

Following up the series of transits, which commenced with that of 1776, at intervals of forty-six years, we find the duration longer and longer. The next transit of the series will occur 1914, November 7, when the path of Mercury will be nearly the same as it was on 1697, November 2.

Among the unobserved transits it is most surprising that that of 1835 appears to have passed unnoticed, although the ingress was visible all over the United States, it having occurred about Washington noon.

The next occasion on which a November transit as near the sun's limb as those of 1776 and 1782 will occur is 1999, when it is probable that Mercury will barely enter upon the sun's northern limb.

In the case of the May transits there will be no remarkably short transit for a number of centuries. That of 1957 will probably be the shortest during the next 300 years. But on the morning of 1937, May 11, Mercury will pass so close to the sun at inferior conjunction that it may almost be seen projected on the chromosphere. The nearest approach to the sun's limb cannot be given without a more careful computation from the tables. It is however certain that it will be only a little more than a minute of arc. The path laid down on the diagram is obtained by simple measurement, and is therefore somewhat uncertain.

Until the question of possible changes in the earth's axial rotation shall be placed on a firm basis, or until the theory of the moon's mean motion shall be so perfected that these changes can be determined with precision from observations of that satellite, transits of Mercury must be regarded with the greatest interest as affording independent determinations of the variations in question. The November transits will long be most favorable for this purpose, for the reason that the series of observed November transits extends back nearly a century before the first well-observed May transit. It is to them therefore that we must principally look for light upon this question. The next November transit will be that of 1894. It will be very favorable for this purpose because it is not far from central. Ingress will be visible over the American continent, and egress at points west of the Alleghenies. The transits of 1907 and 1914 will be less favorable on account of being nearer the limb of the sun. That of 1927 will however be again favorable, and, in may be hoped, will decide the question at issue.

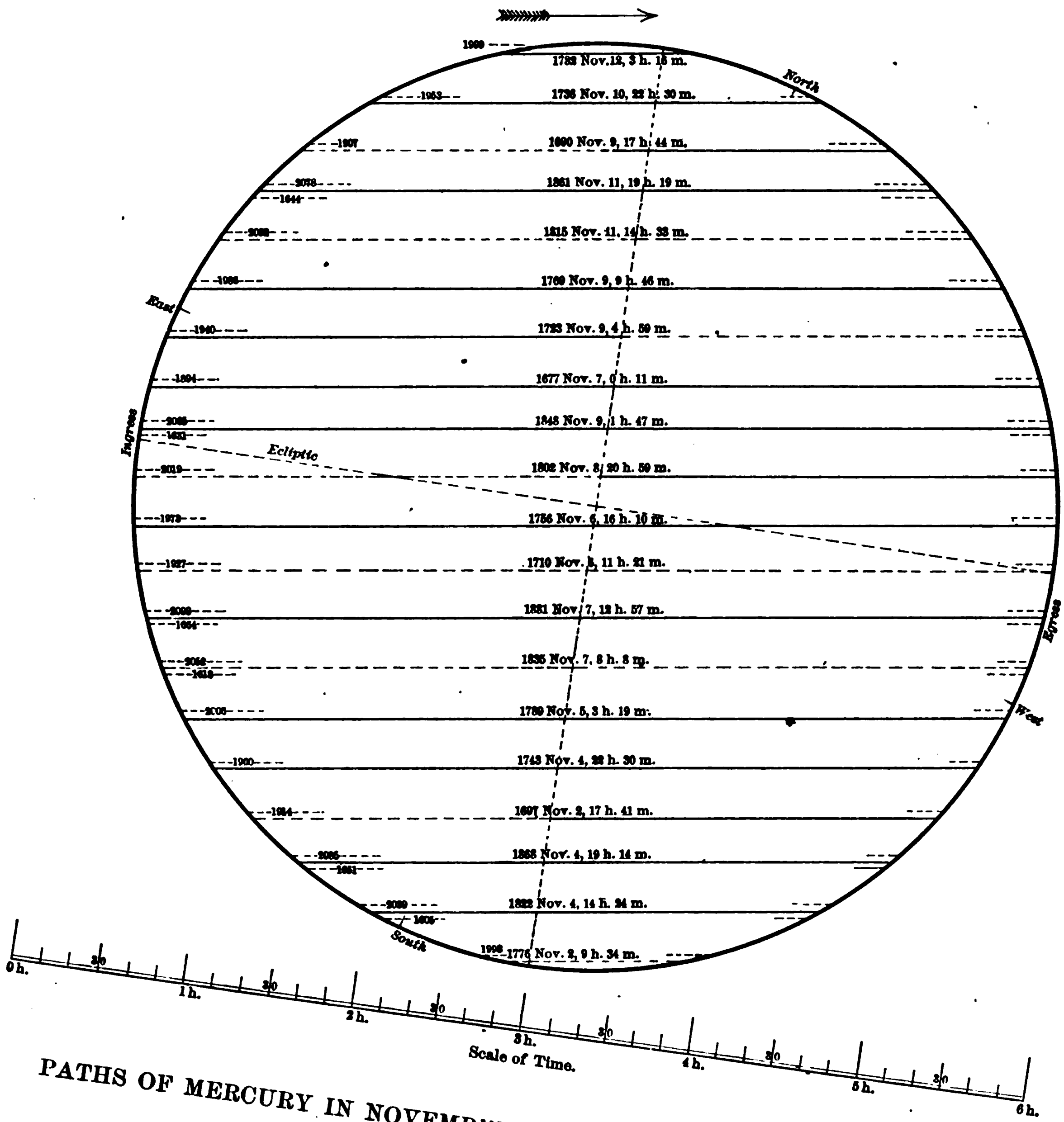
CORRIGENDA.

Page 384, Reduction to geocentric phase for contacts I and II, for $+29^s.2$ read $-25^s.1$, and carry the correction forward.

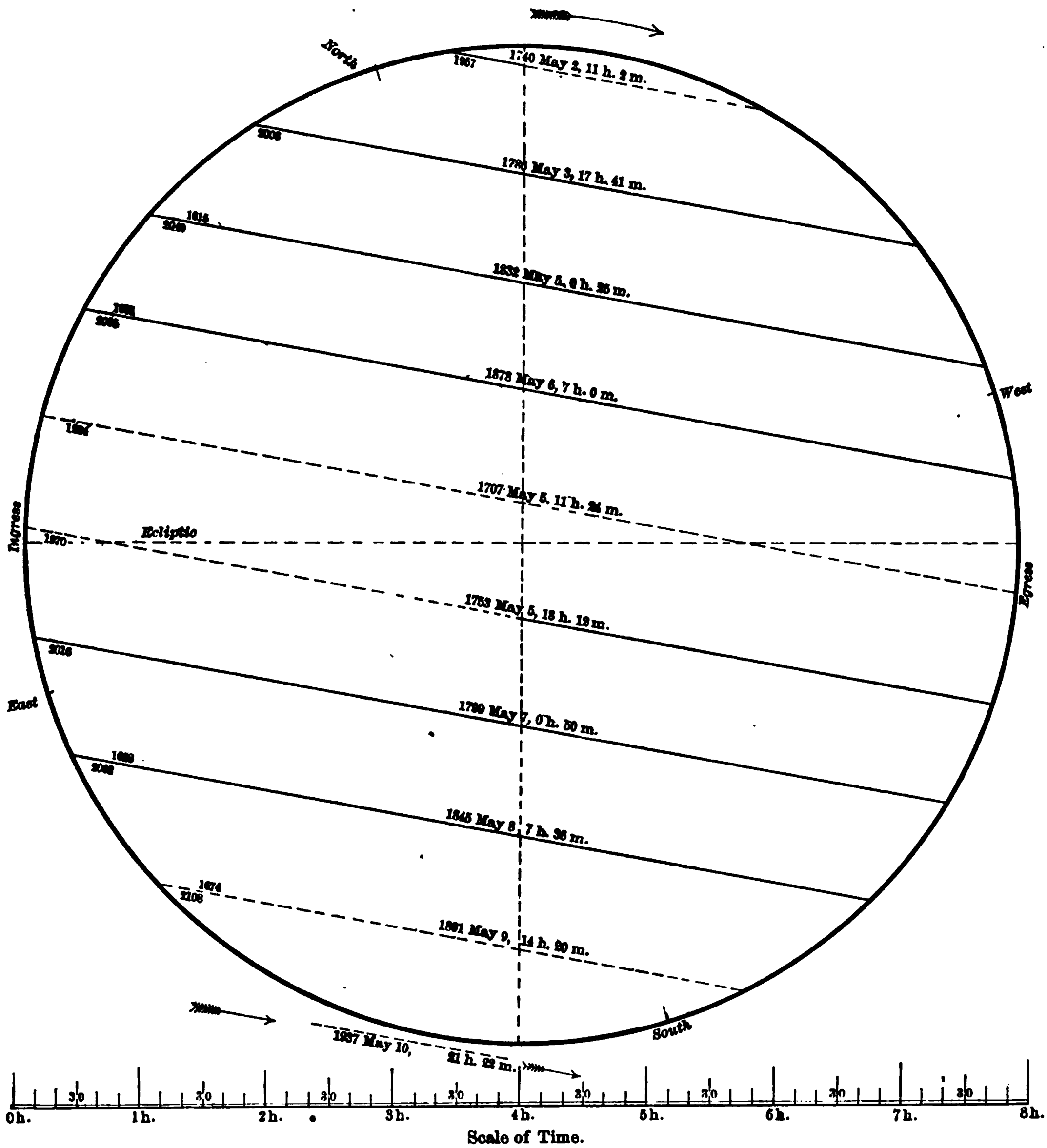
Page 405, Reduction for Altona, for $+58^s$ read $+54^s$, and carry the correction forward.

Page 406, Contacts II, for $21^h\ 3^m\ 32^s$ read $21^h\ 3^m\ 30^s$.

It is also to be remarked that the list of longitudes beginning on p. 374 does not, in all cases, give the longitude of the station actually adopted, it having been sent to press in an imperfect state.



PATHS OF MERCURY IN NOVEMBER TRANSITS OVER THE SUN, 1600–2100.



PATHS OF MERCURY IN MAY TRANSITS OVER THE SUN, 1600–2100.